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USING REDUCE IN SUPERSYMMETRY

by

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We describe a procedure which allows one to do Supersymmetry calculus in REDUCE in a quite trivial way.

Key-words: Algebraic computation; Supersymmetry; Reduce.

1. Introduction

Supersymmetry is a symmetry which relates the two classes of elementary particles known by the physicists as the fermions and the bosons[1].

Global N=1 Supersymmetry is based on the following algebra (called "graded" Lie algebra):

$$
\begin{aligned}
&[P_\mu, P_\nu] = [P_\mu, Q_\alpha] = [P_\mu, \bar{Q}_{\dot\alpha}] = 0 \quad (\mu, \nu = 0, \ldots, 3) , \\
&\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2\sigma^\mu{}_{\alpha\dot\beta} P_\mu \quad (\alpha, \beta = 1, 2 \ ; \ \dot\alpha, \dot\beta = \dot1, \dot2) ,
\end{aligned} \tag{1}
$$

where P_μ is the energy-momentum generator, Q_α is the two-component Weyl spinor Supersymmetry generator, $\bar{Q}_{\dot\alpha}$ its complex conjugate and σ^μ are the 2x2 Pauli spin matrices.

In 1975, Salam and Strathdee[2] introduced an eight-dimensional "superspace" spanned by the four space-time coordinates x^μ and four anticommuting (spinorial) coordinates θ^α, $\bar\theta_{\dot\alpha}$ over which the supersymmetry generators can be represented by differential operators

$$
\begin{aligned}
iQ_\alpha &= \partial_\alpha - i\sigma^\mu{}_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu , \\
i\bar{Q}^{\dot\alpha} &= \partial^{\dot\alpha} + i\theta^\beta\sigma^\mu{}_\beta{}^{\dot\alpha}\partial_\mu .
\end{aligned} \tag{2}
$$

They introduced also "superfields", which represent an entire supermultiplet of particles that transform among themselves. The superfield depends on the coordinates of the superspace and, since θ^α and $\bar\theta_{\dot\alpha}$ anticommute, has a finite Taylor series expansion in them. The coefficients of this expansion are ordinary fields and describe the particles individually.

A scalar chiral superfield may, for example, be written as

$$
\Phi(x, \theta, \bar\theta) = e^{i\theta\sigma\bar\theta} [A(x) + \sqrt{2}\theta^\alpha\Psi_\alpha(x) + \theta^2 F(x)] , \tag{3}
$$

where A(x), Ψ(x), F(x) are a scalar (bosonic), a Majorana spinorial (fermionic), and an "auxiliary" scalar field, respectively.

With these superfields, one can define the covariant derivatives with respect to Supersymmetry as

$$
\begin{aligned}
D_\alpha &= \partial_\alpha + i\sigma^\mu{}_{\alpha\dot\beta}\bar\theta^{\dot\beta}\partial_\mu , \\
\bar{D}^{\dot\alpha} &= \bar\partial^{\dot\alpha} - i\theta^\beta\sigma^\mu{}_\beta{}^{\dot\alpha}\partial_\mu .
\end{aligned} \tag{4}
$$

Of course one can work in Supersymmetry without using superfields, doing the calculations with the component fields, but superfields allow a great economy of work, compactness and elegance.

In dealing with calculations of this kind, one has to multiply superfields referring to different points of superspace, counting powers of θ and $\bar{\theta}$, and deleting all terms with excessive powers of them.

In counting powers, one has also to bear in mind various properties such as

$$(\theta\theta')(\theta\theta') = (\theta^{\alpha}\theta'_{\alpha})(\theta^{\beta}\theta'_{\beta}) = -\tfrac{1}{2}(\theta^{\alpha}\theta_{\alpha})(\theta'^{\beta}\theta'_{\beta}) \qquad (5)$$

(θ and θ' referring to two different points of superspace).

For the sake of compactness, we shall use the following abbreviated notation:

$$\theta^2 = \theta^{\alpha}\theta_{\alpha} = \theta^1\theta_1 + \theta^2\theta_2 = \theta_2\theta_1 - \theta_1\theta_2 = -2\theta_1\theta_2 \, . \qquad (6)$$

2. Computational considerations

From the above description, one can see that the workers on Supersymmetry often find themselves involved with long and tedious though straightforward calculations. There an algebraic manipulator like REDUCE can be of much help.

We have developed this work making use of the version 2 of REDUCE, in which there is no way to attribute an anticommutative character to a given variable.

REDUCE, as a very general algebraic system, has a great flexibility and we succeeded in simulating spinors by means of a judicious combination of the "vector" facility(2) and LET statement.

The vectors in REDUCE (with undefined components, having a internal product indicated by ".") were introduced to represent momentum quadrivectors for high energy calculus, and one can also attribute a mass to each of it such that the substitution

$$\text{vector.vector} = \text{mass}^2 \qquad (7)$$

can be done at the user's will.

Here we have defined variables such as TE, TEP and TEB as "vectors" (regardless of the dimension of the underlying space) in order to represent θ, θ' and $\bar{\theta}$, respectively. We have also defined the operator SIG to represent σ matrices in the following way (in momentum space)

$$\theta\sigma\bar{\theta} \;=\; \theta^{\alpha}\sigma^{\mu}{}_{\alpha\beta}\bar{\theta}'^{\beta}p_{\mu} \;=\; \text{SIG(TE, P, TEPB)} \qquad (8)$$

The desired properties of θ, $\bar{\theta}$ and σ were introduced by a number of LET statements like

```
FORALL T,TP LET (T.TP)*(T.TP) = - (T.T)*(TP.TP)/2; ,
```

corresponding to (5) above, and

```
FORALL T,TB,Q LET SIG(T,Q,TB)*SIG(T,Q,TB) =
                        -(T.T)*(TB.TB)*Q.Q/2;
```

to

$$(\theta^{\alpha}\sigma^{\mu}{}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}q_{\mu})(\theta^{\beta}\sigma^{\nu}{}_{\beta\dot{\beta}}\bar{\theta}^{\dot{\beta}}q_{\nu}) = -1/2\ (\theta^{\alpha}\theta_{\alpha})(\bar{\theta}_{\dot{\beta}}\bar{\theta}^{\dot{\beta}})q.q \quad . \qquad (9)$$

Note that since the "." operation is different from "*", as the first refers to vectors and the second to scalars, the order of the factors in any expression like that which appears on the left side of the first LET statement is not altered without control.

To implement all the needed properties of anticommutativity we attributed a "mass" to each vector variable defined and included statements like

```
T1**3 := 0;

FORALL T LET T1**2*(TE1.T) = 0;

FORALL TB,Q LET SIG(TE1,Q,TB)*T1**2 = 0;
```

where we have previously declared T1 as the "mass" for the TE1 "vector" through the appropriate MASS statement.

Now, with a complete set of statements, the multiplication of θ-quantities is quite trivial. For the integration over the θ-parameter, we have made use of the fact that this operation turns out to be in practice a differentiation, that is,

$$\int W(\theta) d^2\theta \quad = \quad \tfrac{1}{2}\partial^2 W(\theta)/\partial\theta^2 \quad = \quad 1/2*DF(W,T1,2)\ , \qquad (10)$$

where the ½ factor comes from the definition of $d^2\theta$ and DF is the REDUCE differentiation operator.

With this technique we performed all the calculations of the two-loop effective potential for the Wess-Zumino supersymmetric model. This involves the evaluation of three two-loop Feynman superdiagrams with quite complicated propagators[3], from which an example is given below:

$$\tilde{\Gamma}^{(2)}_{\Phi\Phi} = -1/3 \int d^2\theta\ d^2\theta'\ g_{ijk}\ g_{lmn}$$

$$\Delta_{\Phi\Phi}(\theta,\theta')\ \Delta_{\Phi\Phi}(\theta,\theta')\ \Delta_{\Phi\Phi}(\theta,\theta') \qquad (11)$$

with

$$\Delta_{\Phi\Phi} = i \int \frac{d^4p}{(2\pi)^4} \left[A(p) + B(p)\theta^2 + C(p)\theta'^2 + D(p)\theta\theta' + E(p)\theta^2\theta'^2 \right] e^{-\theta \not p \theta} \, e^{\theta' \not p \theta'} \, e^{ip.(x-x')} \qquad (12)$$

In the version 3 of REDUCE one can do that through the declaration NOCOM, but if one tries to use this to give an anticommutative character to θ_1 and θ_2, one has no way to maintain the compact notation (6) above, obtaining the explicit summation instead.

It should be noted that this simple technique does not allow one to do all the covariant derivatives algebra or other more deep supersymmetric calculations. However, in our view, it could be of help to those who are involved in similar calculus.

We feel that this could be a motivation for a theoretical supersymmetry physicist to be "converted" to (algebraic) computation. In fact, this was our case.

3.Acknowledgments

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4. References

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