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THE EFFECTS OF NUCLEAR BINDING ENERGY ON PION YIELDS IN RELATIVISTIC NUCLEAR COLLISIONS

by

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An alternative way for including the nuclear binding energy in intranuclear cascade calculations of relativistic heavy--ion collisions is presented. The present model strictly conserves the total energy and momentum of the system. Pions are produced only via the formation of Δ-resonances and the pion absorption is taken into account through the delta recombination reaction. This simple treatment leads to a correct description of the pion multiplicity in the Ar+KCl reaction case. The proton and pion energy spectra calculated within this model agree with the data.

Key-words: Binding energy; Pion multiplicity; Relativistic intranuclear cascade.

Pion yields and spectra furnish important information for the study of the mechanism of relativistic heavy-ion collisions, i.e. bombarding energies greater than 400 MeV per nucleon. The success of the Intranuclear Cascade (INC) description for the inclusive proton spectra led several authors[1-3] to extend such calculations to pion production phenomena. In these approaches, pions are assumed to be produced in inelastic nucleon--nucleon collisions via the formation of delta resonances. However, it is found that standard INC* calculations tend to overestimate the pion yields[1] even when the processes of pion absorption (π+N $\rightarrow$ Δ) and delta recombination (Δ+N $\rightarrow$ N+N) are introduced[2,3]. This apparent discrepancy between calculated and observed pion multiplicities seems to suggest the existence of many-body effects in the pion production process. Thus several mechanisms have been proposed[4,5].

At present, the most sophisticated microscopic model which is able to describe the dynamical effects of the nuclear mean field at these energies is the Vlasov-Uehling-Uhlenbeck (VUU) model. By using this model, Kruse et al.[6] have shown that the calculated pion multiplicity drops to near experimental results, although it is still sistematically overestimated. One of the main conclusions of their work is that it is at least premature to attribute the original discrepancy between standard INC calculations and experimental results of pion multiplicity to the bulk compression effect of nuclear matter proposed by Stock et al.[4].

The most basic many-body effect should be that derived

*The expression "standard INC" refers hereafter to those INC codes which neglect the binding energy of nucleons in nuclei.

from the nuclear binding energy. The first attempt to elucidate the importance of the nuclear mean field on pion yields in INC calculations was presented by Cahay et al.[5]. The dynamics of the field is included schematically in their work, only to indicate the magnitude of the effect. According to Cahay et al. the maximum effect is of the order of the original discrepancy between predictions by standard INC calculations and experimental data.

The inclusion of binding energy effects in INC calculations of pion multiplicities has been also undertaken by Kitazoe et al.[7,8]. Their INC calculation brings pion multiplicity to near experimental data, even with instantaneous Δ-decay and no pion absorption (this situation gives too high pion multiplicities with standard INC codes[1]). The small difference between their results and experiment has been completely eliminated in a recent work by Kitazoe et al.[9], where they also consider: a) the mass width of Δ-resonance and the corresponding lifetime; b) direct pion production (20% of the inelastic nucleon-nucleon cross section); c) pion absorption and delta recombination.

The works by Kitazoe et al. have apparently solved the problem. However, if one looks carefully at INC calculations of pion yields of the last seven years one finds a somewhat controversial scenery. Cugnon et al.[1-3,5] consider the Δ-resonance lifetime and pion absorption as dominant ingredients to discuss pion multiplicity in the INC context. On the other hand, these same ingredients appear in the work by Kitazoe et al.[9] only to improve their previous results with the inclusion of binding energy effects[7,8].

The inclusion of the nuclear mean field in the relativistic INC scheme is a delicate task by itself. It is very hard to consistently preserve relativistic invariance in such a calculational method. Relativistic non-covariance appears in two different aspects: in coordinate space and in momentum space. The first one is the question of the ordering of the events during the course of INC process. There is no way to put spacially separated events in chronological order in a covariant manner. It was shown by Kodama et al.[10] that in even a simple version of INC calculation the finiteness of nucleon-nucleon cross section causes non-invariant effects. Such effects become more intensified when the spacial extension among events gets larger. Thus, with the introduction of a nuclear mean field, which correlates collision events within nuclear size, we expect a fairly large non-invariant effect in time ordering of events in an INC calculation. However, it has also been shown that these non-invariance effects are washed out by the stochastic nature of INC calculation, causing no practical problems except in some very special cases[10]. The momentum space aspects of non-invariance is more delicate, since it affects directly total energy and momentum conservation. The origin of this non--invariance arises from the well known fact that the global nuclear mean field cannot be covariantly treated unless the proper dynamics of the field is explicitly taken into account. For example, the extension of the non-relativistic recoil effect due to particle emission from the nucleus, as used in previous INC calculations[7,8], spoils total energy and momentum conservations. INC calculation at $E_{lab} < 2$ GeV/A deals with a relatively high frequency of NN collisions near the threshold of delta production. Therefore, we should be very careful in treating the effect of nuclear binding energy in relativistic INC calculations of pion multiplicities.

Here we propose a covariant treatment of this recoil effect and we discuss its kinematical influence on the number of pions produced in relativistic heavy-ion collisions. Our modified INC scheme strictly conserves total energy and momentum in a covariant manner, and has some flexibility to incorporate several possible mechanisms for nuclear excitation.

INC calculations which take into account the nuclear binding energy during its time evolution in general consider the following two additional processes to the standard treatment: a) refraction or reflection of a nucleon whenever it hits the nuclear surface, and b) the redistribution of the recoil energy and momentum among nucleons in the residual nucleus. Let us first examine refraction in the rest frame of a nucleus with A nucleons. Suppose the j-th nucleon has gained a momentum high enough to scape from the nucleus. When this nucleon passes through the nuclear surface, it suffers a change of momentum $\Delta\mathbb{P}$ due to the nuclear potential (normally considered as a rectangular well of depth V_0). The magnitude of $\Delta\mathbb{P}$ is determined by the following energy equation:

$$(m^2 + \mathbb{P}_j^{\,2})^{1/2} - V_0 = [m^2 + (\mathbb{P}_j + \Delta\mathbb{P})^2]^{1/2} \qquad (1)$$

provided that the direction of $\Delta\mathbb{P}$ is specified; m is the nucleon rest mass and $\mathbb{P}_j$ the momentum of the j-th nucleon. From now on all expressions are given in the rest frame of the nucleus. Usually $\Delta\mathbb{P}$ is taken to be normal to the nuclear surface at the impact point. In order to conserve the total momentum of the system, the remaining nucleons suffer the recoil effect

$$\mathbb{P}_i \rightarrow \mathbb{P}_i' = \mathbb{P}_i + \delta\mathbb{P}_i \tag{2}$$

where

$$\sum_{i\neq j} \delta\mathbb{P}_i = -\Delta\mathbb{P} \tag{3}$$

and

$$\sum_{i\neq j} [(m^2 + \mathbb{P}_i^2)^{1/2} - V_0] = \sum_{i\neq j} [(m^2 + \mathbb{P}_i'^2)^{1/2} - V_0] \tag{4}$$

Eqs. (3) and (4) do not determine $\delta\mathbb{P}_i$ uniquely for $A > 2$, therefore one has to introduce some additional assumptions for the redistribution of recoil momentum among the residual nucleons. However, even for a very simplifying assumption, to determine the $\delta\mathbb{P}_i$'s which satisfy eqs. (3) and (4) is a difficult task. The generally adopted prescription,

$$\delta\mathbb{P}_i = -\frac{1}{(A-1)} \Delta\mathbb{P} \tag{5}$$

obviously does not satisfy the energy conservation, eq. (4). Furthermore, if we define the energy of a bound nucleon as

$$E_i = (m^2 + \mathbb{P}_i^2)^{1/2} \quad , \tag{6}$$

then the total energy of the nucleus should be expressed as

$$E = \sum_{i=1}^{A} E_i - AV_0 \tag{7}$$

Therefore, the total momentum of the nucleus cannot be defined as

$$\mathbb{P} = \sum_{i=1}^{A} \mathbb{P}_i \quad . \tag{8}$$

Eqs. (7) and (8) do not constitute a covariant expression of the total four-momentum of the nucleus, unless the momentum carried by the nuclear mean field is explicitly included. Therefore, it is necessary to reformulate the treatment of the nuclear mean--field effect in a covariant manner in order to guarantee strict energy-and-momentum conservations. This could be correctly done only by introducing the dynamics of the nuclear mean field explicitly. However, this is far from being a viable proposal. An alternative may be found by introducing a local mean field approximation. We consider a nucleon and its surroundings as an effective particle having an effective mass μ_i. Let us assume that the four-momentum of a nucleon under the influence of the nuclear mean field has the form

$$p_i = [(\mathbb{P}_i^2 + \mu_i^2)^{1/2}, \mathbb{P}_i] \quad . \tag{9}$$

The total energy and momentum of a nucleus containing A nucleons can be defined by the four-momentum

$$[E, \mathbb{P}] = \sum_{i=1}^{A} [(\mathbb{P}_i^2 + \mu_i^2)^{1/2}, \mathbb{P}_j] \quad . \tag{10}$$

In the ground state of the nucleus all the particles have non--relativistic momenta. We then have

$$E \cong \sum_{i=1}^{A} (\mu_i + \frac{\mathbb{P}_i^2}{2\mu_i}) \cong$$

$$\cong Am + \sum_{i=1}^{A} (\mu_i - m) + \frac{1}{2m} \sum_{i=1}^{A} \mathbb{P}_i^2 \tag{11}$$

where m is the free-nucleon mass. Therefore, the term $\sum_{i=1}^{A} (\mu_i - m)$

represents the total binding energy. In general μ_i-m can be regarded as the potential energy felt by the i-th particle in the nucleus. By introducing the effective mass above, we can require covariantly the rigorous energy-and-momentum conservation in a simple manner. Let us consider again the previous situation of particle emission from a nucleus when it hits the nuclear surface. In this scheme the change of momentum is determined by the equation

$$(\mathbb{P}_j^{\,2} + \mu_j^{\,2})^{1/2} = [(\mathbb{P}_j + \Delta\mathbb{P})^2 + m^2]^{1/2} \qquad (12)$$

Eqs. (12) and (1) coincide in the non-relativistic limit. We may require the "local" conservation of energy in redistributing $\Delta\mathbb{P}$ among the (A-1) remaining nucleons through the change of their effective masses

$$(\mathbb{P}_i^{\,2} + \mu_i^{\,2})^{1/2} = (\mathbb{P}_i'^{\,2} + \mu_i'^{\,2})^{1/2} \quad , \qquad (13)$$

where

$$\sum_{i\neq j} \delta\mathbb{P}_i = \sum_{i\neq j} (\mathbb{P}_i' - \mathbb{P}_i) = -\Delta\mathbb{P} \qquad (14)$$

In this way the total energy and momentum are conserved. Again, eqs. (13) and (14) do not specify μ_i' and $\mathbb{P}_i$. Therefore, we can still introduce some appropriate physical assumptions with respect to the excitation mechanism of the nucleus when it suffers a recoil effect.

Now, in order to incorporate the nuclear mean field effect into the INC calculation we have first constructed a standard INC code. (Pion multiplicities for nearly central collisions of two

^{40}Ca nuclei obtained with this code are in good agreement with the results by Cugnon et al.[2], who have used a similar model.)
We next specify the effective masses of all nucleons at t = 0 as

$$(m^2 + \mathbb{P}_i^{\,2})^{1/2} - V_0 = (\mu_i^{\,2} + \mathbb{P}_i^{\,2})^{1/2} \tag{15}$$

with V_0 = 43 MeV. We then add the following ingredients to the standard code: 1) reflection and refraction when a particle hits the nuclear surface (for the latter, the Coulomb barrier is included for positively charged particles); 2) escape of particles with effective masses greater than their free masses; 3) escape of particles with very high momenta; 4) the Pauli blocking effect — some binary collisions are prohibited in the same sense as in ref. [7].

It should be mentioned here that Δ-particles have been treated at the same level as nucleons, as far as the binding energy effect is concerned. In other words, if a nucleon with a given effective mass appear after the NN collision as a delta, the difference (μ-m) is maintained to the produced delta, the value 1232 MeV being attributed to the free-Δ mass. A similar criterium is applied in the inverse reaction NΔ → NN, and obviously in the elastic scatterings.

Effective masses change when a particle escapes from the nucleus. The outgoing particle has its mass reset to its free-particle value and the remaining particles inside the residual nucleus can suffer a small increase in their effective masses to simulate the nuclear excitation that comes together with the nuclear recoil. We prescribe that only particles moving with momenta such that

$$\mathbb{P}_i \cdot \Delta\mathbb{P} > 0 \tag{16}$$

will change their effective masses. The modulus of the change of momentum of the outgoing particle , $|\Delta\mathbb{P}|$, is determined by eq. (12), and its direction is chosen as being normal to the nuclear surface in the refraction case. For the escape of a particle with very high momentum, $\Delta\mathbb{P}$ is chosen as being parallel to the momentum itself. The recoil momentum, $-\Delta\mathbb{P}$, is absorbed by those particles of the residual nucleus that satisfy eq. (16). Each of them will suffer the change of momentum

$$\delta\mathbb{P}_i = - \frac{\mathbb{P}_i \cdot \Delta\mathbb{P}}{\Sigma\mathbb{P}_i \cdot \Delta\mathbb{P}} \Delta\mathbb{P} \tag{17}$$

where the sum cover only those particles. Now, once the changes of momentum of the particles are specified, eq. (13) determines their new effective masses. These never decrease during the whole nuclear reaction, as it is imposed by eqs. (13), (16) and (17). The time evolution of the average effective mass for bound nucleons in one of the colliding nuclei is shown in Fig. 1 for the reaction $^{40}Ca+^{40}Ca$ at several incident energies. In fact, we have checked the evolution of effective masses for both nuclei individually; they are completely similar, as it should be for a symmetric system. This similarity is also observed in the time evolution of the average number of bound nucleons in each nucleus, which is shown in Fig. 2, and in the time evolution of the average effective mass of bound

Δ-resonances (not shown).

In Fig. 3 we present the main result of our work, the negative pion multiplicity as a function of the incident kinetic energy in the laboratory system, for near-central collisions of two ^{40}Ca nuclei. All results presented in this work were averaged over 240 runs. No systematic difference was observed for the multiplicities of π^-, π^0 and π^+. So, values plotted in Fig. 3 correspond to one third of the average total pion multiplicity. We see that the magnitude of the effect caused by nuclear binding energy on pion multiplicity, as calculated in the present work, is of the order of the original discrepancy between the results of a standard INC code[2] and experimental data, in agreement with the predictions by Cahay et al.[5]. However, there is an apparent discrepancy between our results and those by Kitazoe et al.[7,8] concerning the magnitude of the binding energy effect, when instantaneous Δ-decay ($\tau_\Delta = 0$) is considered. When short-lived deltas are used in a standard INC code too large pion multiplicities are obtained[1]. We have run our code with $\tau_\Delta = 0$, resulting also in too large pion multiplicities, even with the inclusion of the binding effect.

Finally, we would like to show that the introduction of effective masses in the treatment of particle kinematics does not change the INC status in reproducing particle inclusive spectra. In Fig. 4 we present characteristic proton and pion spectra for the reaction $^{40}Ca+^{40}Ca$ at the incident kinetic energy of 0.8 GeV/A. Good agreement is observed between our calculations and experiment in the case of protons. However, the calculated pion spectrum shows a dip at small energies (dashed part of the histogram) which does not appear in the experimental spectrum. This point has already been discussed by Cugnon et al.[3]; it comes from the fact that we have used a fixed mass for the free Δ-resonances.

To conclude this letter, we wish to summarize the main points of the present work. We have used a local mean-field approximation through effective masses as an artifice to incorporate the nuclear binding-energy effect into the INC description of relativistic nuclear collisions. The experimental data on pion multiplicity for the reaction Ar-KCl can be well reproduced under very simplifying assumptions about the mechanism of pion absorption. This latter can be included explicitly in the code, as well as a more realistic treatment of the delta lifetime during these reactions.

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FIGURE CAPTIONS

Fig. 1 - Time evolution of the average effective mass of bound nucleons in any of the two colliding ^{40}Ca nuclei. The numbers near the curves are incident energies in GeV/A. Only nearly central collisions ($b \lesssim 2fm$) were considered.

Fig. 2 - Time evolution of the average number of bound nucleons in any of the two colliding ^{40}Ca nuclei. The energies are the same as in Fig. 1, and again only nearly central collisions are considered.

Fig. 3 - Average pion multiplicity as a function of incident energy for nearly central collisions of two ^{40}Ca nuclei. Both in Cugnon's work[2] and in our results, which include binding energy effects, it is assumed that the deltas survive until the end of the nuclear reaction ($\tau_\Delta > \tau_c$). The uncertainties in our results are less than 3% for $E_{lab} > 0.8$ GeV/A (they are smaller than the symbols). Kitazoe et al.[7,8] also take into account the binding energy effect but assume instantaneous decay of the Δ-resonances ($\tau_\Delta = 0$). The lines were drawn to guide the eye.

Fig. 4 - Proton and pion energy spectra for the system $^{40}Ca+^{40}Ca$ at an incident energy of 800 MeV/A in the laboratory system. Filled circles are the experimental data from ref. [11] and correspond to high multiplicity events (HME) of $^{40}Ar+KCl$. The histograms are the results of the present calculation for nearly central collisions of two ^{40}Ca nuclei (normalized to the data). The uncertainties in our calculation are indicated at three different energies on the histograms.

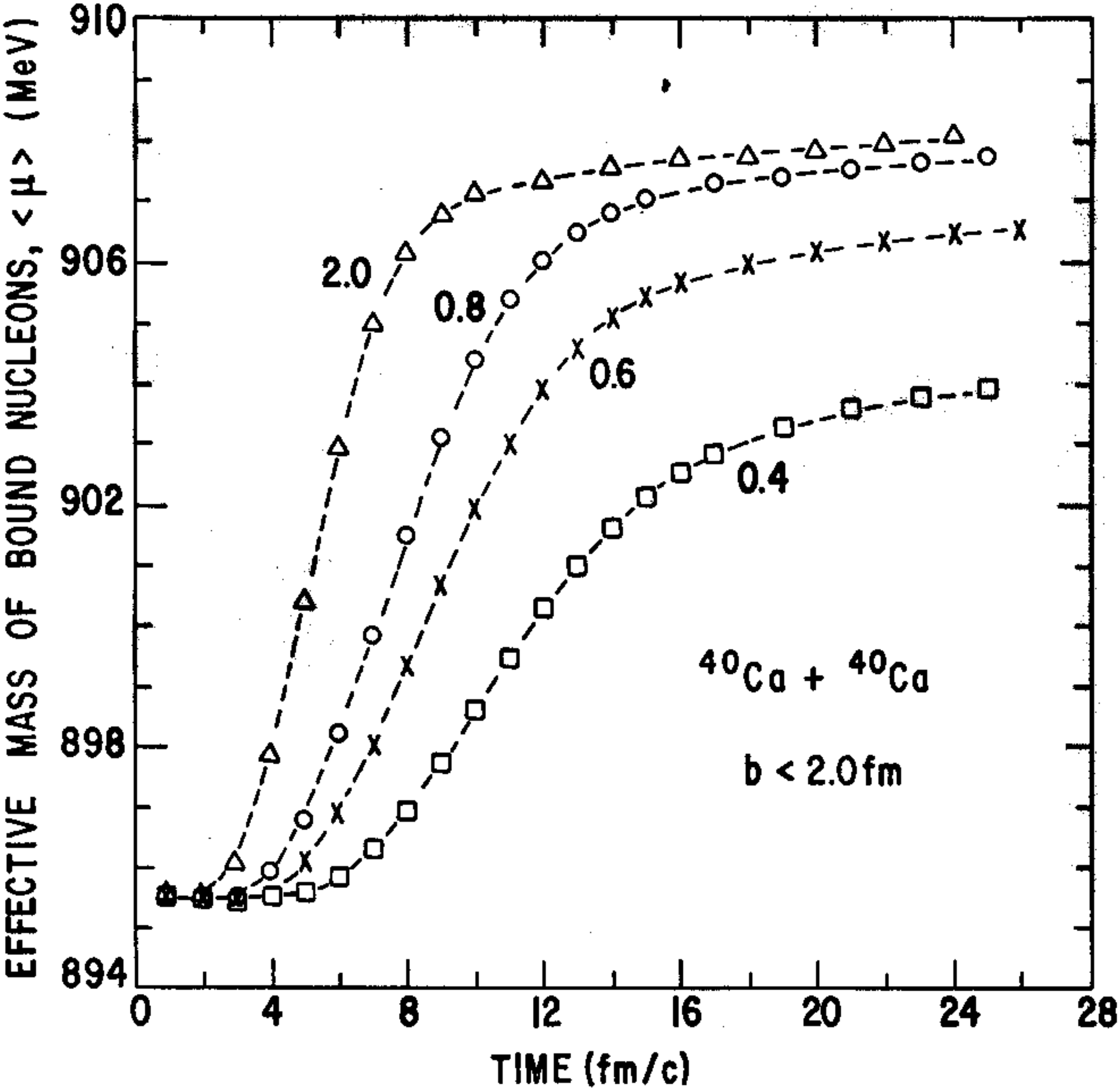
910
906
902
898
894
EFFECTIVE MASS OF BOUND NUCLEONS, <μ> (MeV)
0
4
8
12
16
20
24
28
TIME (fm/c)
2.0
0.8
0.6
0.4
40Ca + 40Ca
b < 2.0fm

Fig. 1

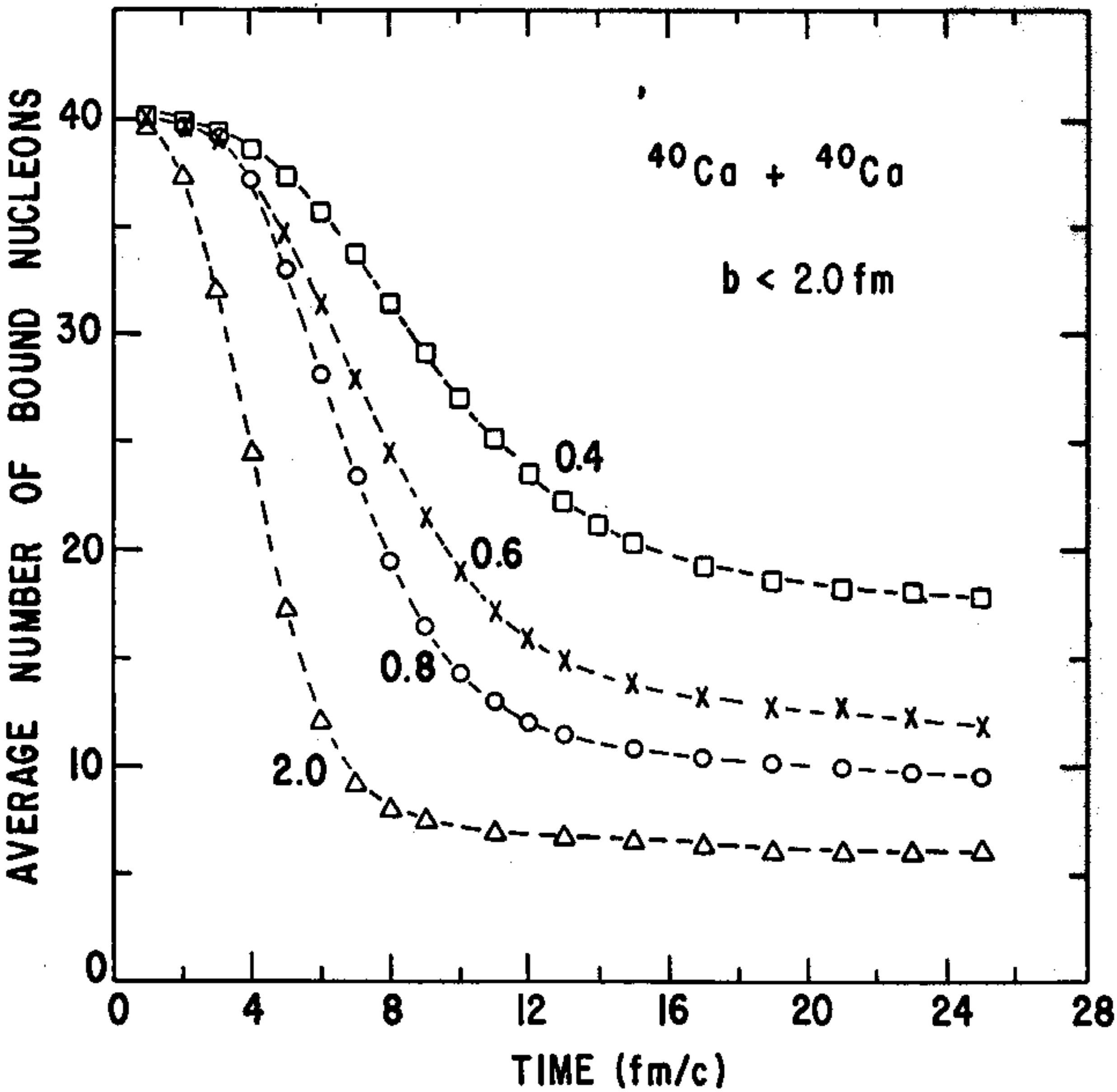
AVERAGE NUMBER OF BOUND NUCLEONS
TIME (fm/c)
^{40}Ca + ^{40}Ca
b < 2.0 fm
0.4
0.6
0.8
2.0
0
4
8
12
16
20
24
28
10
20
30
40

Fig. 2

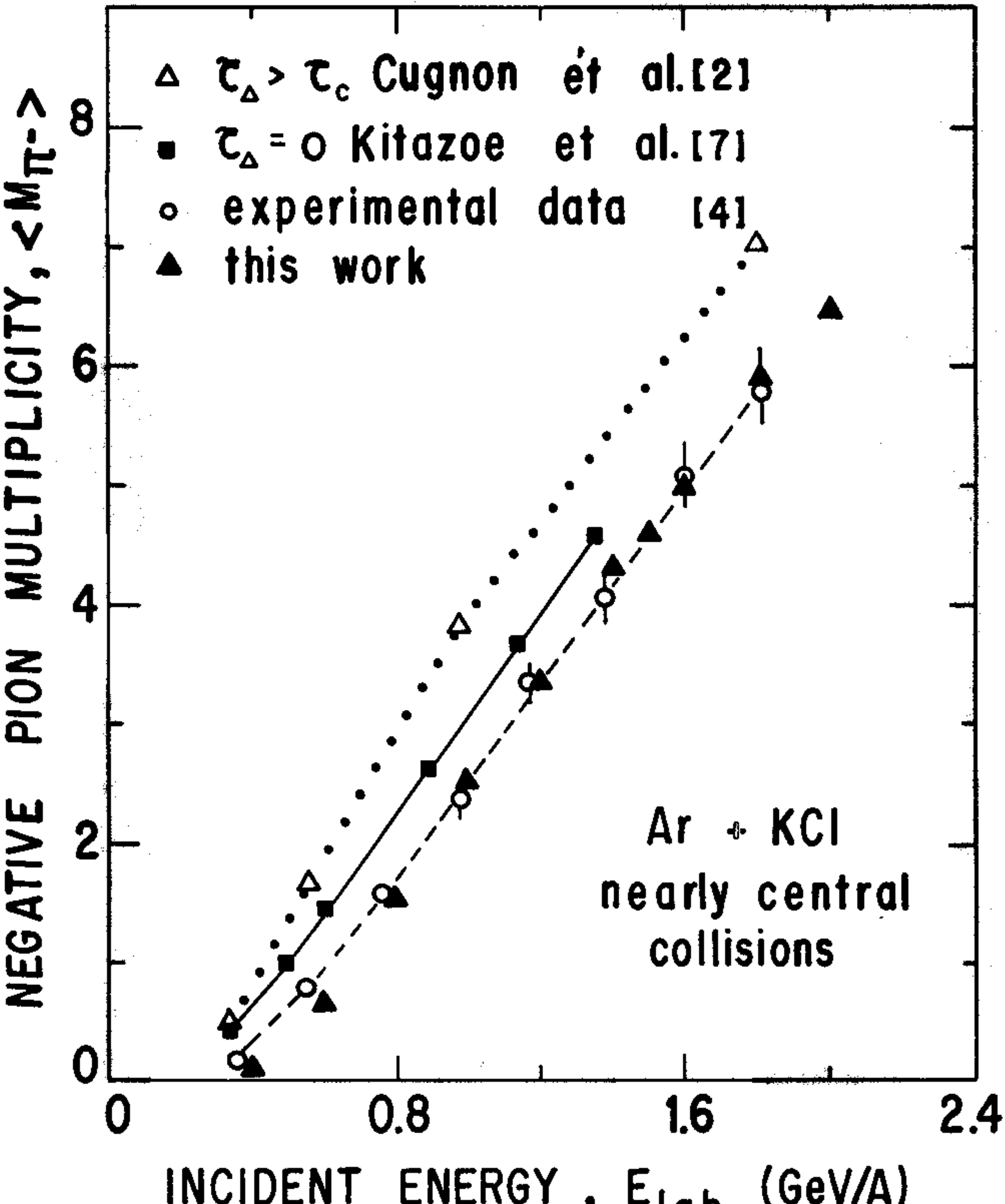
τΔ > τc Cugnon ét al.[2]
τΔ = O Kitazoe et al.[7]
experimental data [4]
this work
NEGATIVE PION MULTIPLICITY, <Mπ->
INCIDENT ENERGY, Elab (GeV/A)
Ar + KCl
nearly central
collisions
0
2
4
6
8
0
0.8
1.6
2.4

Fig. 3

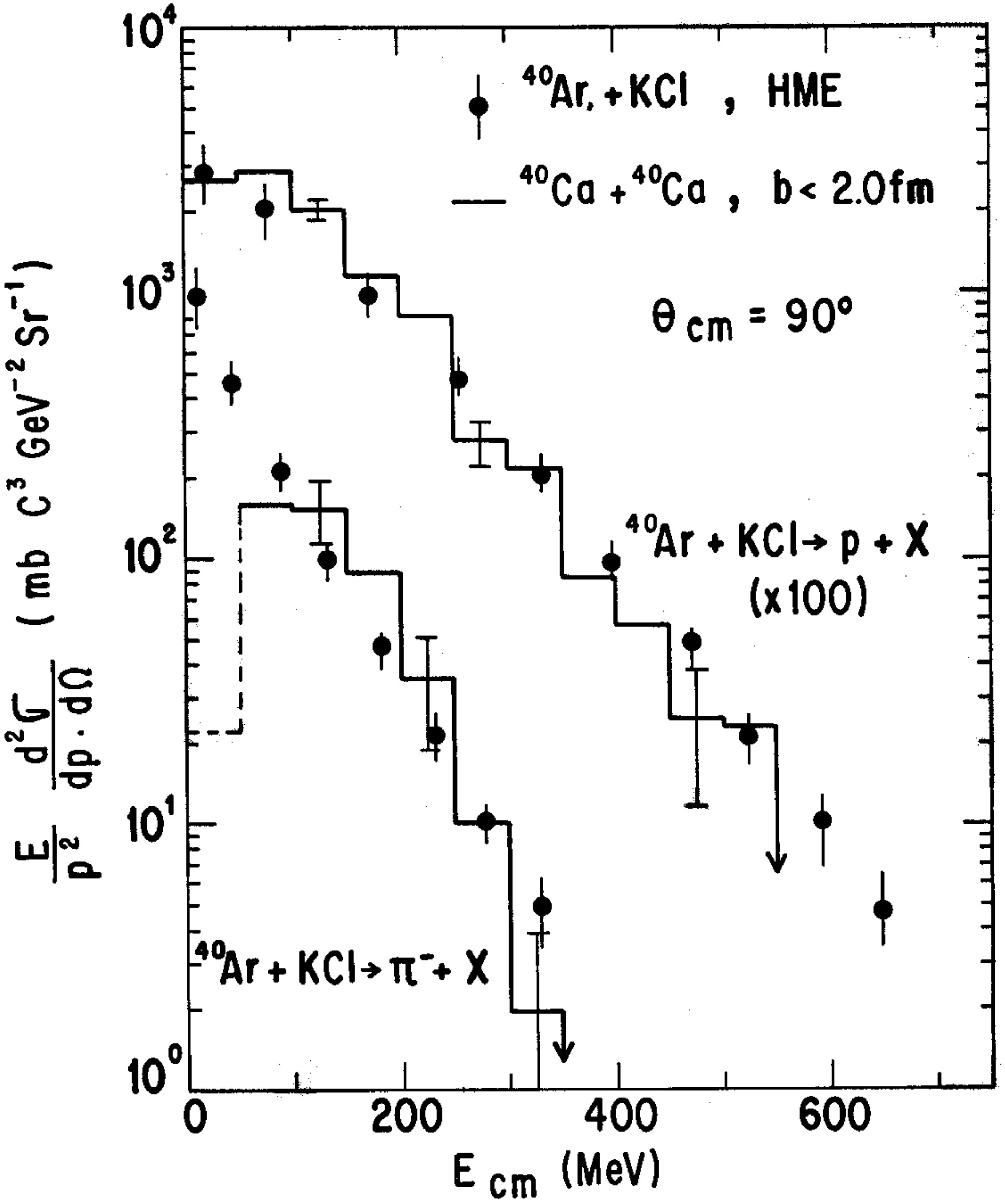
^{40}Ar + KCl , HME
^{40}Ca + ^{40}Ca , b < 2.0fm
θ_{cm} = 90°
^{40}Ar + KCl → p + X
(x100)
^{40}Ar + KCl → π^- + X
$\frac{E}{p^2} \frac{d^2\sigma}{dp \cdot d\Omega}$ (mb c^3 GeV^{-2} Sr^{-1})
E cm (MeV)
10^4
10^3
10^2
10^1
10^0
0
200
400
600

Fig. 4

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