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BRST INVARIANT CLOSED THREE REGGEON (STRING) VERTEX

by

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__Abstract__: We construct a BRST invariant three Reggeon vertex for a closed bosonic string. The result is a simple functional of matter and ghost fields.

Key-words: Field theory; Dual models; String theory.

A BRST invariant formulation of the three string vertex can be of interest both as a starting point in the operatorial construction of BRST invariant loop amplitudes[1,2,3,4] and in string field theory[2,3,5].

In this letter we extend the three Reggeon vertex of Sciuto[6]-Della Selva-Saito[7] for the case of the closed bosonic string and construct its BRST invariant form.

As in the case of the open string[3] the result is a simple functional of the string coordinates $X_\mu(z)$, $\overline{X}_\mu(\overline{z})$ and of the ghost fields $b(z)$, $\overline{b}(\overline{z})$, $c(z)$ and $\overline{c}(\overline{z})$.

To establish the notations let us introduce the string position operator $X^\mu(z,\overline{z})$ solution with boundary condition $X(\tau,0) = X(\tau,\pi)$

$$X^\mu(z,\overline{z}) = X^\mu(z) + \overline{X}^\mu(\overline{z}) \quad ; \quad z = e^{2i(\sigma+\tau)} \quad , \quad \overline{z} = e^{2i(\sigma-\tau)} \tag{1}$$

with

$$\begin{aligned} X^\mu(z) &= \frac{1}{2}\left[q^\mu - \frac{i}{2}\,p^\mu \ln z + i\sum_{n\neq 0}^{\infty}\frac{1}{n}\,\alpha_n^\mu\, z^{-n}\right] \\ \overline{X}^\mu(\overline{z}) &= \frac{1}{2}\left[q^\mu - \frac{i}{2}\,p^\mu \ln\overline{z} + i\sum_{n\neq 0}^{\infty}\frac{1}{n}\,\overline{\alpha}_n^\mu\, \overline{z}^{-n}\right] \end{aligned} \tag{2}$$

The canonical commutation relations are:

$$\begin{aligned} &[\alpha_m,\alpha_n^\nu] = [\,\overline{\alpha}_m^\mu,\overline{\alpha}_n^\nu] = m\, g^{\mu\nu}\,\delta_{m,-n} \\ &[\alpha_m^\mu,\overline{\alpha}_n^\nu] = 0 \qquad \text{and} \qquad [q^\mu,p^\nu] = ig^{\mu\nu} \quad . \end{aligned} \tag{3}$$

The vacuum of the Fock space generated by α_n and $\overline{\alpha}_n$ is annihilated by α_n and $\overline{\alpha}_n$ with $n \geq 0$ $(\alpha_0 = \overline{\alpha}_0 = p)$; then normal order is defined and we get:

$$\underbrace{X^\mu(z)X^\nu(w)} = \frac{g^{\mu\nu}}{4}\left[\frac{1}{2}\ln z - \ln(z-w)\right] \quad ;$$

$$\underbrace{\overline{X}^\mu(\overline{z})\overline{X}^\nu(\overline{w})} = \frac{g^{\mu\nu}}{4}\left[\frac{1}{2}\ln\overline{z} - \ln(\overline{z}-\overline{w})\right] \quad ; \tag{4}$$

$$\underbrace{\overline{X}^\mu(z)X^\nu(w)} = -\frac{g^{\mu\nu}}{4}\frac{1}{2}\ln\overline{z} \quad ; \quad \underbrace{X^\mu(z)\overline{X}^\nu(\overline{w})} = -\frac{g^{\mu\nu}}{4}\frac{1}{2}\ln z \quad ,$$

In the first quantized version of the closed bosonic string theory the conditions defining the physical states can be more conveniently expressed by the single condition[9]

$$Q_{BRST}|\text{phys}\rangle = 0 \tag{5}$$

where Q_{BRST} is the nilpotent (in 26 dimensions) BRST operator for the closed string, acting on a Fock space enlarged by the introduction of ghosts and antighosts.

Following the usual steps[9,10] it is easy to construct the BRST charge for this case we find:

$$Q_{BRST} = Q + \overline{Q}$$

$$Q = 2\oint_0 dz \; :[c(z)T(z)-b(z)c'(z)c(z)]: \quad ^{(*)} \tag{6}$$

$$\overline{Q} = 2\oint_0 d\overline{z} \; :[\overline{c}(z)\overline{T}(\overline{z})-\overline{b}(\overline{z})\overline{c}'(\overline{z})\overline{c}(\overline{z})]:$$

(*) In (6) and in the following we assume that the symbol $\oint$ contains a factor $1/2\pi i$.

where

$$T(z) = -2\, X'(z).X'(z)$$
$$\overline{T}(\overline{z}) = -2\, \overline{X}'(\overline{z}).\overline{X}'(\overline{z}) \qquad (7)$$

with the prime denoting the derivative with respect to z, and $b(z)$, $\overline{b}(\overline{z})$, $c(z)$ and $\overline{c}(\overline{z})$ are the Fadeev-Popov ghost fields:

$$b(z) = \sum_{-\infty}^{\infty} b_n\, z^{n-2} \quad ; \quad \overline{b}(\overline{z}) = \sum_{-\infty}^{\infty} \overline{b}_n\, \overline{z}^{n-2} \quad ;$$
$$c(z) = \sum_{-\infty}^{\infty} c_m\, z^{-m+1} \quad ; \quad \overline{c}(\overline{z}) = \sum_{-\infty}^{\infty} \overline{c}_m\, \overline{z}^{-m+1} \qquad (8)$$

with conformal weights $\Delta_b = \Delta_{\overline{b}} = 2$, $\Delta_c = \Delta_{\overline{c}} = -1$; anti-commutation relations:

$$[c_n, b_m]_+ = \delta_{m,-n}$$
$$[\overline{c}_n, \overline{b}_m]_+ = \delta_{m,-n} \qquad (9)$$
$$[c_n, c_m]_+ = [b_n, b_m]_+ = [\overline{c}_n, \overline{c}_m]_+ = [\overline{b}_n, \overline{b}_m]_+ = [c_n, \overline{b}_m] = $$
$$= [\overline{c}_n, b_m]_+ = 0$$

and hermicity properties:

$$c_n^\dagger = c_{-n} \quad ; \quad \overline{c}_n^\dagger = \overline{c}_{-n} \quad ; \quad b_n^\dagger = b_{-n} \quad ; \quad \overline{b}_n^\dagger = \overline{b}_{-n} \, . \qquad (10)$$

A vacuum state invariant under the projective algebra is obtained by imposing[11]

$$c_m|0\rangle_{gh} = \overline{c}_m|0\rangle_{gh} = b_n|0\rangle_{gh} = \overline{b}_n|0\rangle_{gh} = 0 \qquad \text{for} \begin{cases} m=2,3,4,\ldots \\ n=-1,0,1,\ldots \end{cases} \qquad (11)$$

as a consequence we get

$$_{gh}\langle 0|0\rangle_{gh} = {}_{gh}\langle 0|[c_1,b_{-1}]_+\,|0\rangle_{gh} = {}_{gh}\langle 0|[\bar{c}_1,\bar{b}_{-1}]_+|0\rangle_{gh} = 0 \quad (12)$$

with the non trivial scalar product[11]

$$_{gh}\langle 0|c_{-1}c_0c_1\bar{c}_{-1}\bar{c}_0\bar{c}_1|0\rangle_{gh} = 1 \quad (13)$$

Let us now construct an object depending on two sets of oscillators 1 and 2 giving the vertex for the emission of a state "α" of the closed string on the oscillators 1, when saturated by the state $|\alpha,k\rangle_{(2)}$. Explicitly we search for an operator $W_{1,2}$ such that:

$$_{(2)}\langle q^\mu = 0,0|W_{1,2}|\alpha,k\rangle_{(2)} = V_\alpha^{(1)}(1;k) \quad (14)$$

with $V_\alpha^{(1)}(1;k)$ the vertex for the emission of the state α of the closed string defined on the set of oscillators 1, and $_{(2)}\langle q^\mu = 0,0|$ is the vacuum with vanishing value of the center of mass variable $q^{\mu(2)}$, i.e.

$$(q^{(2)},\alpha_n^{(2)},\bar{\alpha}_n^{(2)})|0,q^\mu{=}0\rangle_{(2)} = 0 \qquad \text{for} \quad n > 0 \ . \quad (15)$$

An operator that satisfies (14) is given by:

$$W_{12} = :\exp J: \quad (16)$$

with

$$J = -4 \oint_0 dw X^{(1)}(1-w) . X'^{(2)}(w) -$$

$$- 4 \oint_0 d\bar{w} \bar{X}^{(1)}(1-\bar{w}) . \bar{X}'^{(2)}(\bar{w}) \qquad (17)$$

with the fields X and $\bar{X}$ defined in (2), and the contour of integration encircling $w = 0$ and $\bar{w} = 0$ respectively, which is the natural extension of the Sciuto[6], Della Selva, Saito[7] vertex.

By performing the integration in (17)

$$J' = ip^{(2)} . \left[X^{(1)}(1) + \bar{X}^{(1)}(1)\right] + 2i \sum_{n>0} \frac{1}{n!} \left[\alpha_n^{(2)} . \partial^{(n)} X^{(1)}(1) + \bar{\alpha}_n^{(2)} . \partial^{(n)} \bar{X}^{(1)}(1)\right] \qquad (18)$$

Saturating with the tachyon state of momentum k in the set of oscillators (2) we get:

$$S_{DSS}|0,k\rangle_{(2)} = {}_{(2)}\langle q^{\mu}=0,0|W_{12}|0,k\rangle_{(2)} = : e^{ik.X^{(1)}(1)} . e^{ik.\bar{X}^{(1)}(1)} : \qquad (19)$$

which is the vertex for the emission of a tachyon from the closed string. Similar analysis can be done for the graviton and the states of higher level of the closed string.

Because of the nilpotency of Q_{BRST} the physical states are cohomology classes of the BRST operator Q_{BRST}. Then the process, described by:

$${}_{(1)}\langle\psi|S_{DSS}|\eta\rangle_{(2)}|\chi\rangle_{(1)}$$

must not depend on which representative is chosen on a cohomology class. This requirement is fulfilled if the vertex S_{DSS} is BRST invariant, that is if:

$$[Q^{(1)}_{BRST}, S_{DSS}] = S_{DSS}\, Q^{(2)}_{BRST} \quad . \tag{20}$$

This ensures that the coupling of the string states depends only on the cohomology class of the BRST operator Q_{BRST} and not on the particular representative chosen within each cohomology class.

In order to satisfy eq. (20) we must extend S_{DSS} given by (16), (17) and (19) introducing conveniently the ghost and antighost fields. It turns out that the correct expression is:

$$S_{DSS} = \langle q^{\mu}=0;0;N_{gh}=\bar{N}_{gh}=3| :\exp J:$$

$$J = \oint_{w=0} dw \left[-4x^{(1)}(1-w).x'^{(2)}(w) - c^{(1)}(1-w)b^{(2)}(w) + b^{(1)}(1-w)c^{(2)}(w)\right] +$$
$$+ \oint_{\bar{w}=0} d\bar{w}\left[-4\bar{x}^{(1)}(1-\bar{w}).\bar{x}'^{(2)}(\bar{w}) - \bar{c}^{(1)}(1-\bar{w})\bar{b}^{(2)}(\bar{w}) + \bar{b}^{(1)}(1-\bar{w})\bar{c}^{(2)}(\bar{w})\right] \tag{21}$$

where the bra denotes the vacuum for the bosonic oscillators and the state with ghost number $N_{gh} = \bar{N}_{gh} = 3$, i.e.

$$|N=\bar{N}=3\rangle = \bar{c}_{-1}\bar{c}_{0}\bar{c}_{1}c_{-1}c_{0}c_{1}|N=\bar{N}=0\rangle \tag{22}$$

with $|N=\bar{N}=0\rangle$ both BRST and projective invariant.

The proof of the BRST invariance of S_{DSS} given in (21) and (22) goes exactly in the same way of ref. [3]. By the use of (4), (8) and (9), integrating over z the O.P.E of $j(z)$ and :exp J: where:

$$Q = \oint_{z=0} dz\ j(z) \tag{23}$$

with Q defined in (6) we get

$$[Q^{(2)}, :\exp J:] = I_1 + I_2 \tag{24}$$

where

$$I_1 = -4 \oint_{w=0} dw{:}e^J\{c^{(1)}(1-w)x'^{(2)}(w).x'^{(2)}(w)+2c^{(2)}(w)x'^{(2)}(1-w).$$
$$.x'^{(2)}(w) - 2c^{(1)}(1-w)x'^{(1)}(1-w).x'^{(2)}(w)-c^{(2)}(w)x'^{(1)}(1-w).$$
$$.x'^{(1)}(1-w)\}: \tag{25}$$

and

$$I_2 = 2\oint_{w=0} dw{:}e^J\{b^{(1)}(1-w)c'^{(2)}(w)c^{(2)}(w) + b^{(2)}(w)c'^{(1)}(1-w)c^{(2)}(w)-$$
$$- b^{(2)}(w)c'^{(2)}(w)c^{(1)}(1-w) + b^{(1)}(1-w)c'^{(1)}(1-w)c^{(2)}(w) -$$
$$- b^{(1)}(1-w)c'^{(2)}(w)c^{(1)}(1-w) - c'^{(1)}(1-w)c^{(1)}(1-w)b^{(2)}(w)\}: \tag{26}$$

Proceeding in the same way with $Q^{(1)}$ and with $\bar{Q}^{(1,2)}$ we get:

$$_{(2)}\langle q^\mu=0;0;N_{gh}=\bar{N}_{gh}=3|[Q^{(1)}_{BRST} + Q^{(2)}_{BRST}, :\exp I:] = 0 \tag{27}$$

or

$$[Q^{(1)}_{BRST}, S_{DSS}] = S_{DSS}\, Q^{(2)}_{BRST}$$

which shows the BRST invariance of S_{DSS}.

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