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GEOMETRIZED INSTANTONS AND THE CREATION OF THE UNIVERSE

by

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Abstract:

A non-singular cosmological scenario is presented in which the currently observed Riemannian configuration of space-time originates from a primordial Weylian "stiff matter" phase driven by a geometrized inflaton-like field. In the particular case of a Friedman-Robertson-Walker line element, it is shown that an unstable original Minkowski vacuum starts to collapse adiabatically at a remote past and bounces when a minimum radius is attained. Throughout the collapsing stage this matter-free Universe is accelerated (or "inflationary"). As in models of spontaneous quantum creation, the bouncing phase can be shown to correspond to the propagation of a Weyl instanton in an Euclideanized, classically forbidden region. In the course of the bouncing period, concurrent to the maximal deviation of the Riemannian configuration, a geometry-driven amplification mechanism operates causing the exponential increase of entropy and matter fluctuations. Once the environment temperature is always bounded, this non-adiabatical process stands for a "Big - but finite - Bang" creation event, followed by a standard radiation-dominated era. The model describes an eternal, Friedman-like open Universe, free of many problems that hinder standard cosmology; the observed existence of a baryon excess also fits naturally within the proposed scheme.

Key-words: Non-standard cosmology; Weyl spaces: Inflationary scenarios; Models of quantum creation.

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I. Introduction

I.1. Introductory remarks

In current standard cosmology the Universe is described as an unique non-stationary system characterized by a Friedman-Robertson-Walker (FRW) homogeneous geometrical configuration, and its history may be traced backwards to a remote stage in which the temperature of the cosmic material medium attained very high (i. e., > 10^{12} GeV) levels. Pushing this extrapolation still further, eventually a peculiar domain would be reached where the temperature of the cosmic medium (and other relevant physical quantities) became infinite. The origin of the Universe is therefore ascribed to the occurrence of a global singular state (the Hot Big Bang initial singularity) and both space-time and matter-energy would have burst into existence at a finite time in the past[1].

In spite of the well-succeeded predictions of the standard Hot Big Bang (HBB) cosmological paradigm, it is widely acquainted today that HBB models also entail some very controversial aspects, known as "standard problems" (see Section II). In consequence, in the course of the last two decades a number of different argumentations has been suggested in search of their resolution. With respect to the aims of the present article, two particularly representative approaches are of main interest: models of spontaneous quantum creation of the Universe and inflationary scenarios.

In distinction of the standard program, in which the provenance of the Cosmos is attributed to an initial singularity of unaccessible nature, different authors have pursued the question of the origin of the Universe through the consideration of creation mechanisms that could derive, at least in part, from the known laws of Physics. This position led, for example, to the idea that the standard picture of a classical (i. e., non-quantum) space-time structure originating from a FRW singularity should be replaced, due to quantum effects that would dominate the cosmic behavior at primeval epochs, when the Universe approached Planckian dimensions ($L_P \sim 10^{-33}$cm). Thus the classical structure of space-time would have been engendered by a primordial process of quantum nature, instead of the standard singularity, and conceivably even the initial conditions for the subsequent cosmic evolution might be derived by means of a quantum analysis.[2]

In many contemporary models of quantum creation, to quote, a primordial quantum "Nothing" state is identified to the physical vacuum and the birth of the Universe understood as a consequence of spontaneous vacuum fluctuations of a given field. There is no unanimity in the literature about the precise characterization of such "Nothing" state. Most approaches, however, coincide upon a specific aspect: that the Universe emerged into existence in a maximally symmetric geometrical configuration of the De Sitter type[3]. In this case, it is also generally admitted that such De Sitter phase cannot be analytically continued (in a classical sense) to a previous stage. Typically, the Universe would initiate its classical evolution endowed with a non-null Friedman scale factor $a(t_0) = a_0$ and thereafter proceed according to the De Sitter solution of Einstein's equations with a "cosmological constant" term as source. The preceding quantum phase is then associated to a De Sitter instanton propagating in a classically forbidden region[4,5].

Even in the case of singular FRW models, interesting modifications have been suggested in order to mitigate some traditional difficulties exhibited by the standard paradigm (see Section II). In particular, due to the consistent arguments provided by *inflationary scenarios*[6] with respect to the solution of the "horizon" and "flatness" problems, the possible existence of a cosmic phase driven by a broken-symmetric primordial scalar field has become widely accepted. Usually, the properties of such "inflaton" field are connected to interaction processes succeeding on an elementary level in matter. In unified theories of elementary particle physics there appears a kind of inflaton, submitted to a self-interaction potential, that can effectively play the role of an unstable vacuum state dominating the cosmic evolution when the extremely high temperatures required for unification are approached[7]. In inflationary models, furthermore, only events taking place inside an effective horizon can have any relation to the observed Universe; thus the consideration of whether an initial singularity actually occurred or not becomes preposterous.

In the present paper we will examine a cosmological scheme in which, similarly to inflationary scenarios, a fundamental role in the cosmic evolution is ascribed to a certain scalar field $\omega(x)$; moreover, some of the results obtained within this scheme display a close analogy to well-known features provided by quantum creation models. However, there exists a crucial distinction: in our approach, the fundamental scalar field $\omega(x)$ is *geometrized*. In fact, convincing arguments have been presented suggesting that profound modifications of the classical geometrical structure probably took place when the Universe attained an extremely dense primordial stage. In particular, *classical* or *semi-classical* structural changes (such as, for instance, geometrical "phase transitions" of the second kind, i. e., continuous) might be expected to happen near to the frontier of the classical and the quantum regimes. Thus a new questioning is brought into cosmological matters: the dynamical determination of the basic nature of space-time structure or "structural" problem.

Of course, many sorts of structural modifications, even very drastic ones, could in principle be considered to describe classical or semi-classical regimes prevailing when the Universe attained temperatures greater than 10^{12} GeV. In the present article we will restrict our investigations to a particular class of *continuous* geometrical transitions in which the traditional Riemannian structure of space-time is enlarged into a *Weylian* affine manifold. As we shall see in what follows, the geometrization procedure of the scalar field $\omega(x)$ explored here is intimately connected to its assimilation, as a fundamental component of the geometrical structure, into a Weyl configuration. Due to its great simplicity, in fact, a Weyl structure appears quite naturally as a preliminary candidate for extending general-relativistic Riemann configurations, once their only distinction consists in that in Weyl space-times vector lengths may be *not* preserved along transportation. In Section III we will present the arguments that motivated the option for limiting our discussion to a particular class of Weyl geometries, known as *Weyl-integrable space-times* (WISTs).

We would like to stress from the outset that a new theory of gravity that would implicate local changes in the behavior of the gravitational field and, in consequence, cosmological effects also, as in scalar-tensorial theories of the Brans-Dicke type[8], is *not* being suggested. Quite on the contrary, we base our argumentation on the hypothesis that the geometrized WIST field $\omega(x)$ has the character of a purely cosmic function acting in a global sense only (as, for example, the cosmological constant does). Its principal activity is then to govern structural transitions of the early Universe geometry in the sense mentioned above.

At first sight, such type of structural modification would be effective on the classical domain only, and thus no direct connection would seem to exist with quantum fluctuations and quantum creation models. However, as we shall see below (Section IV), the particular type of structural change that appears in the present scenario is associated to the evolution of an instanton of a massless inflaton-like field, tunneling between asymptotic Riemann configurations. This is in close analogy with the mechanisms of quantum creation quoted above (and with axion cosmology also).[9] In other words, we will show that space-time "Weylization", at least in the case of a specific model, is equivalent to a semi-classical description of a quantum tunneling process, as given by the standard bouncing solution (instanton) provided by Euclideanized field equations describing the dynamics of a Friedman scale factor generated by a massless inflaton field, in a Riemannian space-time. We do not claim, of course, this equivalence to be valid for all possible processes of quantum creation; nevertheless, it could suggest a deeper connection, underlying these simple considerations, between quantum mechanisms and Weyl structures. We will return to these matters in the Conclusion.

I.2. A synthesis of the model

The surmise of a Weyl background configuration leads to a non-standard cosmological scenario which, in the case of a homogeneous and isotropic line element, admits a non-singular, eternal Friedman-like solution exhibiting the following main features:

1. the evolution of the Universe begins at the infinitely remote past due to the unstability of a spatially infinite, empty Minkowski space-time;

2. this matter-free open Universe, driven by the energy associated to a geometrized homogeneous Weyl field $\omega(t)$, collapses adiabatically until a minimum radius a_0 is approached;

3. in the course of this everlasting collapse, the Universe is always

accelerated (or "inflationary") and any occasional matter fluctuation is exponentially suppressed;

4. near to the phase of maximum contraction, the cosmic evolution enters in a non-adiabatical regime in which the collapse is reverted to an expansion;

5. this bouncing may be associated to the propagation of a Weyl instanton ("Wiston") in an Euclideanized, classically-forbidden region;

6. as the expanding phase initiates, matter (e. g., photons) and entropy fluctuations are exponentially amplified at the expenses of the energy of the Weyl field;

7. an eventual baryon excess taking place at the start of the expanding era may be amplified as well;

8. this mechanism of matter-entropy production saturates soon, and the cosmic evolution attains a standard, radiation-dominated Friedman configuration.

Given the properties outlined above, this Friedman-like cosmological model describes an eternal, bouncing Universe, created from a Minkowskian "Nothing", in which the singularity, horizon and flatness problems of standard Cosmology do not occur; the model also provides a geometry-driven mechanism able to control the production of large amounts of matter and entropy, due to the amplification of vacuum fluctuations. Once the environment temperature is always bounded, this creation process stands for a "Big - but finite - Bang" event. The observed presence of a baryon excess also fits naturally within the proposed scheme.

To achieve the demonstration of the above statements, the contents of the present paper are arranged as follows. In Section II we discuss the physical and cosmological motivations and present the basic assumptions of the proposed scenario. Section III provides a brief survey on the necessary mathematical machinery of Weyl space theory. In Section IV we derive a Friedman-like non-singular solution and comment upon its properties. In Section V, the construction of a mechanism of matter-entropy production is detailed and other issues concerning thermodynamical processes are discussed. Section VI, finally, contains a short account of the results obtained previously and some concluding remarks. Throughout the article, use is made of "geometric" units such that $k_{einstein} = c = 1$.

II. Motivations

II.1. Primordial Cosmology

In standard HBB models, with conventional matter as source, the Universe has a singular origin. This means that the scale factor a(t) of a spatially homogeneous and isotropic FRW line element

$$ds^2 = dt^2 - a^2(t)\, d\sigma^2 \tag{2.1}$$

vanishes at a finite time t_0 in the past[10]. Due to this distinctive feature, in addition to its well-known observational successes - the incorporation, in a natural way, of the evidence concerning the Hubble expansion, the presence of a cosmic microwave background radiation and the primordial relative abundances of the chemical elements - the HBB program also leads to a bunch of difficult questions. In the literature, the lists of "standard troubles" usually comprise the following items:[10,11]

- the occurrence of causal limitations to the cosmic homogeneity ("horizon" problem);

- the apparently Euclidean nature of space ("flatness" problem);

- the explanation of the prevalence of matter against anti-matter and of the observed ratio of entropy per baryon ("baryon asymmetry" problem);

- the elaboration of an accurate perturbative scheme to allow for galaxy formation.

These issues are seen to be related to the fact that very specific initial conditions are required in order to guarantee a proper cosmic evolution to the later stage we observe today; and, particularly remarkable,

- the "singularity" problem, which concerns the absolutely unscrutable provenance of the physical world from the HBB initial singularity: no causal description of the behavior of the Universe could be expected to include the singular origin in view of the divergent (infinite) values assumed by physical quantities at the creation instant t_0. The application of the notion of thermodynamical equilibrium to the cosmic "fluid" under such extreme conditions also seems doubtful.[12]

These difficult issues motivated, of course, many proposals of solution. With the purpose of eliminating the irksome initial singularity there have appeared suggestions concerning, for example, modifications of the geometrical nature of space-time in connection with spin properties[13]; the adoption of non-minimal couplings of scalar or electromagnetic fields to gravity[14]; the inclusion of higher order curvature terms in the gravitational Lagrangian[15]; the occurrence of viscous effects in the matter-energy content[16]; the introduction of a time-dependent "cosmological constant" term[17], to quote just a few.

In recent years the horizon and "flatness" problems have been attacked by means of various sorts of conjectures based on a rapidly expanding primordial phase ("inflation") of the cosmic evolution, associated to a De Sitter solution[7]. Most inflationary scenarios have in common the fact that the De Sitter phase starts from a non-vanishing value of the cosmic radius. The introduction of such finite radius, however, does not necessarily contradict the occurrence of an initial singularity; in many approaches the inflationary phase is spread between two standard radiation-dominated eras, eventually preceded by a standard singularity. Nevertheless, different authors were motivated to consider non-singular, inflationary models in order to explore the appealing possibility of generating a classical structure - such as the De Sitter space-time - from a typically quantum process such as quantum vacuum tunneling[4,5]. Let us sketch briefly their argumentation.

The De Sitter solution is provided by Einstein's equations for the "vacuum", generically represented by a (positive) cosmological constant $\Lambda = 3\zeta^2$. Accordingly, the scale factor a(t) satisfies the Friedman equation

$$\dot{a}^2 - \zeta^2 a^2 = - \varepsilon \tag{2.2}$$

where $\varepsilon = \pm 1$. Some authors privilege closed worlds ($\varepsilon = 1$) since basic material properties such as total mass and charge can be made null[3,18]. In the closed case, a typical De Sitter solution is obtained as

$$a(t) = 1/\zeta \cosh(\zeta t) \ . \qquad (2.3)$$

Now this solution exhibits an apparent deficiency: the occurrence of a primeval collapsing phase of *infinite* duration, thus implying an everlasting cosmic history. This is an already traditional difficulty of bouncing eternal universe models in general, irrespective of whether one or many bounces are allowed[19]. The problem is to conceive the behavior of matter in the course of, say, a collapse-expansion sequence of unlimited duration. If gravity can somehow produce particles, for instance, an infinite amount of matter - and entropy - must have been produced during the past collapsing phase. Thus such eternal longevity, whereas it could be helpful in resolving some "standard troubles" (the horizon problem, for example)[11,20], is hardly conciliated with finite values of entropy and matter production, unless some type of saturation mechanism has been in action throughout the infinite past evolution (see Sections IV and V).

This difficulty can be easily surmounted through the assumption that the infinitely old collapsing era simply did not exist, due to quantum effects dominating the cosmic behavior near the region of maximum contraction. The overall picture is that the Universe began its spatio-temporal evolution directly in a classical metric configuration of the De Sitter type, endowed with a minimum radius a_0. Once this typical dimension a_0 is taken to be of the order of the Planck length L_P, quantum processes shall be invoked to lay the physical foundations for the emergence of the classical De Sitter stage. An ingenious way to supply a framework in which the quantum generation of a classical structure can be depicted is to resort to a quantum version of the theory embodied in Eq. (2). A semi-classical approximation of this theory[21] is provided by an Euclideanization procedure in which Eq. (2) is interpreted as a dynamical process consisting of a particle with position $q = a$, submitted to a potential $V(q) = - \zeta^2 q^2$. One then obtains that the minimum value q_0 classically allowed is given by $a_0 = 1/\zeta$, and the corresponding classically forbidden region is described by the Euclideanized equation

$$\dot{a}^2 = 1 - \zeta^2 a^2 . \tag{2.4}$$

In this way, a quantum tunneling process - represented by an instanton solution of Eq. (4) - may provide a connection between the quantum and the classical regimes: a De Sitter space-time appears as the "terminal point" of a De Sitter instanton propagating in the classically forbidden region[4]. The interpretation of the physical status of the Euclideanized region is controversial: some authors argue in favor of its physical reality - therefore admitting, implicitly, a non-Lorentzian phase of the cosmic evolution[22] - whereas others support the opinion that such region is purely virtual once it does not define an actual space-time structure, at least in a classical sense.[23] According to this view, the virtual Euclidean stage is to be associated to a structureless quantum vacuum state denominated "Nothing"[3,4]. Thus the Universe was created, through a quantum tunneling process, from a "Nothing" state identified to a instanton solution of the Euclideanized equation Eq. (4).

II.2. Unstabilities of Minkowski space-time

In the quantum creation models outlined above the Universe seemingly manifests a very specific preference to tunnelate into a De Sitter configuration - and not, for instance, into Minkowski space-time, which in the general-relativistic context is understood as an empty (i. e., completely matter-free) Universe, and so, in this sense, a truly "fundamental" state of Einstein's dynamics[24]. Both configurations, moreover, display the maximum number of symmetries admitted by Einstein's theory. A possible explanation is that the De Sitter solution (due to the presence of a cosmological constant term) allows for the occurrence of vacuum fluctuations - which is a indispensable condition for the ulterior appearance of matter - while Minkowski space is classically as well as quantum-mechanically stable against statistical perturbations.[25]

However, the statements about the stability of the Minkowski vacuum quoted above are ultimately model-dependent, since they rely on perturbative schemes related, in different ways, to specific descriptions of matter properties. On the other hand, it could be argued that *unstable* Minkowski spaces constitute very appealing candidates to perform the role of a cosmic proto-structure: since Minkowski spaces bear no distinctive trace, they possess no causal "memory" - whatever the conjectural process leading to a Minkowski vacuum, its effects would be utterly erased of any causal chain established subsequently. Therefore, an unstable Minkowski configuration is indeed "original"; the choice of the precise type of perturbations yielding such unstability requires, of course, further discussion.

The problem may be posed as follows: if the assumption that the actual, observed Universe developed from fluctuations of an unstable empty space-time is accepted, could we devise a sufficiently generic (i. e., independent of matter properties) perturbative scheme so as to provide for a smooth evolution to the present Friedmanian era? This question probably does not have an unique, definitive answer; nonetheless, we may attempt to shed some light upon it through the examination of a particular model.

According to the Special Theory of Relativity, Minkowski space-time constitutes the fundamental descriptive arena in which inertial observers shall compare their measurements of distances and durations in order to supply an absolute meaning to the laws of Physics[26]. Once Minkowski spaces are devoid of any matter-energy content, their characterization depends exclusively on the spatiotemporal determination of physical events by a class of ideal observers through the *gedanken* exchange of light signals. Given the assumption that, in the sake of generality, fluctuations of the matter-energy content shall be discarded, the only remaining physical system available to be perturbed consists of the basic framework of the measurement procedure itself, that is, the idealized apparatuses of clocks and rods employed to quantify separations and intervals. *Our working hypothesis therefore addresses the induction of unstabilities of Minkowski space through perturbations of the system of measure units.* More specifically, we will consider "structural" fluctuations of Minkowski geometry in the general form

$$\delta(g_{\mu\nu;\lambda}) = (\delta\omega_\lambda)\ g_{\mu\nu}\ , \tag{2.5}$$

in which $\omega_\lambda = \partial_\lambda\ \omega$, $\omega(x)$ being a scalar field defined on the background manifold, and the semi-colon stands for covariant differentiation. In Section III we will show how fluctuations of this type may be ascribed to variations of measuring scales.

II.3. The "Structural Problem"

In the context of the standard formulation of space-time dynamics put forth by Einstein's General Theory of Relativity the hypothesis underlying Eq. (5) is rendered inconsistent from the outset, once no room is left for variations of measuring apparatuses of the kind outlined above. This may be seen as a consequence of the stringent requirements imposed upon the characteristics of space-time by the rules of General Relativity, according to which the behavior of clocks and measuring rods must be determined exclusively by the metric properties (i.e., the metric tensor) of the underlying manifold. This implies, in turn, that the structure of physical space-time must correspond unequivocally to that of a Riemannian manifold, in which covariant derivatives of the metric tensor vanish (i. e., $g_{\mu\nu;\lambda} = 0$). Indeed, if this condition is fulfilled, the manifold affine connections $\Gamma^\alpha_{\mu\nu}$ become identical to the Christoffel symbols $\{^{\alpha}_{\mu\nu}\}$ of Riemann geometry[26]. The same result may be obtained *a posteriori* by means of the

Palatini variational method (see Section III).

Therefore, the adoption of perturbations of Minkowski space-time in the form of Eq. (5) requires (or, conversely, induces) modifications of the affine nature of space-time - which, according to the rules of General Relativity, should be *(either on a priori or a posteriori grounds)* strictly Riemannian. In the course of the last decades, however, these requirements have been questioned both from the axiomatical and the observational standpoints. Indeed, both types of approaches lead to the conclusion that space-time structure is *not* completely described by a simple Riemannian manifold. Following the attempts of Ehlers, Pirani and Schild[27] to supply, through the consideration of ideal operations of elementary clocks and rods, an axiomatical foundation for the geometrical nature of space-time, one is led to the assertion that the "... Weylian structure of space-time is axiomatically well-founded, whereas its Lorentzian structure is not" [28]. On the other hand, the *observational* determination of the behavior of measuring instruments, besides the metric tensor $g_{\mu\nu}(x)$, must also involve a scalar function $\omega(x)$ in order to guarantee the conformal invariance of null light cones (not to be confused with the principle of conformal invariance of *all* physical laws - see Section III) - which constitute the most important observational aspects of the background geometry; in consequence, a *conformally-Riemannian* structure is involved, rather than a Riemannian one[29].

These considerations compel us to conclude that a new, much deeper problem is embodied in an eventual change of the affine nature of space-time, as suggested in Eq. (5): accepting that currently the structure of space-time is indeed Riemannian, are there sound reasons to believe that it has always been so? Or, in a more rigorous, formal sense: given the fact that space-time structure is such that on a certain hypersurface Σ_0 the covariant derivative of the metric tensor vanishes, $g_{\mu\nu;\lambda}\ (\Sigma_0) = 0$ - which corresponds to a Riemannian configuration - what can be said about the value of $g_{\mu\nu;\lambda}\ (\Sigma_1)$ on another hypersurface Σ_1? Thus, in addition to the "standard troubles" quoted before, a new component must be brought to our cosmological investigations: the determination of the evolutionary pattern of the geometrical background affine character. This is a restricted form of what we may call a *"structural problem"*, which in its broadest scope concerns any kind of possible variations of the basic nature of the structure of space-time.

There is, in principle, an unlimited number of physical scenarios in which some kind of structural change might take place - so, put in a completely unrestricted form, "structural problems" seem hopelessly vague. In order to address them in a proper way, a definite conceptual context - i. e., a cosmological model - for the description of such structural transitions must be provided. In the literature, different sorts of modifications have been proposed, within either classical or quantum approaches, thereby resulting effects such as, for instance, variations of topological properties[30] or changes in the signature of the metric[31]. In the present paper, likewise, the hypothesis conveyed in Eq. (5) with respect to scale fluctuations of the Minkowski vacuum corresponds to a specific assumption about the evolution of the value of $g_{\mu\nu;\lambda}$ (Σ) and, accordingly, of the non-Riemannian character of the geometrical background. In view of the axiomatical and observational arguments mentioned previously, and in accordance with Eq. (5), we will assume thereof that the basic structure of space-time is of a *conformally-Riemannian* type. Conformally-Riemannian geometries are more commonly acknowledged as *Weyl-integrable space-times* (WISTs). In Section III we will supply a brief account of the essentials of the theory of Weyl spaces[32].

In summary, we will deal here with a specific structural problem in which the evolution of the Universe is provoked by unstabilities of an "original" empty Minkowski space, due to measuring scales perturbations associated to a WIST background manifold. On the grounds of the good experimental status currently enjoyed by Einstein's General Theory of Relativity, objections could be raised against the surmise of the abandonment of the Riemannian configuration - particularly if modifications of the well-tested *local* characteristics of the gravitational field are implied. However, the enlargement of the structure of space-time to a WIST configuration proposed here does not lead to a new theory of gravitational phenomena, once *global* effects only (in the sake, say, of the inclusion of a cosmological constant term into Einstein's equations) are induced - as we will see in what follows.

III. A Brief Review of the Theory of Weyl Spaces

III.1. Introduction to Weyl spaces

In view of the reasons put forth in the previous section, we are interested in exploring the suggestion that space-time structure exhibits a Weylian character. A Weyl geometry is an affine manifold specified by a metric tensor $g_{\mu\nu}(x)$ and a "gauge" vector $\omega_\mu(x)$ which participate in the definition of the manifold affine connection $\Gamma^\alpha_{\mu\nu}(x)$. Besides the MMG group of Riemannian structures, Weyl geometries admit internal ("gauge") transformations which are intimately connected to point-dependent variations of measuring scales. Due to this property, such geometries have been considered, for example, in abelian gauge theories - as in Weyl's original attempt to unify, on a geometrical basis, electromagnetism and gravitation[32]; and in theories addressing the conformal invariance of physical processes - such as in the scale-invariant theories of Dirac[33] and Canuto[34]. It is important to remark that both attempts have failed, mainly due to the fact that physical laws are *not* conformally invariant.[35] Moreover, the most generic cases of Weyl geometries provoke the so-called "second clock effect", leading to observational inconsistencies. Before demonstrating how such difficulties can be circumvented, let us provide the reader with some necessary definitions and notations.

In Weyl geometries the rule of parallel transport of a given vector requires a non-vanishing covariant derivative of the metric tensor $g_{\mu\nu}$:

$$g_{\mu\nu;\lambda} = g_{\mu\nu}\,\omega_\lambda \qquad (3.1)$$

in which $\omega_\lambda(x)$ is the gauge vector and the semi-colon denotes covariant differentiation in a general affine sense. This implies that vector lengths may vary along transport or, equivalently, that the units of measure may change locally. Remark, in contrast, that one of the attractive results of Einstein's theory of gravitation is that it contains *a posteriori* the Riemannian characterization of space-time structure. The argument is simple and is commonly related to the Palatini variational procedure[36] as follows:

Consider the theory given by Einstein's Lagrangian

$$L_E = \sqrt{-g}\, R \; ; \qquad (3.2)$$

varying in the Palatini fashion, that is, taking both the metric tensor $g_{\mu\nu}$ *and* the (as yet unspecified) affine connection $\Gamma^\alpha_{\mu\nu}$ as independent geometric variables, one obtains

$$[\delta g_{\mu\nu}]: \qquad R_{\mu\nu} = 0 \qquad (3.3)$$

$$[\delta\Gamma^\alpha_{\mu\nu}]: \qquad g_{\mu\nu\|\lambda} = 0 \; , \qquad (3.4)$$

where $R_{\mu\nu}$ is the Ricci tensor and the double bar denotes covariant differentiation in a Riemannian sense, i. e., making use of the Christoffel symbols

$$\left\{ {\alpha \atop \mu\nu} \right\} = 1/2 \; g^{\alpha\lambda} \left[g_{\mu\lambda',\nu} + g_{\nu\lambda',\mu} - g_{\mu\nu',\lambda} \right] \tag{3.5}$$

of Riemann geometry (commas indicate simple differentiation). Therefore, a Riemann configuration - characterized by Eq. (4), which implies that vector lengths do not change under parallel transport - is obtained as a direct consequence of the variational procedure.

III.2. WISTs

However, this is a model-dependent result. Other Lagrangians will yield different geometrical configurations. Consider, for instance, the theory of a scalar field $\phi(x)$ in the form

$$L = \sqrt{-g}\; f(\phi)\; R + \mathcal{L}(\phi) \; . \tag{3.6}$$

Variation *a la* Palatini (with ϕ, $g_{\mu\nu}$ and $\Gamma^{\alpha}_{\mu\nu}$ as independent variables) now gives Eq. (1) in place of Eq. (4), with

$$\omega_{\lambda} = - \; [\ln f(\phi)]_{,\lambda} \; . \tag{3.7}$$

Thus the variational principle leads to a special kind of Weyl geometry and *not* to a Riemann space[36]. This particular type of Weyl geometries - in which the gauge vector is the gradient of a scalar function - is called a *conformally-Riemannian* or *Weyl-integrable space-time* (WIST), and in fact constitutes the basis of the cosmic scenario examined here. Its fundamental importance for the present developments stems from the following reason: according to the definition of a Weyl space, variations of the units of measure are controlled by the gauge vector $\omega_\mu(x)$. Weyl suggested that in the course of an infinitesimal parallel transport dx^α the length $L = g_{\mu\nu}\, \ell^\mu\, \ell^\nu$ of a given vector $\ell^\mu(x)$ is changed according to the first-order expression

$$dL = L\, \omega_\alpha\, dx^\alpha\ . \tag{3.8}$$

This result implies, in general, observational difficulties. Suppose, for instance, that at a given space-time point A two identical clocks are synchronized. According to General Relativity, if these two clocks travel to another point B through distinct paths, gravitational effects may cause them to lose their synchronization. This is the "first clock effect". In Weyl spaces, due to the distinct variation of the units of measure along the two different paths, the discrepancy between time measurement units at B might add a supplementary contribution to the loss of synchronization - called the "second clock effect". This effect was the root of Einstein's criticism against Weyl's original proposal of unification, once in the case of *closed* circuits such additional synchronization loss would disagree with well-known observations.[28] To overcome this objection, one has to impose the coincidence of the units of measure of both observers at A, regardless of the particular closed path chosen; this implies that

$$\oint dL = 0 \; . \tag{3.9}$$

But according to Stoke's theorem this condition leads precisely to the result that $\omega_{\mu',\nu} - \omega_{\nu',\mu} = 0$, that is,

$$\omega_\mu = \omega,_\mu \; . \tag{3.10}$$

Thus the corresponding Weyl structure is characterized by a gauge vector which is the gradient of a scalar function - a Weyl geometry in which length variations are integrable along closed paths or, in short, a WIST. It is interesting to remark that the variational procedure sketched above (Eq. (6)) does not lead to a general Weyl space, but specifically to a WIST configuration, in which the "second clock effect" results eliminated.

III.3. Conformal invariance

From Eq. (1) it is simple to derive the expression of the Weyl affine connection $\Gamma^\alpha_{\mu\nu}$:

$$\Gamma^\alpha_{\mu\nu}(x) = \{^{\alpha}_{\mu\nu}\} - 1/2\;[\omega_\mu\delta^\alpha_\nu + \omega_\nu\delta^\alpha_\mu - g_{\mu\nu}\omega^\alpha] \; . \tag{3.11}$$

Consider now a conformal mapping of the metric tensor $g_{\mu\nu}$ such as

$$\tilde{g}_{\mu\nu} = \Omega^2(x)\; g_{\mu\nu} \tag{3.12}$$

in a given *Riemann* geometry. It then follows that the corresponding transformed connection is given by

$$\tilde{\Gamma}^{\alpha}_{\mu\nu} = \{^{\alpha}_{\mu\nu}\} + (1/\Omega)\ [\Omega,_{\mu}\delta^{\alpha}_{\nu} + \Omega,_{\nu}\delta^{\alpha}_{\mu} - g_{\mu\nu}g^{\alpha\varepsilon}\Omega,_{\varepsilon}]\ . \qquad (3.13)$$

Setting $\omega(x) = -\ln \Omega^2(x)$, one obtains that connections Eqs. (11, 13) are equivalent when Eq. (10) holds. Thus Weyl-integrable space-times are also called conformally-Riemannian, since a conformal transformation maps a Riemann geometry into a WIST one. If the laws of physics were invariant with respect to conformal transformations, the WIST scalar function $\omega(x)$ would be unobservable and both structures could not be distinguished by any physical effect. The hypothesis of the conformal invariance of all physical processes, in fact, provided the basis for the approaches of Dirac ("Large Number Hypothesis") and Canuto and co-workers ("Scale-invariant theory"), who advocated the introduction of a new general symmetry (besides MMG) in Physics: the gauge invariance of measuring units[34]. Despite the elegancy of these proposals, eventually astrophysical observations brought in decisive evidence against the assumption of a general conformal symmetry of physical laws[35]. Therefore the WIST field $\omega(x)$ cannot, in principle, be discarded by a convenient gauge choice; it suffices to dynamically break the global conformal invariance of a given WIST theory in order to distinguish it, under conformal transformations, from its Riemannian counterpart. In consequence, $\omega(x)$ constitutes a true (i. e., observable) field and Riemann and WIST configurations are physically distinguishable.

III.4. Some useful quantities

Given the Weyl connection Eq. (11), it is straightforward to write Weylian expressions for geometrical objects with the use of the corresponding Riemannian formulae; the covariant differentiation of a vector field V^α reads, for instance,

$$V^{\alpha}{}_{;\mu} = V^{\alpha}{}_{,\mu} + \{^{\alpha}_{\mu\nu}\} V^{\nu} - 1/2 \, [\omega_{\mu} \delta^{\alpha}_{\nu} + \omega_{\nu} \delta^{\alpha}_{\mu} - g_{\mu\nu} \omega^{\alpha}] \, V^{\nu} =$$

$$= V^{\alpha}{}_{\Vert\mu} - 1/2 \, [\omega_{\mu} V^{\alpha} + \omega_{\nu} V^{\nu} \delta^{\alpha}_{\mu} - g_{\mu\nu} V^{\nu} \omega^{\alpha}] \, . \qquad (3.14)$$

In particular, the contracted (Ricci) curvature tensor $R_{\mu\nu} \equiv R^{\alpha}_{\mu\alpha\nu}$ can be written in terms of its Riemannian counterpart $\hat{R}_{\mu\nu}$ and the gauge vector ω_μ as follows:

$$R_{\mu\nu} = \hat{R}_{\mu\nu} - \frac{3}{2}\, \omega_{\mu\Vert\nu} + \frac{1}{2}\, \omega_{\mu\Vert\nu} - \frac{1}{2}\, \omega_{\mu}\, \omega_{\nu} - \frac{1}{2}\, g_{\mu\nu}\, [\omega^{\lambda}{}_{\Vert\lambda} - \omega_{\lambda}\, \omega^{\lambda}] \qquad (3.15)$$

which, in the case of a WIST, reduces to

$$R_{\mu\nu} = \hat{R}_{\mu\nu} - \omega_{\mu\Vert\nu} - \frac{1}{2}\, \omega_{\mu}\, \omega_{\nu} - \frac{1}{2}\, g_{\mu\nu}\, [\hat{\Box}\, \omega - \omega_{\lambda}\, \omega^{\lambda}] \, , \qquad (3.16)$$

where $\omega_\mu = \partial_\mu \omega$, $\Box$ is the D'Alembertian operator and the symbol ^ denotes objects constructed in the associated Riemannian structure (i. e., making use of Christoffel symbols only). Contracting Eq. (16) one obtains the WIST scalar curvature R:

$$R = \hat{R} - 3\, \omega^{\mu}{}_{\Vert\mu} + \frac{3}{2}\, \omega_{\mu}\, \omega^{\mu} = \hat{R} - 3\, \hat{\Box}\, \omega + \frac{3}{2}\, \omega_{\mu}\, \omega^{\mu} \, . \qquad (3.17)$$

IV. A Friedman-like Cosmological Model

IV.1. Dynamical equations

Let us then proceed to the implementation of the investigative program discussed in Section II. We consider the veritable primordial phase of the evolution of the Universe to correspond to a "Nothing" state described by an empty Minkowski space-time. In order to provoke the unstability of this basic configuration we resort to perturbations of the system of measuring units as in the form given in Eq. (2.5),

$$\delta(g_{\mu\nu;\lambda}) = (\delta\omega_\lambda)\ g_{\mu\nu}\ . \tag{4.1}$$

Since this is a particular case of Eq. (3.1), we are explicitly assuming that the background geometry is endowed with a Weylian structure.

The subsequent evolution of the Cosmos depends, of course, on the behavior of the perturbations $\delta\omega_\lambda$. Thus a dynamical framework is required in which the pair $(g_{\mu\nu}\ ,\ \omega_\lambda)$ constitutes the set of fundamental geometrical variables. A simple action involving this pair is given by

$$S_c = \int \sqrt{-g}\ [\ R + \xi\ \omega^\lambda{}_{;\lambda}\ ] \tag{4.2}$$

in which ξ is a dimensionless parameter. Two points deserve comment here: firstly, in view of the arguments put forth in Section III about the "second clock" effect, we will restrict our considerations to a WIST configuration, i. e., we assume that $\omega_\lambda(x) = \partial_\lambda\ \omega(x)$ in Eq. (2). Then the set of independent variables is actually reduced to $(g_{\mu\nu}\ ,\ \omega)$. Secondly, the attentive reader will be aware of the presence of a total divergence term in the action S_c; in the usual Riemannian context of General Relativity, the contribution of total divergence terms to the dynamical equations vanishes. Remark, however, that according to Eq. (3.14) one has for the divergence of the gauge vector $\omega_\lambda(x)$ the expression

$$\omega^\lambda{}_{;\lambda} = \omega^\lambda{}_{\|\lambda} - 1/2\ \omega^\lambda\ \omega_\lambda = 1/\sqrt{-g}\ \ (\sqrt{-g}\ \omega^\lambda)_{,\lambda} - 1/2\ \omega^\lambda\ \omega_\lambda \qquad \textbf{(4.3)}$$

in the WIST case, and so a non-vanishing contribution to the dynamics is obtained. Decomposition Eq. (3) also shows that if a term proportional to ω^λ ω_λ is included in the Lagrangian in Eq. (2) the net result is just a renormalization of parameter ξ.

Variation of action S_c with respect to the pair $(g_{\mu\nu}\ ,\ \omega)$ of independent WIST variables yields the equations of motion

$$R_{\mu\nu} - 1/4\ R\ g_{\mu\nu} + \omega_{,\mu\|\nu} = 0\ , \qquad (4.4)$$

and, consequently,

$$\hat{\square}\ \omega = 0\ , \qquad (4.5)$$

in which the double bar denotes Riemannian covariant differentiation and $\hat{\Box}$ is the D'Alembertian operator written in the associate Riemann space-time, i. e., Eq. (5) reads

$$\hat{\Box}\,\omega = 1/\sqrt{-g}\;\;(\sqrt{-g}\;\omega,_{\alpha}\;g^{\alpha\beta}),_{\beta} = 0\;. \tag{4.6}$$

Let us at this point remind the reader that we do not aim to associate action S_c to a new theory of gravity. We treat S_c, instead, as an effective canonical action which results of a combination of geometrical components (metric $g_{\mu\nu}$ and gauge vector ω_{λ}) of distinct nature. Nevertheless, in order to simplify our understanding of the cosmological consequences of the present model it is useful to recast the set Eqs. (4,5) of WIST dynamical equations in terms of a Riemannian configuration plus an external source term represented by the scalar field $\omega(x)$. This re-interpretation is legitimate due to the decomposition Eq. (3.16) of the WIST contracted curvature tensor $R_{\mu\nu}$ in terms of the associated Ricci tensor $\hat{R}_{\mu\nu}$ and functions of the scalar field $\omega(x)$. In this vein, Eqs. (4, 5) can be rewritten as follows:

$$\hat{R}_{\mu\nu} - 1/2\;\hat{R}\;g_{\mu\nu} - \lambda^2\;\omega_{\mu}\;\omega_{\nu} + \lambda^2/2\;\omega_{\alpha}\;\omega^{\alpha}\;g_{\mu\nu} = 0\;, \tag{4.7}$$

$$\hat{\Box}\,\omega = 0\;, \tag{4.8}$$

in which $\lambda^2 = [\frac{4\xi - 3}{2}\;]$.[37] Eq. (7) is thus equivalent to an Einstein equation in which the WIST field ω provides the source of the Riemannian curvature.

Once in the present paper we will be concerned exclusively with spatially homogeneous FRW cosmologies, described by the line element Eq. (2.1), it is natural to make the WIST field ω a function of the cosmic time t only: $\omega = \omega(t)$. The gauge vector ω_λ then becomes

$$\omega_\lambda = \partial_\lambda\ \omega(t) = \dot{\omega}\ \delta^0_\lambda\ , \qquad (4.9)$$

where the dot denotes simple differentiation with respect to the time variable. In this case, the ω-dependent "source" term in the Einstein equation Eq. (7) may be seen to represent a "stiff matter" state of a perfect fluid, endowed with a *negative* energy, once if Eq. (7) is rewritten in the form

$$\hat{R}_{\mu\nu} - 1/2\ \hat{R}\ g_{\mu\nu} = -\ T_{\mu\nu}(\omega) = -\ [(\rho_\omega + p_\omega)\ V_\mu\ V_\nu - p_\omega\ g_{\mu\nu}]\ , \qquad (4.10)$$

in which $\{V_\mu\}$ is a set of unit time-like vectors, one obtains for the energy density ρ_ω and the isotropic pressure p_ω the values

$$\rho_\omega = p_\omega = -\ \lambda^2/2\ \dot{\omega}^2\ , \qquad (4.11)$$

and so the equation of state of the "ω-fluid" indeed corresponds to a "stiff matter" state[38].

Use of Eq. (9) into the field equation Eq. (8) yields a first integral for the function $\omega(t)$:

$$\dot{\omega} = \gamma\ a^{-3}\ , \tag{4.12}$$

where γ = constant. In turn, Einstein's equations Eq. (7) for the Friedman scale factor a(t) consists of the system

$$\dot{a}^2 + \varepsilon + \lambda^2/6\ (\dot{\omega}\ a)^2 = 0 \tag{4.13}$$

$$2\ a\ \ddot{a} + \dot{a}^2 + \varepsilon - \lambda^2/2\ (\dot{\omega}\ a)^2 = 0\ , \tag{4.14}$$

where $\varepsilon = (0, +1, -1)$ is the 3-curvature parameter of the FRW geometry. From Eqs. (13, 14) we see that if $(3 - 4\xi) = -\lambda^2 < 0$ an *open* Universe is obtained (i. e., $\varepsilon = -1$).

A combination of Eqs. (12) and (13) supplies the fundamental dynamical equation

$$\dot{a}^2 = 1 - [a_0/a]^4\ , \tag{4.15}$$

with a_0 = constant = $[\gamma^2\lambda^2/6]^{1/4}$. It is straightforward to show that Eq. (14) results of Eqs. (11, 12, 13).

Prior to the elaboration of the solution of the system of structural and dynamical equations Eqs. (12, 15) let us comment on the consequent cosmological model and list some interesting results.

IV.2. Aspects of the model

- *The age of the Universe:* it is an immediate consequence of Eq. (15) that the scale factor a(t) cannot attain values lesser than the minimum limit a_0. The singularity problem, one of the most fundamental difficulties of standard cosmologies, is solved in the present theory.

Let us consider a time reversal operation and run backwards into the past of the cosmic evolution. As the cosmic radius a(t) decreases, the temperature of the material medium grows. In HBB models such increment is unlimited; in the present theory, on the other hand, there is an epoch of greatest condensation in the vicinity of the minimum radius a_0. Close to this period, there occurs a *continuous* "phase transition" in the geometrical background: a Weylian structure is activated, according to Eq. (12), and in consequence an unbounded growth of the temperature is inhibited. The Universe attains the minimum radius a_0 at (t = 0), beckoning to a previous collapsing era. Once the Universe had this infinite collapsing era to become homogeneous, in the present scenario the "horizon" problem of Standard Cosmology does not happen also.

- *The Riemannian structure of space time:* for very large times the scale factor behaves as $a \sim t$. Thus, asymptotically, the geometrical configuration assumes a Riemannian character (once $\dot{\omega} \to 0$) in the form of a flat Minkowski space (in Milne's coordinate system). In consequence, in the present model the evolution of the Cosmos may be assigned to a primordial unstability of Minkowski space, at the remote past, against Weylian perturbations of the Riemann structure in the guise of Eq. (2.5). In order to prescribe the behavior of these perturbations, knowledge is required on the time dependence of the gauge vector ω_λ. As we will see, once the WIST function $\dot{\omega}$ has a maximum at (t = 0), the largest deviation of the Riemannian configuration corresponds to the epoch of greatest contraction near to the value a_0; we shall postpone this development, though, to a subsequent part of this section.

- *The flatness concern:* in the standard theory one faces the following problem. Defining the critical density ρ_c of the Friedman model as $\rho_c = 3H^2 = 3[\dot{a}/a]^2$, current observations show that the value of the ratio $|\rho - \rho_c|/\rho_c$ is rather large - where ρ is the density of the matter-energy sources. However, this quantity could assume a very small value at the beginning of the present expanding era. In fact, it is a consequence of Einstein's equations for radiation (according to the equation of state $p = 1/3\ \rho$) that $|\rho - \rho_c|/\rho_c \sim 1/\dot{a}^2 \sim 0$ when $(t \to 0)$; this in turn implies a "flat" or Euclidean ($\varepsilon = 0$) configuration. How could such an enormous difference have occurred? In other words, why should the Universe display such a fine-tuning (i. e., $\rho \sim \rho_c$) of its initial conditions?

In the present scenario this becomes a false problem. Indeed, from Eqs.

(11, 13) it follows that close to the era of maximum condensation the matter-energy distribution is dominated by the energy of the WIST field $\omega(t)$ - which, according to Eqs. (11, 12), is described by a "stiff matter" state such that $\rho_\omega = p_\omega \sim a^{-6}$. In this case, near to the minimum value a_0 one has, in view of Eq. (15),

$$|\rho - \rho_c|/\rho_c \sim 1/\dot{a}^2 \sim (1 - [a_0/a]^4)^{-1} , \qquad (4.16)$$

- which is a rather large quantity. To guarantee the compatibility of Einstein's equations, it suffices that $\varepsilon = -1$ (open solution). Hence, no resource to a specific set of initial conditions is required.

- *The accelerated Universe:* in the present model the Universe starts to evolve due to Weylian perturbations of an empty Minkowski space-time; thus, the most remote image of the cosmic history is that of a collapsing primordial Universe of infinite radius. Throughout this collapsing era the cosmic evolution is driven by the energy of the WIST field $\omega(t)$; in consequence, in the course of the entire collapse the Universe is accelerated - or "inflationary" - once $\ddot{a} = 2/a\ [a_0/a]^4 > 0$. In fact, were the Universe always dominated by the ω-energy only, it would accelerate forever. However, as we will see in the next section, in the neighborhood of the maximally condensed epoch a significant amount of matter may come to appear, therefore implying important modifications of space-time curvature in the ensuing expanding era. Nevertheless, it is remarkable that during the whole collapsing era the Universe manifested such inflationary behavior.

- *A quest for stability:* among the difficult questions concerning "eternal", bouncing Universes one may count the problem of their survival with respect to eventual *metric* perturbations. With the use of Eqs. (12, 15), it is straightforward to show that way of the stage of greatest condensation the Universe is *stable*[39]. As we will show in the next section, this result is consistent with the behavior of matter fluctuations prior to the phase of greatest contraction.

IV.3. The exact solution

Let us proceed to the solution of the fundamental dynamical equation Eq. (15) for the Friedman scale factor a(t). The desired solution is given in terms of the elliptic functions[40]

$$\begin{cases} F(\alpha, \sqrt{2}/2) = \int_0^{\alpha} (1 - 1/2 \sin^2 z)^{-1/2}\, dz \\ \\ E(\alpha, \sqrt{2}/2) = \int_0^{\alpha} (1 - 1/2 \sin^2 z)^{1/2}\, dz \end{cases} \tag{4.17}$$

by the expression

$$t = a_0 \left[\sqrt{2}/2\, F(\alpha, \sqrt{2}/2) - \sqrt{2}\, E(\alpha, \sqrt{2}/2) + \sin\beta\, (\cos\beta)^{-1/2} \right], \tag{4.18}$$

in which $\alpha = \arccos\,(a_0/a)$ and $\beta = \arccos\,(a_0/a)^2$. The new variable β ranges over the compact domain $[-\pi/2, \pi/2]$. Figure (**4.1**) below illustrates the qualitative behavior of the scale factor a(t).

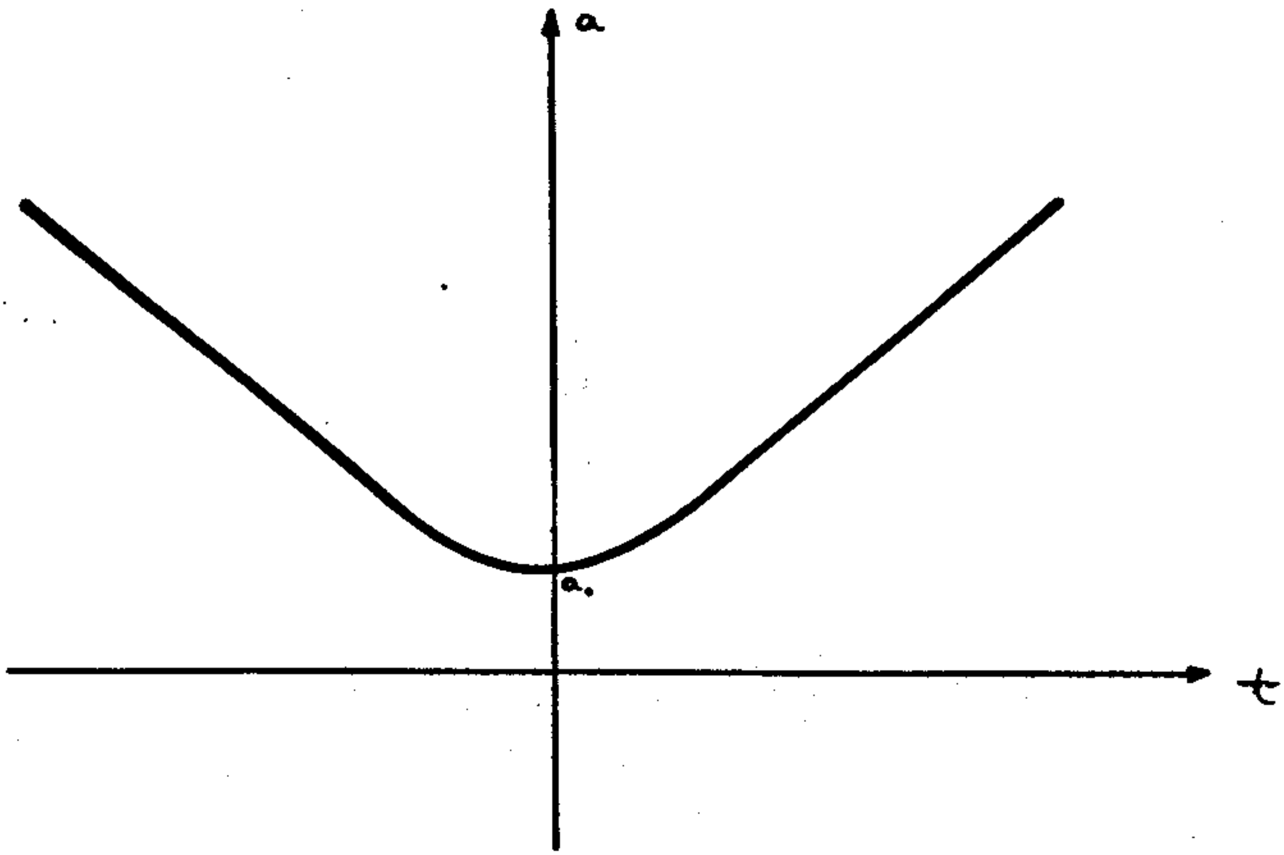

FIGURE (4.1)

Fig. (4.1): Qualitative behavior of the scale factor a(t) according to the implicit solution Eq. (22). Given that $|\dot{a}| < 1$, the angle of inclination of the curve a(t) is always less than $\pi/4$.

The scale factor has a minimum for $a = a_0$, which corresponds to (t=0). Thus the Universe had a collapsing era for (t < 0), attained its minimum dimension at (t = 0), and thereafter initiated an expanding era. Both the collapse and the expansion run adiabatically, i. e., in a very slow pace.

The correlate behavior of the Hubble expansion parameter - or Hubble "constant" - $H \equiv (\dot{a}/a)$ is worth of mention, since it provides a valuable clue to the understanding of the model.

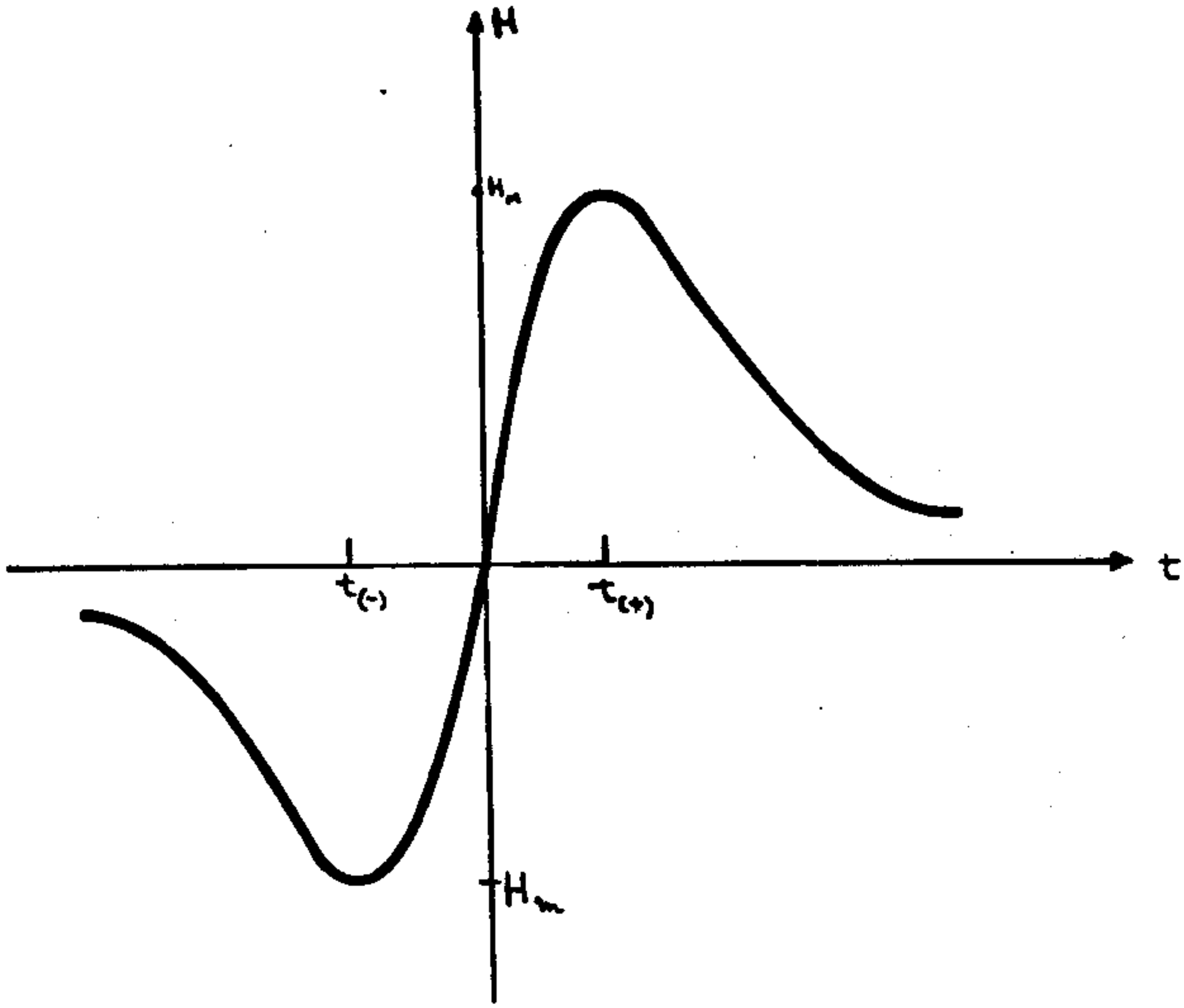

FIGURE (4.2)

Fig. (4.2): The Hubble parameter $H = 1/a\ (1 - [a_0/a]^4)^{1/2}$. Here, $H_M = H(t_{(+)}) = [3^{3/4}\ \sqrt{2}/a_0] = -\ H_m = -\ H(t_{(-)})$, where $t_{(\pm)}$ corresponds to $a_{(\pm)} = 3^{1/4}\ a_0$.

A direct inspection of Fig. (4.2) allows a remarkable conclusion: in distinction of singular HBB standard models, here we have a sort of "Big - although not infinite! - Bang". Indeed, the Hubble parameter H is always a smooth function of the cosmic time t and does *not* diverge at the origin of the expanding era; quite on the contrary, it *vanishes* at ($t = 0$). The corresponding evolution of the Cosmos may be outlined as follows: the Universe stays for a long period in a phase of slow adiabatic contraction, until parameter H attains the minimum value $H_m = -(3^{-3/4}\sqrt{2}/a_0)$. Then an abrupt transition occurs: a fast compression turns into a fast expansion until H attains the maximum value $H_M = -H_m$; afterwards the expansion proceeds in an adiabatic slow pace again. The time lapse between these two extrema, H_m and H_M, is in fact very short, once the value of the scale factor at both occasions, $a(t_-) = a(t_+) = 3^{1/4} a_0$, keeps very close to the minimum a_0. Figure (4.3) displays the behavior of H with respect to the scale factor $a(t)$.

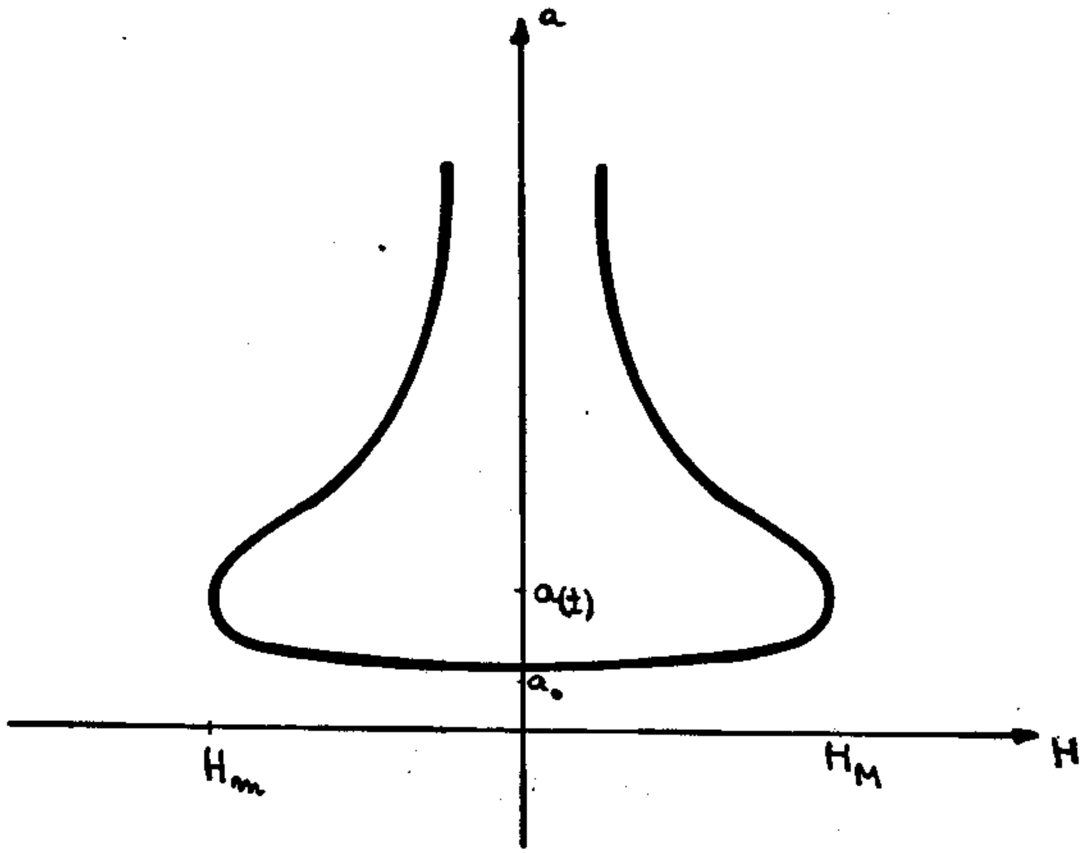

FIGURE (4.3)

Fig. (4.3): Behavior of the Hubble parameter H with respect to the scale factor a(t). Here, $a_{(\pm)} = a(t_{(\pm)}) = 3^{1/4}\ a_0$.

This peculiar behavior can be depicted by means of an analogy with a thermodynamical system consisting of a gas, filling a box, submitted to the action of a piston. During a long period the gas is compressed adiabatically until its volume attains a value close to $V_M = 3^{1/4}\ V_0$ (where $V_0 = a_0^3$); suddenly the compression rate increases fast, reaching a maximum when the volume is precisely V_M. The pressure then starts to decrease rapidly, vanishing when the minimum volume V_0 is achieved. At this point a fast expansion begins, attains its maximum rate when the volume is once more V_M and afterwards it slows down to an adiabatical regime again.

This fictional image, whereas it can be helpful in supplying an analogical picture of the behavior of an Universe driven by the WIST function $\omega(t)$, is however incomplete. Such incompleteness is due precisely to the occurrence of a period of strong non-adiabaticity. During this period, which extends from $t_{(-)}$ to $t_{(+)}$ - or, equivalently, from $a(t_{(-)})$ to a_0 and then back to $a(t_{(+)}) = a(t_{(-)})$ - crucial new phenomena may come to occur, namely, the production of large amounts of matter and entropy - as we shall see in the next section.

IV.4. The WIST function $\omega(t)$: structural transitions

According to the basic conception of the present scenario discussed previously, the WIST function $\omega(t)$ governs the structural characteristics of the cosmic background manifold. Taking into account the solution Eq. (18) for the scale factor a(t), the first integral Eq. (12) yields for $\omega(t)$ the expression

$$\omega = \gamma/2a_0^2 \text{ arc cos } [a_0/a]^2 \, . \qquad (4.19)$$

The behavior of $\omega(t)$ is qualitatively portrayed in Fig. (4.4). Observe that when $a \to \pm \infty$ (i. e., for large times), $\omega \to \pm \gamma\pi/4a_0^2$ = constant, which is consistent with the assumption that the Universe originated from a Minkowskian "Nothing" state.

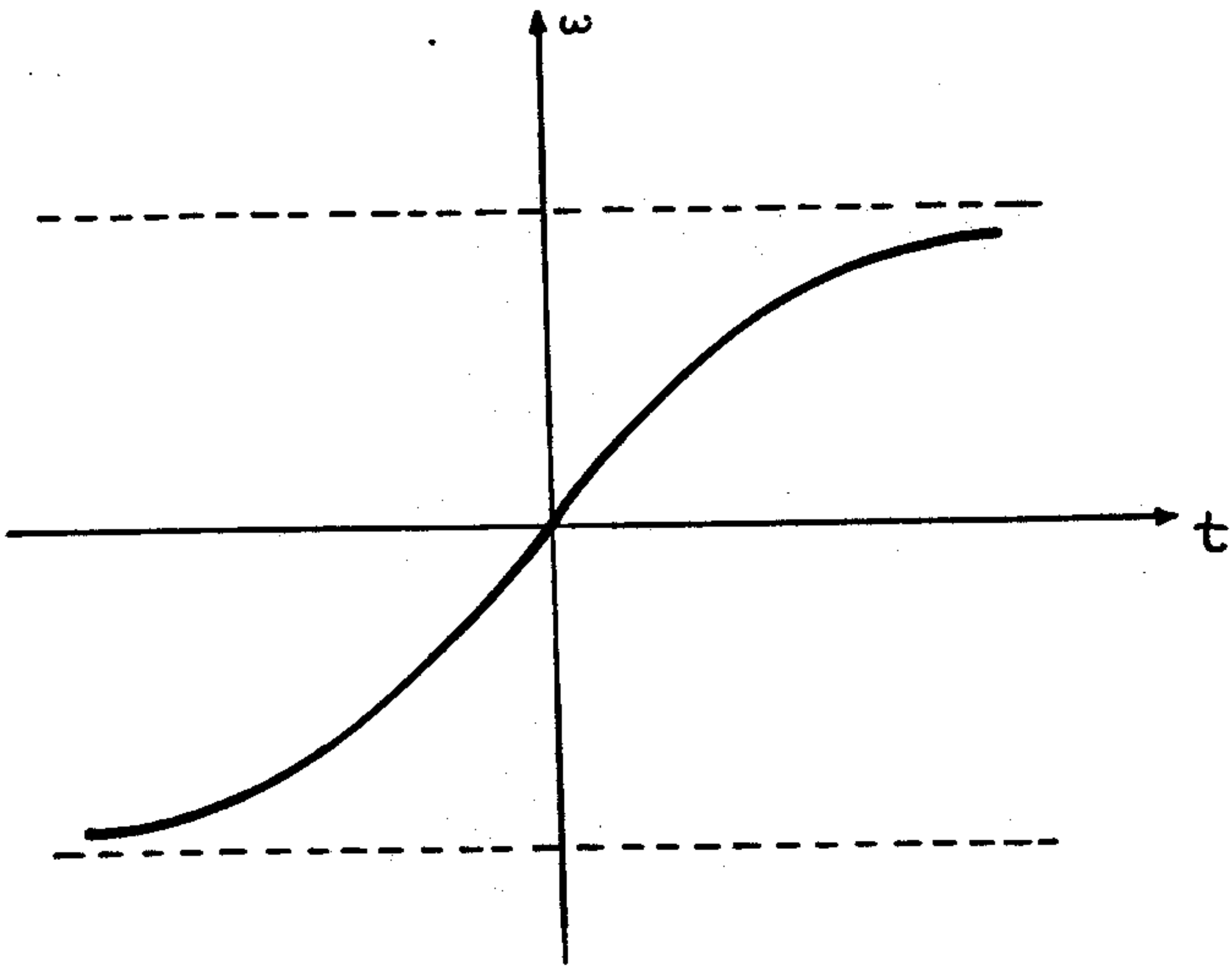

FIGURE (4.4)

Fig. (4.4): Behavior of the WIST function $\omega(t)$.

It is also interesting to show (Fig. (4.5)) the behavior of the time derivative $\dot{\omega} = \gamma/a^3$, which appears in the expression Eq. (11) of the energy density ρ_ω of the "stiff matter" state associated to the WIST field.

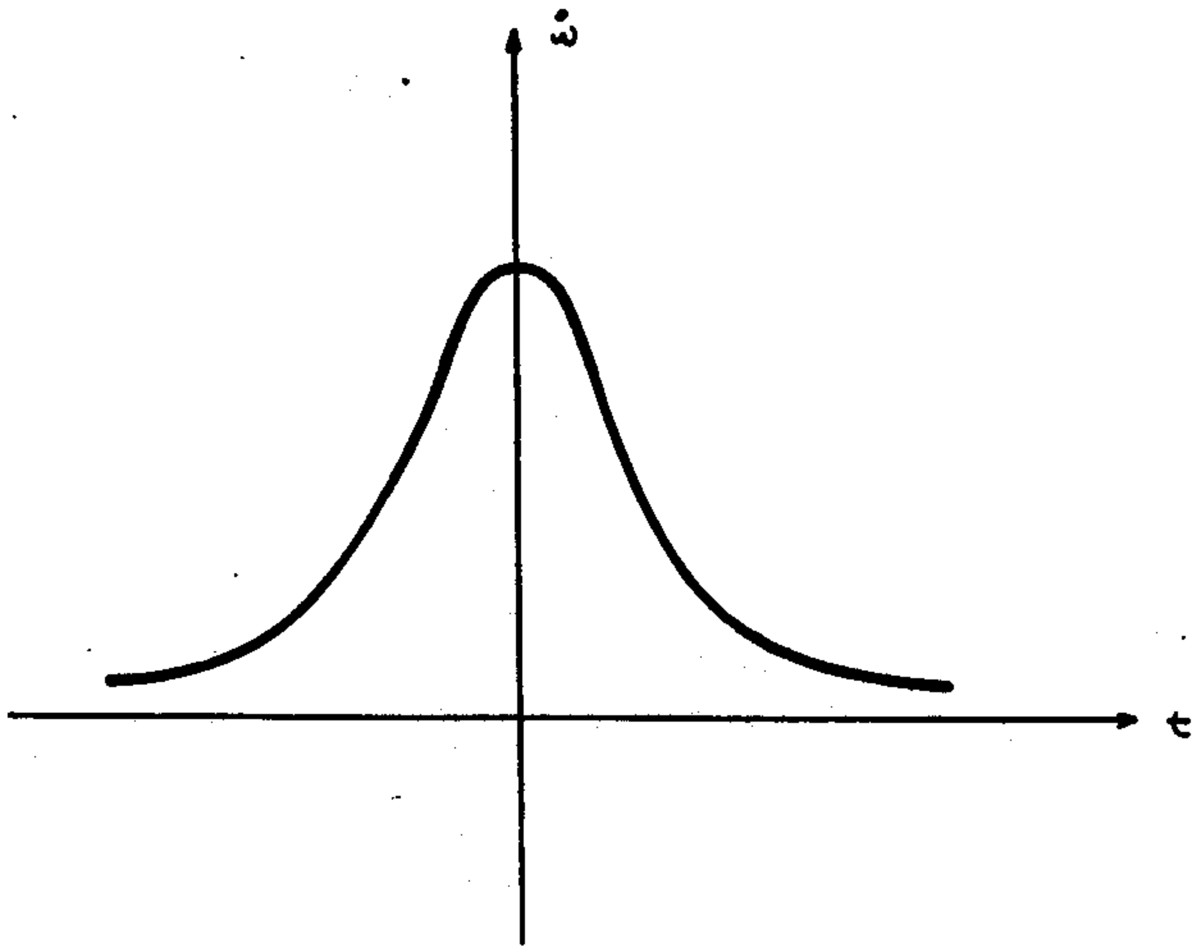

FIGURE (4.5)

Fig. (4.5): Behavior of the WIST function $\dot{\omega} = \gamma/a^3$.

Once this function has a strong peak in the neighborhood of the minimum radius a_0, there occurs in this region the greatest deviation from the Riemannian configuration. In this sense, a sort of structural "phase transition" takes place when the Universe approaches its maximally condensed state. The rapid increase of the energy of the WIST "fluid", according to the interpretation set forth at the beginning of this section (see Eqs. (10, 11)), hinders a further collapse to a singularity and reverses the cosmic evolution into an expansion. One could say, alternatively, that variations of measuring scales are excited when the temperature of the cosmic medium approaches a critical limit associated to the minimum size $V_0 = a_0^3$, the net effect of these scale variations being the reversion of the collapsing process. On the other hand, the "kinky" aspect of the behavior of the WIST function $\omega(t)$ in Fig. (4.4) suggests a similarity between the Weylian structural transition described above and the propagation of instantons in Euclideanized models of quantum creation. This is our next step.

IV.5. "Wistons" and "Anti-Wistons": On the geometrization of instantons

In the derivation of solution Eq. (19) of the WIST "structural" function $\omega(t)$ no attention was paid to the sign of the constant γ. However, the only information we have about γ is that $\gamma^2 = 6a_0^4/\lambda^2$, according to Eqs.(12, 15). Therefore γ in fact admits both a positive and a negative value, given by

$$\gamma^{(\pm)} = \pm \sqrt{6}\, a_0^2/|\lambda| \quad . \tag{4.20}$$

Hence, Eqs. (19, 12) actually have two corresponding expressions, as follows:

$$\omega^{(\pm)} = \omega_0^{(\pm)} \text{ arc cos } [a_0/a]^2 \tag{4.21}$$

$$\dot{\omega}^{(\pm)} = \gamma^{(\pm)}/a^3 \ , \tag{4.22}$$

in which $\omega_0^{(\pm)} = \gamma^{(\pm)}/2a_0^2 = \pm \sqrt{3/2}\ |\lambda|^{-1}$. Thus the amplitude of the solutions $\omega^{(\pm)}$ depends exclusively on the dimensionless parameter ξ . Figs. (4.6) and (4.7) show the symmetric configurations of the WIST functions $\omega^{(\pm)}(t)$ and $\dot{\omega}^{(\pm)}(t)$.

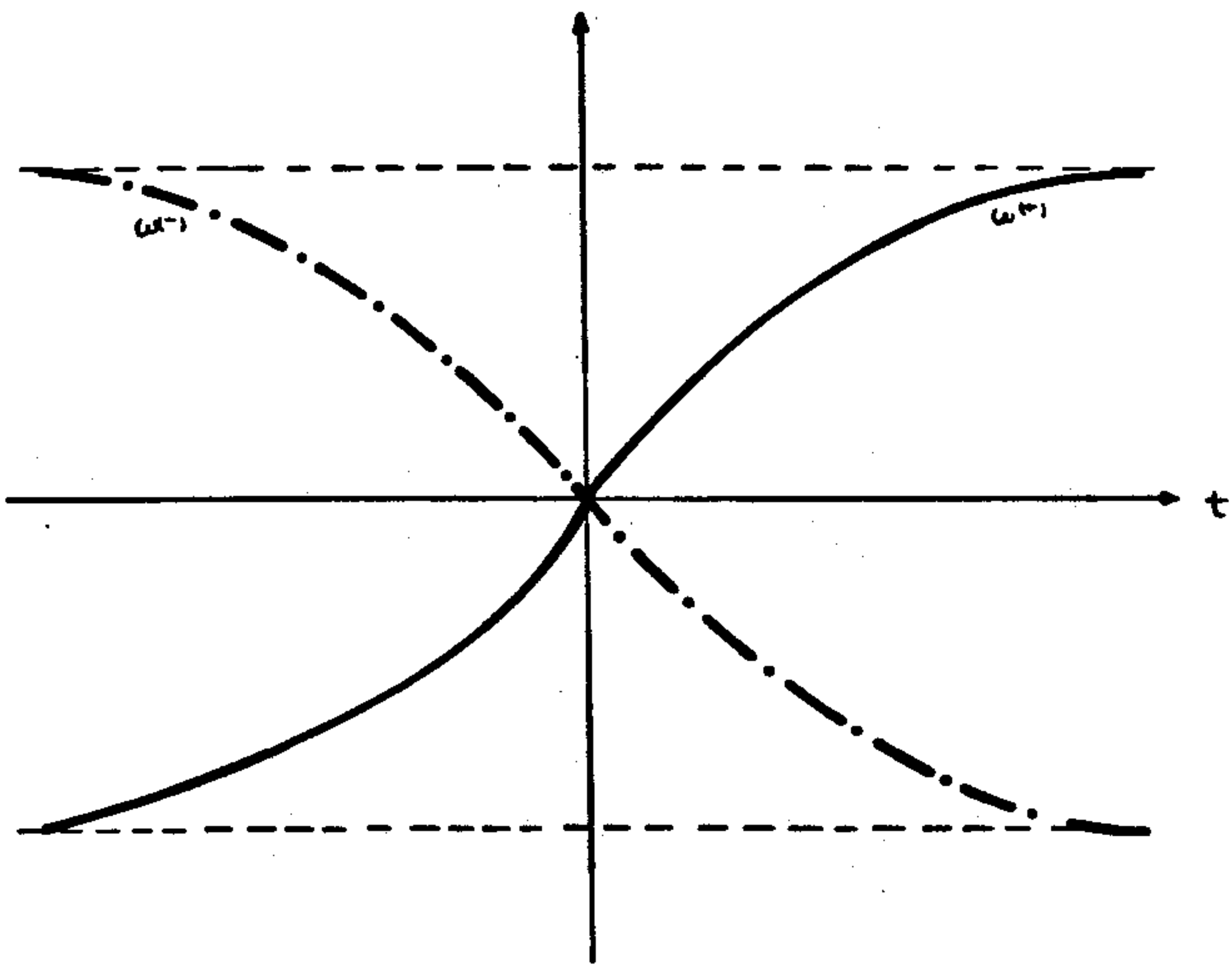

FIGURE (4.6)

Fig. (4.6): Behavior of the WIST "structural" functions $\omega^{(+)}(t)$ and $\omega^{(-)}(t)$.

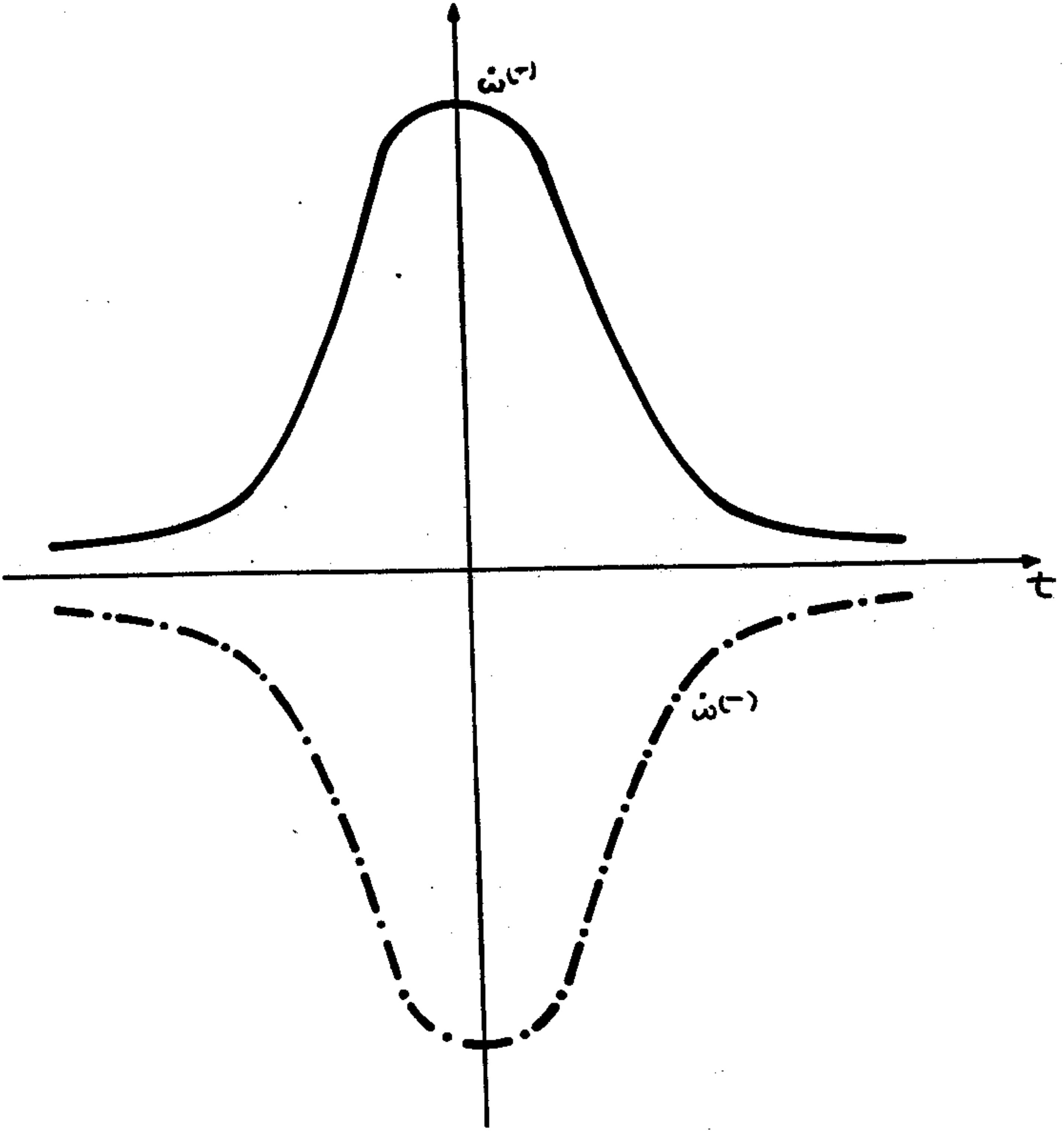

FIGURE (4.7)

Fig. (4.7): Behavior of the WIST "energy" functions $\dot{\omega}^{(+)}(t)$ and $\dot{\omega}^{(-)}(t)$.

Remark, however, that the energy density ρ_ω of the "stiff matter" state associated to the WIST field $\omega(t)$ is the same in both cases, once in view of Eqs. (11, 12) we have

$$\rho_\omega = - \lambda^2/2\ \dot{\omega}^2 = - 3\ [a_0^4/a^6]\ . \tag{4.23}$$

Thus, in spite of the fact that the pairs of WIST functions $(\omega^{(+)}, \dot{\omega}^{(+)})$ and $(\omega^{(-)}, \dot{\omega}^{(-)})$ have different characteristics, they induce the same type of cosmological evolution: solution Eq. (18) does not distinguish which pair is the source of Einstein's equations Eq. (10). Their only distinction, in fact, is connected to length variations, once according to Eq. (3.8) one has now $\Delta L^{(\pm)} = L\ \dot{\omega}^{(\pm)}\ \Delta t$.

It is interesting to observe that the system is invariant with respect to the time reversal operation $t \rightarrow (-t)$ if $\omega^{(+)}$ is concurrently mapped into $\omega^{(-)}$ – and reciprocally. In this sense, the WIST instanton-like functions $\omega^{(+)}$ and $\omega^{(-)}$ may be called "*Wiston*" and "*anti-Wiston*" solutions, respectively, since an anti-Wiston may be described as a Wiston running backwards in time. According to Eq. (19), Wistons are defined up to an additive constant.

A closer inspection of the equations governing the behavior of $\omega(t)$ reveals an instanton-like behavior typical of non-linear theories of self-interacting scalar fields. Of course, the root of such non-linearity is the fact that $\omega(t)$ is taken as the actual source of the curvature of the metric structure, which in turn modifies the D'Alembertian operator $\Box$ due to the introduction of ω-dependent terms. A direct way to clarify this issue is to make explicit the hidden non-linearity of the system Eqs. (12, 15) of equations of motion involving the scale factor a(t) and the WIST function $\omega(t)$ by means of a change of variables. Define the new variable $\sigma(t) = \dot{\omega}(t)$. Using Eq. (12), we have

$$\begin{cases} \dot{\sigma} + 3\gamma\, a^{-4}\, \dot{a} = 0, \\ \\ a^3 - \gamma\, \sigma^{-1} = 0\ . \end{cases} \tag{4.24}$$

Taking $\sigma(t)$ to represent a generalized coordinate associated to a one-particle dynamical system, use of Eq. (15) then yields the conservation equation

$$1/2\ (\dot{\sigma})^2 + V(\sigma) = 0\ , \tag{4.25}$$

in which the associate potential $V(\sigma)$ is given by

$$V(\sigma) = 9/2\gamma^2\ [\ a_0^4\, \sigma^4 - \gamma^{4/3}\, \sigma^{8/3}] =$$

$$= 3\lambda^2/4\ [\sigma^4 - b^2\sigma^{8/3}]\ , \tag{4.26}$$

with $b^2 = 6\lambda^{-2}\gamma^{-2/3}$. Thus the evolution of field σ is equivalent to a unit mass particle moving in a potential with vanishing total energy. In view of the non-linear character of this potential, the instanton-like aspect of functions $\omega^{(\pm)}(t)$ is not surprising. Fig. (4.8) shows the behavior of $V(\sigma)$. The potential vanishes at $\sigma = 0$ and at $\sigma_B^{(\pm)} = \gamma^{(\pm)}\ a_0^{-3}$; its extrema are at $\sigma = 0$, and at $\sigma_m^{(\pm)} = (2/3)^{3/4}\ \gamma^{(\pm)}\ a_0^{-3}$ (which are also minima). However, the system cannot remain at the stable states $V(\sigma_m^{(\pm)}) = -\ 2/3\ \gamma^2\ a_0^{-8}$, once in this case $\dot{\sigma} \neq 0$; this in turn implies, of course, a non-trivial, evolving cosmic configuration.

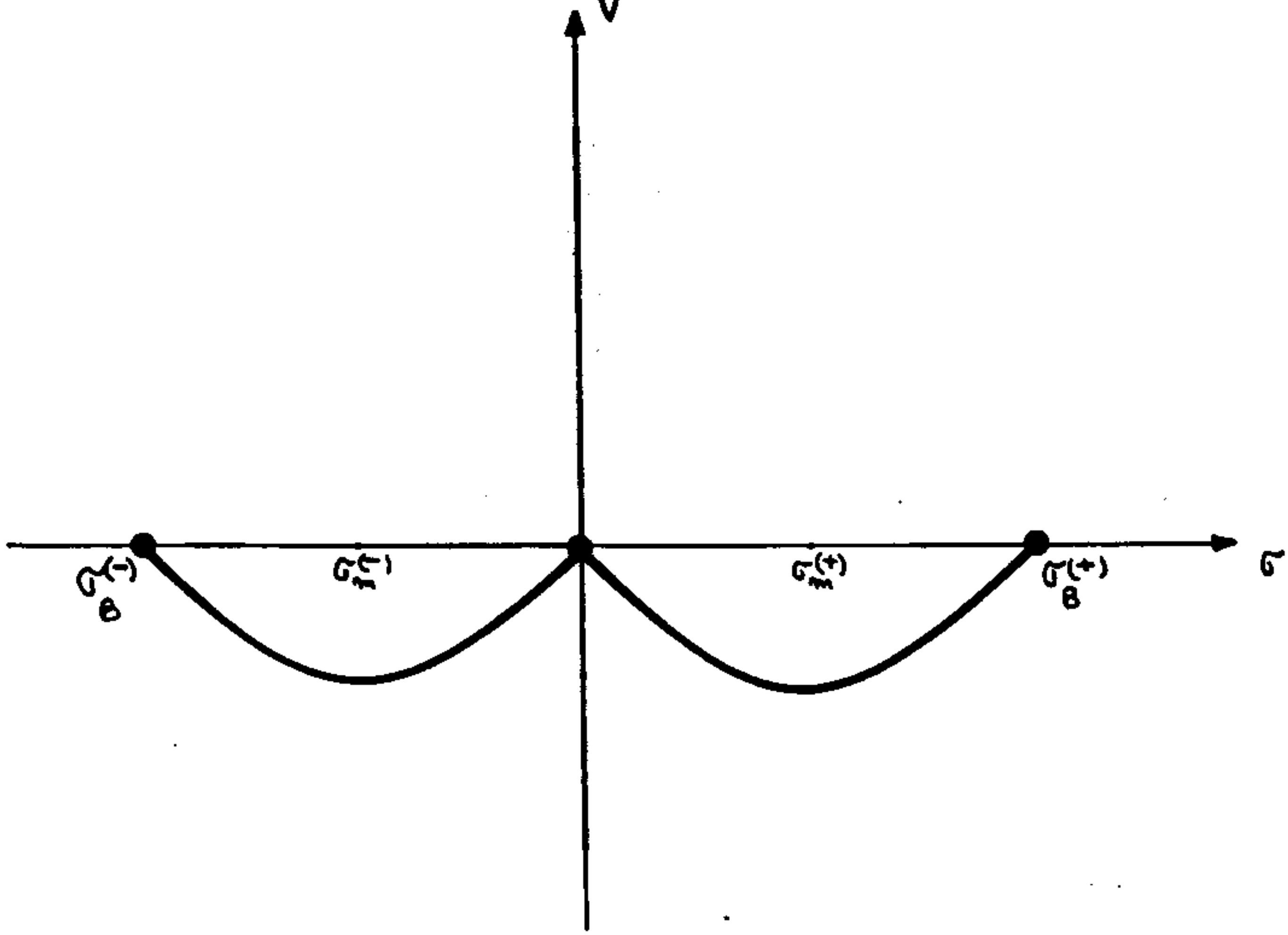

FIGURE (4.8)

Fig. (4.8): Behavior of the non-linear potential $V(\sigma)$. Here, $\sigma_B^{(\pm)} = \gamma^{(\pm)}\ a_0^{-3}$ and $\sigma_m^{(\pm)} = (2/3)^{3/4}\ \sigma_B^{(\pm)}$.

This non-linear scheme provides a succinct picture of the evolution of the Universe: its development is initiated at $\sigma = 0$ (which corresponds to Minkowski space-time at $t \to -\infty$), attains its minimum radius $a(t = 0) = a_0$ at either $\sigma_B^{(+)}$ or $\sigma_B^{(-)}$ and returns back to $\sigma = 0$ (which now corresponds to a Minkowski space-time at $t \to +\infty$). According to whether the system proceeds along the right or the left branches (i. e., from $\sigma = 0$ to $\sigma_B^{(+)}$ or $\sigma_B^{(-)}$) of the figure, the cosmic evolution is driven by a Wiston or an anti-Wiston, respectively.

The appearance of the instanton-like configurations called Wistons is a direct consequence of the fundamental dynamical equation Eq. (15), in combination with the "structural" equation Eq. (12) which prescribes the degree of "Weylization" of space-time. Let us then demonstrate the equivalence between such "structural transitions" and quantum tunneling processes in models of quantum creation from "Nothing". Consider a generic Einstein equation for a Friedman scale factor,

$$\dot{a}^2 = - \varepsilon + 1/3\, \rho\, a^2 \; . \tag{4.27}$$

Coleman has shown[21] that a semi-classical description of a quantum tunneling event is given by the bounce solutions of Euclideanized field equations, i. e., of field equations in which the time parameter t is changed into (-it). Applying such Euclideanization procedure to Eq. (27), one obtains

$$\dot{a}^2 = + \varepsilon - 1/3\ \rho\ a^2 . \tag{4.28}$$

For instance, in the case of a closed (ε = +1) Universe driven by a (positive) cosmological constant $\Lambda = 3\zeta^2$, Vilenkin[4] made use of this approach to obtain, instead of the classical De Sitter solution Eq. (2.3), the solution

$$a_E(t) = (1/\zeta) \cos (\zeta t) , \tag{4.29}$$

corresponding to a De Sitter instanton - a "kink" configuration, propagating with negative classical energy, which bounces at the classical turning point $a = a_0 = (1/\zeta)$ - interpreted as representing the tunneling to classical De Sitter space Eq. (2.3) from "Nothing".

Now consider Einstein's equation Eq. (27) in the case of a closed Universe driven by the energy density $\rho = 3\ [a_0^4/a^6]$. Euclideanized Eq. (28) then gives

$$\dot{a}^2 = 1 - [a_0^4/a^4] . \tag{4.30}$$

But this is precisely the fundamental dynamical equation Eq. (15) of the present cosmological scenario. In this way, an equivalence is established between the Euclideanization of a *closed* Universe model driven by a *positive* energy density and a "structural transition" to a Weyl configuration which results in an *open* Universe model driven by a "stiff matter" state of *negative* energy. Just as in models of quantum creation the propagation of an instanton is seen to represent the tunneling of the Universe from a primordial quantum "Nothing" state, in the present scenario the propagation of a Wiston (i. e., a deviation of the Riemannian structure) is tantamount to the development of the Universe from a primordial empty Minkowski space[41].

As we have pointed out in Section II, it has been argued that solutions obtained through Euclideanization are in fact non-realistic, once they are to be interpreted as instantons, field configurations which tunnel across a classically forbidden region. Other authors supported, inversely, that such solutions correspond to an actual primordial phase of the cosmic evolution in which the basic Lorentzian nature of space-time is changed into an Euclidean one. According to the present reasonings, a different interpretation may be ascribed to these solutions, once an enlargement of the space-time structure to a Weyl configuration - in which the geometry is characterized by the pair $(g_{\mu\nu},\ \omega_{\lambda})$ of fundamental variables - supplies, at least in a particular case, the same basic behavior. It then becomes possible to conciliate the opposing interpretations of an "abstract soliton configuration"[23] and of a truly observable Euclidean cosmic phase[22]. Our WIST solution is observable, whereas its basic nature is always Lorentzian. It is the Riemannian character of space-time structure that results altered; allegorically, the opposition now has been changed: no longer Euclid versus Lorentz, but rather Riemann versus Weyl.

V. Matter and Entropy Production

V.1. Matter and entropy production in the standard context

Until now we have been studying structural and gravitational aspects implied by the basic assumption of the present model: that space-time geometry is dynamically determined by the interaction of a pair of fundamental geometrical components, namely, the metric $g_{\mu\nu}(x)$ and the WIST field $\omega(x)$ - which we assimilated to the geometrical background in the sense that its behavior effectively controls the affine character of space-time. According to the previous developments, the Universe evolved from scale fluctuations of a primordial Minkowski vacuum; by the same token, its matter content should also originate from dynamical processes involving the fundamental pair $(g_{\mu\nu}\ ,\ \omega)$. Let us then turn our attention to the material substance of the Universe.

Let us first remind the reader that matter (and entropy) creation in the standard context relies ultimately on the occurrence of an initial singularity. Consider, for instance, the standard HBB model in which the Friedman scale factor is given by $a(t) \sim t^n$ with $n < 1$. Thus the Hubble expansion is represented by a monotonic function of a very regular behavior, corresponding to a completely adiabatic configuration. The initial singular state, then, is the only occasion in which there exists a non-monotonic behavior able to engender matter and entropy. Strictly speaking, when the cosmic temperature is within a few orders of magnitude of the Planck temperature $T_p \sim 10^{32}$ K the Friedman expansion is fast enough in order to allow for the creation of particle - anti-particle pairs[42]. However, such mechanism cannot explain the observed asymmetry between matter and anti-matter (one of the aspects of the "baryon asymmetry problem" of standard cosmology[10]) without making appeal to unaccessible initial conditions issued at the singularity.

In inflationary scenarios, the energy density associated with the "inflaton" scalar field dominates the evolution of the Universe at primordial epochs of great condensation. In homogeneous models, the behavior of the inflaton field $\phi(t)$ is described by the evolution equation

$$\dot{\rho}_\phi + 3H\ (\rho_\phi + p_\phi) = 0\ , \qquad (5.1)$$

in which $(\rho_\phi,\ p_\phi)$ are the energy density and the isotropic pressure associated to the inflaton and $H = [\dot{a}/a]$ is the Hubble parameter. In this case, couplings of the inflaton field $\phi(t)$ to other fields may give rise, via vacuum excitations, to particle generation, once as the inflaton oscillates its energy can be converted to produce other particles. This effect can be taken into account through the addition of a term such as $(-\Gamma_\phi\ \dot{\phi}^2)$ to the energy conservation equation, where Γ_ϕ is the total decay width of the inflaton field[43]; in the case of, for example, the production of relativistic particles (e. g., photons), the evolution equations for the energy densities ρ_ϕ of the inflaton and ρ_γ of the photons become, respectively,

$$\dot{\rho}_\phi = -\ (3H + \Gamma_\phi)\ \mu\ \rho_\phi\ ,$$

$$\dot{\rho}_\gamma = -\ 4H\ \rho_\gamma\ +\ \mu\ \Gamma_\phi\ \rho_\phi\ , \qquad (5.2)$$

where use was made of the equations of state $p_\phi = (\mu - 1)\ \rho_\phi$ and $p_\gamma = 1/3\ \rho_\gamma$. Integrating these equations one obtains that if the value of the decay width Γ_ϕ is sufficiently large then the inflaton energy will be rapidly converted into photon energy, and its contribution to the total energy which drives the metric evolution will become negligible exponentially.

V.2. Thermodynamical comments

In the present scenario, on the other hand, one should expect the creation of matter and entropy to occur in the course of the non-adiabatical regime correlate to the reversion of the collapsing to the expanding phase (Section IV). Indeed, according to Quantum Field Theory in curved space-times the number of particles of a given species is an adiabatical invariant. This means that if the effects of the background geometry can be characterized as an infinitesimally slow thermodynamical process, then no particles are created by the gravitational field. This result was shown[42] in the case of field theories minimally coupled to gravity within the standard Riemannian context, and it may be generalized immediately to arbitrary affine configurations as far as the main properties of the equations of motion of test-fields are retained. Nevertheless, in the present approach these standard adiabaticity arguments must be reconsidered in view of the non-adiabatical phase taking place when the Universe bounces at the minimum radius a_0, once in this case the energy of the WIST field $\omega(t)$ could be converted into matter.

What could we say, on an intuitive basis, about particle production in the eternal Universe dominated by the WIST function $\omega(t)$? The theory of chemical reactions, for instance, offers a model of a mechanism in which the variation of the number N of particles of a given chemical species is controlled by the environment temperature T and by the number N_0 of such particles already existing, i. e., $\Delta N \sim N_0\ T$. From the standpoint of macroscopic Thermodynamics, in turn, taking into account the "stiff matter" behavior associate to the WIST field $\omega(t)$ a very naïve application of Gibb's law yields a temperature proportional to the inverse of the volume $V = a^3$, that is, $T \sim 1/a^3$. Combining these ideas, a rough estimate of the rate of particle creation in the course of the "structural transition" which characterizes the present scenario is given by

$$dN/dt = \text{const.}\ N/a^3\ . \tag{5.3}$$

Supposing that this is indeed the case, a straightforward use of solution Eq. (4.23) would give the total amount of particles produced in the present model. On the other hand, these considerations suggest the association of the WIST "energy" field $\dot{\omega}(t)$ to a thermal bath of temperature $T \sim a^{-3} \sim \dot{\omega}$ (note that once $\dot{\omega}(t)$ has a maximum at $(t = 0)$, temperature T is never divergent). As in conventional scalar field theory, this assumption leads to the induction of a Landau-type phase transition[44].

V.3. Particle creation in a WIST background

In order to provide more reliable arguments to support formula Eq. (3), however, we must consider once more the observed status of the known laws of physics with respect to point-dependent scale (conformal) transformations. According to the reasonings of Section III, in fact, convincing observational evidence indicates that physical quantities describing relevant properties of matter are *not* preserved under conformal mappings. This result is truly of importance once it guides us in the establishment of a gauge-independent methodology to generalize the Riemannian expressions of the laws of physics to the present WIST context. Indeed, following the standard prescription based on the Minimum Coupling Principle - which rules the extension of special-relativistic formulae, written in flat Minkowski space, to general-relativistic covariant expressions in curved space-time[20] - we will assume that in a WIST scenario the energy-momentum tensor $T^{\mu\nu}$, the entropy flux S^{μ} and the particle number current N^{μ} describing a given fluid satisfy the evolution relations

$$\begin{cases} T^{\mu\nu}{}_{;\nu} = 0 \\ S^{\mu}{}_{;\mu} = 0 \\ N^{\mu}{}_{;\mu} = 0 \end{cases} \qquad (5.4)$$

where, as stipulated in Section III, the semi-colon denotes covariant differentiation in a Weyl manifold. In the particular case of a Riemann configuration, i. e., when the WIST field $\omega(t)$ vanishes, these expressions reduce to the usual conservation laws of General Relativity; for example, the particle current N^{μ} obeys in this case $N^{\mu}{}_{\|\mu} = 0$, and thus the conservation of the particle number density $n = N/V$, in a Friedman background, follows as usual:

$$\dot{n} + 3n\,H = 0\ . \tag{5.5}$$

In the WIST case, on the other hand, Eq. (4) yields precisely the intuitive formula Eq. (3) for particle production, due to the dissipative effects induced by the presence of the WIST field $\omega(t)$. Indeed, according to to the formula Eq. (3.14) of WIST covariant differentiation, in the case of a relativistic fluid (photons) in a Friedman background the evolution relations Eq. (4) may be written as

$$\begin{cases} \dot{\rho} + 4H\rho - 3\dot{\omega}\rho = 0 \\ \dot{n} + 3Hn - 2\dot{\omega}n = 0 \\ \dot{s} + 3Hs - 2\dot{\omega}s = 0 \end{cases} \tag{5.6}$$

in which $s = S/V$ is the entropy density. We see that the WIST "energy" function $\dot{\omega}(t)$ plays the role of a (time-dependent) total decay width Γ_{ω} of the bosonic field $\omega(t)$ into photons, in the likeness of the inflationary case described by Eq. (2). Integrating Eq. (6) one obtains

$$\begin{cases} \rho = \rho_0 \; a^{-4} \; \exp \; [3\omega] \\ n = n_0 \; a^{-3} \; \exp \; [2\omega] \\ s = s_0 \; a^{-3} \; \exp \; [2\omega] \end{cases} \qquad (5.7)$$

where the symbol (o) denotes the values of small fluctuations of these magnitudes that supposedly occurred at some occasion in the past. It is interesting to observe that in spite of the fact that the present theory has two free parameters, namely, the dimensionless parameter ξ in the Lagrangian Eq. (4.2) and the minimum radius a_0 of Eq. (4.15), according to the Wiston solution Eq. (4.26) the efficiency of the mechanism of matter-entropy production represented in Eq. (7) is sensitive only to the value of ξ - besides, of course, the seminal input supplied by the original fluctuations.

V.4. The baryon asymmetry problem

A remarkable consequence of the introduction of dissipative effects induced by the WIST character of the space-time background is the exponential dependence of matter properties on the behavior of WIST field $\omega(t)$. Indeed, according to the solution Eq. (4.24) it follows that any fluctuation $(\Delta\psi)_0$ experienced by a given matter field ψ at the remote past is strongly damped in the course of the collapsing phase ($t < 0$); then there occurs a sudden transition from suppression to stimulation around ($t = 0$), and a equally strong amplification begins as the expanding phase takes place ($t > 0$). This production mechanism, however, saturates very rapidly, and for later times ($t \gg 0$) it becomes insignificant (note that these conclusions concern a Universe driven by a Wiston; in the case of an anti-Wiston, of course, this account shall be inverted). Thus, in distinction of other eternal, bouncing cosmologies, the infinite span of the contracting phase in the present model does not imply a boundless matter-energy production.

Due to the exponential damping of any primeval irregularity, only fluctuations taking place near (t = 0) do care for the subsequent evolution; but these fluctuations are exponentially amplified for a short period, so as to allow for arbitrarily large amounts of matter - e. g., particles - and entropy to be created. This period of intense creation is tantamount to a non-equilibrium process; notwithstanding this fact, after the amplification mechanism has been shut down one might expect the WIST field declining contribution to the source of Einstein's equations to be rapidly outmatched by the newly produced matter content. In this way, the primordial "stiff matter" state associated to the the "Big - but not infinite - Bang" described here could be straightforwardly continued to a standard sequence of radiation-dominated and matter-dominated phases; the addition of a standard inflationary phase, if required, is also not excluded.

The operation of this amplification mechanism also provides a fresh perspective with which the standard baryon asymmetry problem may be envisaged. It is well known that the prevalence of matter (e. g., baryons) against anti-matter in the observed Universe - as well as the observed ratio of entropy per baryon - is not explained in standard cosmology except with the use of fine-tuned initial conditions[45]. In the present scenario, on the other hand, an eventual baryon excess fluctuation $\Delta N_0 = (N_B - N_{\bar{B}})$ taking place shortly after the stage of maximum contraction at $(t = 0)$ may be exponentially increased up to a convenient amount, since in this case we have

$$\Delta N_B = \Delta N_0 \exp [2\omega] \ . \qquad (5.8)$$

While the production rate depends on the free parameter ξ only, the initial spectrum of fluctuations which become the subject of the amplification mechanism must bear a relationship to the relative size of the Universe - and thus to the minimum radius a_0. Elementary particle theory, for instance, requires the set of specific baryonic species contained in ΔN_0 to be regulated by the environment temperature T_E - which in the present non-singular scenario is always bounded (that is, $T_E \leq T_M \sim a_0^{-3}$). According to the value chosen for the minimum radius a_0, different species may be selected for amplification; conversely, a particle physics analysis could in principle yield an evaluation of this parameter. We shall leave these matters, though, to a subsequent investigation.

VI. Conclusion and Further Discussions

VI.1. The cosmic evolution

On the basis of the above results, the complete history of the cosmic evolution in the case of an homogeneous and isotropic metric configuration may be outlined as follows: due to Weylian scale fluctuations ruled by Eq. (2.5), a primordial empty Minkowski space-time begins to collapse at a remote past. This collapsing phase of indefinite duration is driven by the WIST field $\omega(t)$, whose effects are thermodynamically equivalent to a "stiff matter" state of a perfect fluid with energy density given by $\rho_\omega \sim a^{-6}$ (Eq. (4.11)). Throughout the collapse, the Universe is accelerated - or "inflationary". In agreement with the stability arguments discussed in Section IV, any eventual matter-energy fluctuation is exponentially suppressed in the course of the entire collapsing phase. This resembles the "memory loss" of inflationary scenarios, in which the problem of the singular origin may be circumvented due to the presence of an effective horizon limiting the present observational scope.[7] The collapse proceeds adiabatically in a very slow pace until a stage of greatest condensation - corresponding to the minimum a_0 of the cosmic radius - is approached. In fact, in the neighborhood of this maximally condensed stage the contraction is accelerated to an acme and then decreases suddenly, reverting to an expansion when the minimum radius a_0 is attained.

In the likeness of quantum creation models, the infinite collapsing phase of the present scenario may be associated to the propagation of a Weyl instanton - or "Wiston" - in an Euclideanized, classically forbidden region (Section IV); according to this interpretation, the Universe - as a classical entity - emerged from "Nothing", endowed with a minimum radius a_0, in a "stiff matter" state characterized by the absence of a matter content (e. g. , baryons and leptons), except for small fluctuations. However, as the Universe begins to expand, a non-adiabatical amplification mechanism starts to operate, driven by the energy of the WIST field $\omega(t)$, in such a way that matter-energy fluctuations may come to be converted, in an exponential rate, into large amounts of particles and radiation. An eventual baryon excess may be amplified in the same fashion. This "Big - but not infinite -Bang" stage lasts for a very short period, once the energy of the produced material soon dominates the energy of the WIST field; in this way, the Universe enters in the Friedmanian radiation-dominated and matter-dominated regimes which characterize the standard evolution.

VI.2. Concluding remarks

From an empirical point of view one could argue that the occurrence of a non-adiabatical cosmic stage is required by the observed existence of huge quantities of matter. However, according to each particular cosmological scenario, the details of matter production mechanisms can in principle be rather different. In effect, in order to excite a process in which enough entropy could be produced the standard HBB program makes appeal either to non-controlled initial conditions issued at the explosive beginning or, alternatively, to an intermediary instance provided by the inclusion of an inflationary era into the standard frame of the cosmic evolution.[7,10]

In the scenario proposed here, matter-entropy production appears as a natural consequence of a non-adiabatical regime driven by the brisk change from a collapsing phase to an expanding one, correlate to the maximal deviation from the Riemannian structure[46]. As we saw in Section V, a straightforward application of the Minimum Coupling Principle supplies a simple description of dissipative effects, induced by the WIST background, which appear in the evolution equations of matter fields. Similarly to inflationary approaches, these dissipative effects are expressed in terms of a (time-dependent) total decay width associated to the WIST field $\omega(t)$. Throughout the creation period, the environment temperature is never divergent; in this way, elementary particle problems - such as, for instance, quark confinement - can be addressed from a new angle.

With respect to the inclusion of a non-standard primordial "rigid matter" phase of the cosmic evolution, current trends in High Energy Physics suggest that the standard radiation-dominated era should be preceded by a primeval cosmic domain in which all fundamental interactions were unified. So far, the exact constituents of this unified state remain undetermined; nevertheless, given the circumstance that such unique mode of energy exchange should have a *global* character, in analogy with well-known issues of Field Theory[26] one is led to conceive that a "stiff" or "rigid matter" state, described by the ultra-relativistic equation of state $p = \rho$, might provide a suitable representation of this one-interaction configuration. In the present article we have shown that the hypothesis of a WIST background manifold leads to a dynamical scheme in which *geometry* itself accounts for the existence of a primordial "stiff" state[47].

In effect, in the present approach geometry generates *everything*. The conceptual cost to be paid for the election of one such unique physical matrix is, evidently, the widening of the traditional Riemannian space-time structure of General Relativity, once the geometrization of the basic inflaton-like field $\omega(t)$ implies the modification of the affine connections of the underlying manifold due to the contribution of ω-dependent terms; in turn, the behavior of the WIST "structural" function $\omega(t)$ is regulated, in a non-linear fashion, by the metric evolution (Section IV). The explicitly non-linear character of the system of fundamental dynamical equations describing the cosmic development, on the other hand, points to the most intriguing aspect put forth by the present WIST scenario: the equivalence of structural disturbances due to the curved WIST background (namely, the propagation of Wistons) to a semi-classical description of a *quantum* process.

Actually, Weyl spaces and quantum processes are not entirely foreign matters according to the literature. Since the early days of London[48], a curious connection of Weyl's length transport theory to certain aspects of quantum mechanics has been indicated, suggesting that quantum rules could be obtained from a classical formalism based on Weyl spaces. In other words, geometries in which length variation under transport may occur - such as Weyl's - seemingly constitute a well suited classical foundation upon which a successful interpretation of typical microscopic processes could be built. It has been shown, for instance, that the non-relativistic Schrödinger equation can be derived from a stochastic formulation in which quantum "forces" are due to curvature effects associated to the gauge vector ω_μ in a Weyl geometry. In this vein, quantum mechanical behavior would arise from a feedback relationship of the geometrical structure with dynamics.[49]

The analogies drawn in the text between Euclideanized solutions of quantum creation models and the present WIST theory, however, require further consideration. The dynamical activity of the WIST background is particularly pronounced close to the phase of greatest contraction, when the scale factor a(t) approaches the minimum value a_0. The enlargement of the geometry to a WIST configuration could then be ascribed, if this value is sufficiently small, to a first approximation of a quantum description of gravitational processes. The excitation of a Weyl structure would therefore represent the initial response of the background manifold to the structural transition from classical to quantum regimes.

In the manner of Utiyama[50], one could also consider the engaging - but rather speculative - possibility that WIST effects could become relevant on *microscopic* dimensions. In the likeness of the "vacuum bubbles" examined in some current approaches of quantum gravity theory[51], such "Weyl bubbles" would constitute microscopic domains endowed with a non-Riemannian internal structure, which could interact with an embedding Riemann manifold either by analytical continuation or by discontinuous jumps at the frontier (for instance, while current observations conclude that conformal invariance is broken at large scales, no similar statement has been achieved with respect to the microworld; thus, localized microscopic domains of Weylian character, embedded in a Riemann background, could in principle exhibit an invariant behavior[52]). In a certain sense, one such WIST domain (of an infinite extension, though), enclosed between two asymptotic Riemann configurations, is described in the present scenario. Albeit unclear the relation of quantum processes to WIST structures may be as of now, the equivalence - at least in a particular case - of Wiston propagation to Euclideanized semi-classical solutions noticed in Section IV certainly deserve further investigation.

In conclusion, the assumption of a WIST manifold leads to a cosmological solution describing an eternal Friedman-like open Universe which does not exhibit the usual difficulties of standard models, e. g., the singularity, horizon and flatness problems. In fact, the presence of the WIST background accounts for both the unstability of a primordial Minkowskian "Nothing" and the operation of a matter-entropy creation mechanism, so as to dispense with the cumbersome standard initial singularity and/or fine-tuned initial conditions. This geometrization procedure also supplies a *dynamical* explanation of the origin of the observed baryon excess over anti-baryons in the Universe today; it may be conjectured, furthermore, that the occurrence of a WIST-driven non-adiabatical phase could provide a suitable basis for the derivation of an appropriate primordial spectrum of density fluctuations in order to allow for galaxy formation. This subject, as well as other complementary aspects of the scenario discussed here, shall be addressed in a forthcoming study.

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VII. References and Notes

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$$\omega \to (\omega + \delta\omega) \ , \ a \to (a + \delta a)$$

to Eqs. (12, 13) one obtains

$$(\delta\omega)^{\cdot} = - (3\,\gamma/a^4)\ \delta a \ ; \ \delta\dot{a} \sim 4 \ [a_0^4/a^5\dot{a}]\ (\delta a).$$

Hence,

$$(\delta\omega)^{\cdot} = - (\dot{a}/\,\gamma\,\xi^2)\ a^3\ (\delta\dot{a}) \ , \ (\delta\dot{a})/\delta a \sim a^{-5}\ [a^4 - a_0^4\ ]^{-1/2}.$$

Far from a_0 (i. e., for large periods) we have $a >> a_0$; then,

$$(\delta\dot{a})/\delta a \sim a^{-7} \ , \ (da \sim dt),$$

so with $(\delta a)_i$ being the initial spectrum of perturbations one obtains

$$(\delta a) \sim (\delta a)_i \exp\,[a^{-6}] << (\delta a)_i \ .$$

The solutions of the system Eqs. (12, 15) are therefore stable against metric perturbations in the course of the infinite collapsing phase.

[40]. I. S. Gradshtein and I. M. Ryzhik, *"Table of Integrals, Series and Products"*, Academic Press, NY (1980).

[41]. *Note.* The fundamental dynamical equation Eq. (4.15) is also strongly reminiscent of Euclideanized field equations in axion cosmology, which are meant to represent the tunneling of an Euclidean wormhole to a Lorentz-signature space-time describing the further evolution of a newly

produced "baby universe"[9].

[42]. L. Parker, *Fund. Cosm. Phys.* **7**, 201 (1982), and references therein.

[43]. L. F. Abbott, E. Farhi and M. B. Wise, *Phys. Lett.* **117B**, 29 (1982).

[44]. *Note.* Indeed, the net effect of the interaction of a scalar test-field $\phi(x)$ with the Weyl field $\omega_\mu(x)$ is precisely equivalent to the conventional result [A. D. Linde, *Rep. Prog. Phys.* **42**, 25 (1979)] if the interpretation of $\dot{\omega}(x)$ as inducing a thermal bath of temperature T is acknowledged. Introducing in the Lagrangian L describing the dynamics of the field ϕ,

$$L = 1/2\ (\partial_\mu\ \phi)^2 - 1/2\ \alpha^2\ \phi^2 + 1/4\ \beta\ \phi^4\ ,$$

an interaction term such as

$$L_{(\phi\omega)} = \sqrt{-g}\ [\tau\ \omega_\mu \omega^\mu\ \phi^2 + \chi\ (\partial_\mu\ \phi)\ \omega^\mu],$$

we note that the in the homogeneous WIST case explored here - in which $\omega_\mu = \partial_\mu\ \omega = \dot{\omega}(t)\ \delta^0_\mu$ - the crossed term in the r. h. s. of the above equation does not contribute to the dynamics, once upon integration it leads to Eq. (4.8) and can thus be discarded. Setting $\dot{\omega}(t) = \nu\ T$, ν = constant, one obtains the effective Lagrangian

$$L = 1/2\ (\partial_\mu\ \phi)^2 - 1/2\ \alpha^2\ \phi^2 + 1/4\ \beta\ \phi^4 + \nu^2/8\ \ T^2\ \phi^2\ .$$

The non-trivial stable solution $\phi = \sigma$ = constant is then given by

$$\sigma^2 = \nu^2/4\beta\ (T_c^2 - T^2),$$

where $T_c = 2\alpha/\nu$. In consequence, when temperature T is greater than the critical value T_c stable non-trivial solutions $\phi = \sigma$ = constant cannot occur. Therefore, the association of a WIST background to a thermal bath does have the effect of inducing a Landau phase transition.

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[46]. A different approach to the entropy per baryon problem is suggested in M. Novello, L. A. R. Oliveira and J. M. Salim, *Class. Quantum Grav.* 7, 51 (1990).

[47]. The fact that the "stiff matter" phase has a *negative* energy should cause no dismay; in this connection, see J. V. Narlikar and T. Padmanabhan, *Phys. Rev.* D32, 1928 (1985).

[48]. F. London, *Z. Physik* **42**, 375 (1927); see also J. Godfrey, *Phys. Rev.* **52**, 1365 (1984).

[49]. E. Santamato, *Phys. Rev.* D29, 216 (1984).

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[51]. See, for instance, S. W. Hawking, I. G. Moss and J. M. Stewart, *Phys. Rev.* D26, 2681 (1982); for a general review of quantum gravity, see *"Quantum Theory of Gravity"*, S. M. Christensen, Ed.; Adam Hilger, Bristol, UK (1984).

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