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THE TRIVIALITY OF THE ABELIAN THIRRING QUANTUM FIELD MODEL

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ABSTRACT

By using a grassmanian polymer representation for the Fermionic functional determinant we argue the triviality of the vectorial four fermion interaction for space-time with dimensionality greater than two.

Key-words: Quantum field theory; Abelian Thirring model; Fermionic functional determinant.

One of the most interesting problem in D-dimensional Euclidean field theories is the appearance of a critical dimensionality where above this value the associated field theory becomes trivial ([1], [2]).

Our aim in this comment is to present the Parisi geometrical analysis [4] generalized to the fermionic case by analyzing the critical space-time dimension for the vectorial four fermion interaction (the Abelian Thirring model).

Let us start our analysis by considering the Thirring model Euclidean partition functional in $\mathbb{R}^D$ with the fermionic fields integrated out:

$$Z[g] = \int DA_\mu \exp\left(-\frac{1}{2}\int dx^D \; A_\mu^2(x)\right) \det \not{D}(A_\mu) \quad , \qquad (1)$$

where $\not{D}(A_\mu) \equiv \gamma_\mu(\partial_\mu + g A_\mu)$ is the Euclidean Dirac operator in the presence of the external auxiliar vectorial field and g is the bare theory's coupling constant.

We aim to show that $Z[g] = Z[g=0]$ when $D > 2$ since this result will lead (formally at least) to the triviality of eq. (1).

By using the fermionic loop representation for $\det \not{D}(A_\mu)$, as displayed in [3] we can write this functional determinant as a grassmanian path integral:

$$\det \not{D}(A_\mu) = \sum_{\{\chi_\mu^F(\xi,\theta)\}} \exp\left(\int_0^1 d\xi \int d\theta \; A_\mu(\chi_\mu^F(\xi,\theta)\; D\chi_\mu^F(\xi,\theta)\right)$$

$$= \sum_{\{\chi_\mu^F(\xi,\theta)\}} \int d^D x \; A_\mu(\chi) \; J_\mu^F(\chi_\mu^F(\xi,\theta)) \quad , \qquad (2)$$

where the sum $\sum_{\{\chi_\mu^F(\xi,\theta)\}}$ is defined by eq. (4) in [3] and $J_\mu^F(\chi_\mu^F(\xi,\theta))$ is the current associated to the Grassmanian loop $\chi_\mu^F(\xi,\theta) = \chi_\mu(\xi) + i\theta\psi_\mu(\xi)$, ($\theta^2 = 0$; $0 \le \xi \ge 1$). Througth a g-power series expansion and integrating the gaussian $A_\mu(\chi)$ functional integral we get, for instance, for its first coefficient $\left.\frac{dz[g]}{dg}\right|_{g=0} = z_1$, the following expression:

$$z_1 = \sum_{\{\chi^F_\mu(\xi,\theta)\}} \exp \frac{1}{2}\int_0^1 d\xi d\theta \int_0^1 d\xi' d\theta' \, D\chi^F_\mu(\xi,\theta)\,\delta^{(D)}(\chi^F_\mu(\xi,\theta) - \chi^F_\mu(\xi',\theta')$$

$$. \, D\chi^F(\xi',\theta') \tag{3}$$

We can understand eq. (3) as the partition functional associated to a gas of closed polymers $\{\chi^F_\mu(\xi,\theta)\}$ possessing a grassmanian structure and interacting among themselves by a self-avoiding interaction $\delta^{(D)}(\chi^F_\mu(\xi,\theta) - \chi_\mu(\xi',\theta'))$.

In order to argue the triviality of the fermionic polymer gas we follow Parisi [4] by assigning a Hausdorff dimension d_H for the "set" $\{\chi^F_\mu(\xi,\theta);\ \theta^2=0;\ 0 \le \xi \le 1\}$. A natural Hausdorff dimension for this set is given by the exponent of the fermion free field propagator in the momentum space which is 1, so $d_H(\{\chi^F_\mu(\xi,\theta)\} = 1$.

By using now the geometrical intersection rule $d_H(A \cap B) = d_H(A) + d_H(B) - D$ ([4]), with D being the space-time dimensionality we obtain that the support set of the self-avoiding interaction $\delta^{(D)}(\chi_\mu(\xi,\theta) - \chi_\mu(\xi',\theta'))$ has a negative Hausdorff dimension for $D > 2$ which means that this set is empty.

As a consequence we have the analitical relation

$$\int_0^1 d\xi d\theta \int_0^1 d\xi' d\theta' \, D\chi^F_\mu(\xi,\theta)\,\delta^{(D)}(\chi^F_\mu(\xi,\theta) - \chi^F_\mu(\xi',\theta'))D\chi^F_\mu(\xi',\theta') = 0 \quad , \tag{4}$$

which means by its turn theory's triviality, since this argument can be straight forwardly applied for any arbitrary coefficient z_n and leading to the result $z_n = z_1$.

Finally we remark that by reformulating the Thirring theory in the loop space we can in principle define the theory for any general manifold m as space-time by only including the constraint $\{\chi_\mu^F(\xi,\theta)\} \subset m$ in the path-integral eq. (3). Note that m may be fluctuating [5].

Work in this direction is in progress and will appear elsewhere.

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