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HELICITY FORMALISM AND SPIN EFFECTS

by

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ABSTRACT

The helicity formalism and the technique to compute amplitudes for interaction processes involving leptons, quarks, photons and gluons are reviewed. Explicit calculations and examples of exploitation of symmetry properties are shown. The formalism is then applied to the discussion of several hadronic processes and spin effects: the experimental data, when related to the properties of the elementary constituent interactions, show many not understood features. Also the "nucleon spin problem" is briefly reviewed.

Key-words: Spin; Helicity formalism; Hadronic processes; Nucleon spin problem.

Introduction

In high energy particle physics hadronic interactions should be described in terms of interactions amongst constituents, quarks and gluons; such interactions can be computed in the framework of perturbative QCD. Whereas in inclusive interactions the hadronic cross-sections are obtained by summing the elementary cross-sections, to describe exclusive processes one must coherently sum the elementary amplitudes to obtain the hadronic ones. It is then crucial to master a technique to compute amplitudes for quark and gluon interactions; in fact, to make physical predictions for each given exclusive process involving composite hadrons in the high energy region, one must know *all* contributing amplitudes, *including* their relative phases.

In this paper we first review the helicity technique to compute amplitudes for lepton, quark, photon and gluon interactions; the emphasis will be on explicit expressions for helicity spinors and polarization vectors, with detailed examples of computations. The properties of the amplitudes under parity, time-reversal and crossing transformations will also be discussed and used in particular cases.

We will then present a critical analysis of many recent spin effects in exclusive hadronic interactions, which pose a severe challenge to all existing theories. We discuss them in the language of helicity amplitudes, which are written in terms of the elementary ones, trying to relate the spin effects to the nature of the elementary constituents and their couplings.

There exist in the literature some excellent review works on spin and spin physics [1.1-3], but none of them deals with the technical details necessary to actually compute amplitudes; as a consequence the helicity technique is not widely known. Also, the majority of elementary particle physicists are not familiar with the problems encountered when dealing with polarized hadronic interactions and with the difficulties of explaining most spin effects. We trust that our paper will fill this gap in the literature.

The plan of the work is as follows. In Chapter 1 we recall the helicity formalism and give the explicit expressions of helicity spinors and polarization vectors. Their relationship with the usual canonical states and some useful formulae are collected in Appendix A. In Chapter 2 we give an explicit example of amplitude calculations using the helicity technique and show how to exploit parity, particle exchange, time-reversal and crossing transformation properties. In Chapter 3, as an application of the helicity formalism, we discuss many spin effects in exclusive hadronic interactions; the properties of the constituent amplitudes are shown to be in apparent contradiction with several observed spin effects. Finally, in Chapter 4, we give a brief review of another spin problem involving a composite hadron, which is still being much debated, the so called "proton spin problem".

Throughout the paper we will be using the same metric conventions and the same Dirac γ matrices representation as in Bjorken&Drell, "Relativistic Quantum Mechanics" (McGraw Hill, 1964).

1 - Helicity formalism

As we said in the Introduction we often need, when computing high energy interactions amongst hadrons, to sum all the amplitudes describing the constituent interactions contributing to the same process. We have then to compute expressions like

$$\overline{\Psi}(p';s')\,\Gamma\,\Psi(p;s) \tag{1.1}$$

where $\Psi(p;s) = u(p;s), v(p;s)$, the Dirac spinors for quarks or antiquarks with four-momentum p and covariant spin vector s. In such a case we cannot resort to the familiar trace technique, useful when computing cross-sections

$$\begin{aligned}\sum_{s,s'} \left[\overline{\Psi}(p';s')\,\Gamma\,\Psi(p;s)\right]^* \left[\overline{\Psi}(p';s')\,\Gamma\,\Psi(p;s)\right] = \\ = Tr\left[(\not p \pm m)\,\bar{\Gamma}\,(\not p' \pm m)\,\Gamma\right]\end{aligned} \tag{1.2}$$

where $\pm m$ refers to quarks and antiquarks of mass m respectively and $\bar{\Gamma} = \gamma^0\Gamma^\dagger\gamma^0$.

Analogously, Γ in Eq.(1.1) might contain products of gluon or photon polarization vector components , $\epsilon^{\alpha}(k,\lambda)\epsilon^{\beta}(k',\lambda')$. Again, we cannot exploit the usual prescription, used when computing unpolarized cross-sections (in the Feynman gauge)

$$\sum_{\lambda} \epsilon^{\alpha}(k,\lambda)\epsilon^{*\alpha'}(k,\lambda) = -g^{\alpha\alpha'} \tag{1.3}$$

One simple way of handling expression like (1.1) is that of using an explicit particular representation for the spinors and the polarization vectors. The representation we will be using is the helicity one [1.4,5], which is particularly suitable to describe high energy spin 1/2 particle and massless spin 1 bosons.

For a complete definition of helicity states we refer the readers to the original Jacob and Wick paper [1.4], or, e.g., to Ref.[1.1]. We will be following the same conventions as in Ref.[1.1]. Let us simply recall here the main points and give, for practical purposes, the explicit expressions of the helicity spinors and polarization vectors.

1.1 Helicity Dirac spinors

Let us start from the Dirac spinors for a particle at rest with mass m and spin quantized along the z axis

$$|1/2, s_z> \equiv u_{s_z}(m,\vec{0}) = \sqrt{2m}\begin{pmatrix} 1/2+s_z \\ 1/2-s_z \\ 0 \\ 0 \end{pmatrix} \tag{1.1.1}$$

The helicity states can be built from the canonical states at rest in two different ways. One may first rotate the rest states so that the quantization axis is along the $\hat{p}(\theta,\varphi)$ direction and then boost the system along $\hat{p}$; or, equivalently, one may first boost the rest states along the z-axis and then rotate the system to the $\hat{p}$ direction. By performing either of the above successions of operations on the states (1.1.1) one gets the helicity spinors for a spin 1/2 particle moving along $\hat{p}$:

$$u_{\lambda}(p) = \frac{1}{N}\begin{pmatrix} E+m \\ 2|\vec{p}|\lambda \end{pmatrix} \mathcal{D}^{1/2}(\theta,\varphi)\chi_{\lambda} \tag{1.1.2}$$

where $p = (E, \vec{p}), \hat{p} = \vec{p}/|\vec{p}|$, and $N = \sqrt{m+E}$ is the normalization factor such that $\bar{u}u = 2m$. The matrices $\mathcal{D}^j(\theta, \varphi)$, showing the transformation properties under rotations of spin j states, are given explicitly (for $j = 1/2$ and $j = 1$) in the Appendix. Finally

$$\chi_\lambda = \begin{pmatrix} 1/2+\lambda \\ 1/2-\lambda \end{pmatrix} \quad , \tag{1.1.3}$$

and the non relativistic states

$$\begin{aligned} \mathcal{D}^{1/2}(\theta,\varphi)\chi_\lambda &= \sum_{\lambda'} \mathcal{D}^{1/2}_{\lambda'\lambda}(\theta,\varphi)\chi_{\lambda'} \\ &\equiv \chi^{\hat{p}}_\lambda \end{aligned} \tag{1.1.4}$$

represent spin 1/2 particles with spin projection λ along the $\hat{p}(\theta, \varphi)$ direction,

$$\vec{\sigma} \cdot \hat{p}\chi^{\hat{p}}_\lambda = 2\lambda\chi^{\hat{p}}_\lambda \tag{1.1.5}$$

where the $\vec{\sigma}$'s are the usual Pauli matrices. It is then immediate to check that

$$\hat{p} \cdot \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} u_\lambda(p) = 2\lambda u_\lambda(p) \tag{1.1.6}$$

showing that, indeed, the helicity spinors are eigenfunctions of the four-dimensional helicity operator (spin projection into the direction of motion) with eigenvalues 2λ.

It is also easy to see that Eq.(1.1.2) can be written as

$$u_\lambda(p) = \frac{\not{p} + m}{N} \begin{pmatrix} \chi^{\hat{p}}_\lambda \\ 0 \end{pmatrix} \tag{1.1.7}$$

which shows explicitly that the helicity spinors $u_\lambda(p)$ are solutions of the Dirac equation $(\not{p} - m)u = 0$.

The spinors for Dirac antiparticles can be obtained by application of the charge conjugation operator

$$v_\lambda(p) = i\gamma^2 u^*_\lambda(p) \tag{1.1.8}$$

From Eqs.(1.1.8), (1.1.2) and the identity

$$\sigma^2 \left(\chi_\lambda^{\hat{p}}\right)^* = 2\lambda i \chi_{-\lambda}^{\hat{p}} \tag{1.1.9}$$

one gets the helicity antispinors

$$v_\lambda(p) = \frac{1}{N} \begin{pmatrix} -|\vec{p}| \\ 2\lambda(E+m) \end{pmatrix} \chi_{-\lambda}^{\hat{p}} \tag{1.1.10}$$

Again, one can check that

$$\hat{p} \cdot \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} v_\lambda(p) = -2\lambda v_\lambda(p) \tag{1.1.11}$$

(remember that $v_\lambda(p)$ represents a negative energy Dirac particle, with momentum $-\vec{p}$ and, according to Eq.(1.1.11), spin parallel to $-2\lambda\vec{p}$. It then corresponds to a positive energy state with momentum $\vec{p}$ and spin parallel to $2\lambda\vec{p}$, that is a Dirac antiparticle with helicity 2λ) and Eq.(1.1.10) can be written as

$$v_\lambda(p) = \frac{\not{p} - m}{N} \begin{pmatrix} 0 \\ -2\lambda\chi_{-\lambda}^{\hat{p}} \end{pmatrix} \tag{1.1.12}$$

Let us finally recall that Eqs.(1.1.6) and (1.1.11), in the $m/E \to 0$ limit, simply read

$$\gamma_5 u_\lambda(p) = 2\lambda u_\lambda(p) \qquad \gamma_5 v_\lambda(p) = -2\lambda v_\lambda(p) \tag{1.1.13}$$

In the Appendix we give the relationship between the helicity spinors and the canonical ones (spin quantized along the $\hat{z}$ direction). We also specialize, for an easy and convenient use, Eqs.(1.1.2) and (1.1.10) to the particular frequent kinematical case of $2 \to 2$ exclusive processes in the center of mass frame.

1.2 Helicity polarization vectors for spin 1 particles

Let us consider first the case of massive spin 1 particles. We start again, as in the spin 1/2 case, from the expressions of the polarization vectors for particles at rest and spin quantized along the $\hat{z}$ direction

$$\epsilon_{s_z}^\mu(m, \vec{0}) = \begin{pmatrix} 0 \\ \vec{\epsilon}_{s_z} \end{pmatrix} \tag{1.2.1}$$

with

$$\vec{\epsilon}_{s_z=\pm 1} = \frac{-s_z}{\sqrt{2}} \begin{pmatrix} 1 \\ is_z \\ 0 \end{pmatrix} \tag{1.2.2a}$$

$$\vec{\epsilon}_{s_z=0} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{1.2.2b}$$

By performing a Lorentz boost and rotating to the $\hat{p}(\theta,\varphi)$ direction we then have the helicity polarization vectors

$$\epsilon^{\mu}_{\lambda=\pm 1}(p) = \begin{pmatrix} 0 \\ \vec{\epsilon}^{\,\hat{p}}_{\lambda=\pm 1} \end{pmatrix} \tag{1.2.3a}$$

$$\epsilon^{\mu}_{\lambda=0}(p) = \frac{1}{m} \begin{pmatrix} |\vec{p}| \\ E\vec{\epsilon}^{\,\hat{p}}_{\lambda=0} \end{pmatrix} \tag{1.2.3b}$$

where

$$\vec{\epsilon}^{\,\hat{p}}_{\lambda} = \sum_{\lambda'} \mathcal{D}^{1}_{\lambda'\lambda}(\theta,\varphi)\vec{\epsilon}_{\lambda'} \tag{1.2.4}$$

The full expression of the rotation matrix $\mathcal{D}^1(\theta,\varphi)$ is given in the Appendix, Eq.(A.1.9), together with the explicit expressions of $\epsilon^{\mu}_{\lambda}(p)$ for particular typical center of mass scattering momenta.

In the case of massless spin 1 particle we cannot, of course, start from the rest frame and we do not have the longitudinal polarization vectors, (1.2.2b) and (1.2.3b). The helicity vectors for spin 1 massless particles are then simply those given by Eqs.(1.2.3a) and (1.2.4) (with $\lambda' = 0$ still included in the sum).

Eqs.(1.2.1) and (1.2.2a) represent the helicity vectors for photons or gluons moving along the z-axis. The helicity formalism is, obviously, natural when dealing with massless particles for which the spin is always pointing in the direction of the motion.

Let us finally add that the actual spin four-vector s^{μ} can be obtained, for a spin 1 particle of mass m and four-momentum p, from the polarization vector $\epsilon^{\mu}(p)$ using

$$s^{\mu} = \frac{1}{m}\epsilon^{\mu\alpha\beta\gamma}p_{\alpha}\,\mathrm{Im}(\epsilon^{*}_{\beta}\epsilon_{\gamma}) \tag{1.2.5}$$

One can check, for example, that, indeed, the transverse polarization vectors (1.2.1)-(1.2.2a) correspond to a longitudinal spin pointing along the $\hat{z}$ direction.

2 - Explicit examples

In this Chapter we explicitly compute some scattering amplitudes for exclusive processes involving two initial and two final Dirac fermions and massless spin 1 particles, as an application of the helicity formalism developed in the previous Chapter. We also give and exploit the properties of the helicity scattering amplitudes under parity, time reversal and crossing transformations. Let us start from the two photon annihilation process, $\gamma\gamma \to e^+e^-$.

2.1 Helicity amplitudes for $\gamma\gamma \to e^+e^-$

At lowest order in perturbative QED the $\gamma\gamma \to e^+e^-$ process is described by the two Feynman diagrams of Fig.2.1, where we also define the kinematics

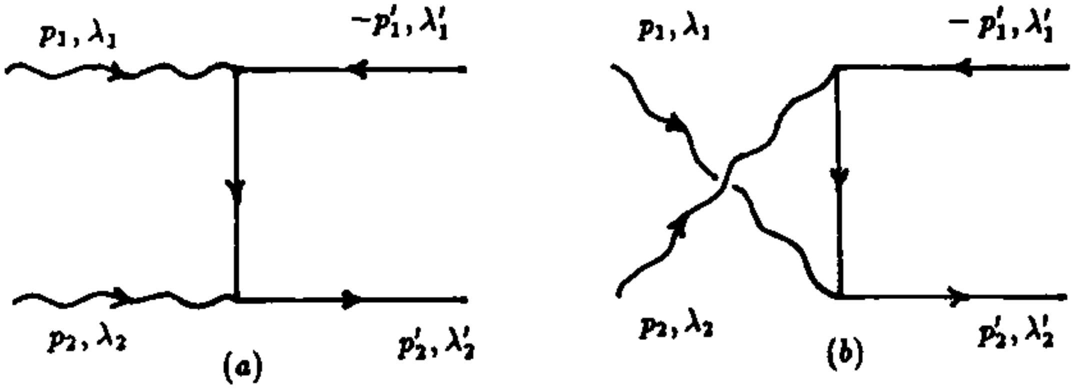

Fig.2.1 *Lowest order Feynman diagrams contributing to* $\gamma\gamma \to e^+e^-$

The corresponding Feynman helicity amplitudes are given by

$$H^{(a)}_{\lambda_1'\lambda_2';\lambda_1\lambda_2} = \frac{ie^2}{2p_1\cdot p_1'}\bar{u}_{\lambda_2'}(p_2')\not\epsilon_{\lambda_2}(p_2)(\not p_1 - \not p_1' + m)\not\epsilon_{\lambda_1}(p_1)v_{\lambda_1'}(p_1') \quad (2.1.1a)$$

$$H^{(b)}_{\lambda_1'\lambda_2';\lambda_1\lambda_2} = \frac{ie^2}{2p_2\cdot p_1'}\bar{u}_{\lambda_2'}(p_2')\not\epsilon_{\lambda_1}(p_1)(\not p_2 - \not p_1' + m)\not\epsilon_{\lambda_2}(p_2)v_{\lambda_1'}(p_1') \quad (2.1.1b)$$

By using the explicit expressions for the Dirac spinors and the polarization vectors listed in the Appendix A.4 one finds, for the 16 center of mass system helicity amplitudes of Eq. (2.1.1a):

$$
\begin{aligned}
H^{(a)}_{+-;11}(\theta) &= -H^{(a)}_{-+;-1-1} = -e^2\sin\theta \\
H^{(a)}_{-+;11}(\theta) &= -H^{(a)}_{+-;-1-1} = e^2\sin\theta \\
H^{(a)}_{+-;1-1}(\theta) &= -H^{(a)}_{-+;-11} = -e^2\frac{\beta\sin\theta(1+\cos\theta)}{1-\beta\cos\theta} \\
H^{(a)}_{-+;1-1}(\theta) &= -H^{(a)}_{+-;-11} = -e^2\frac{\beta\sin\theta(1-\cos\theta)}{1-\beta\cos\theta} \\
H^{(a)}_{++;11}(\theta) &= H^{(a)}_{--;-1-1} = -e^2\frac{m}{E}(1+\cos\theta)\frac{\beta+1-\beta\cos\theta}{1-\beta\cos\theta} \\
H^{(a)}_{--;11}(\theta) &= H^{(a)}_{++;-1-1} = -e^2\frac{m}{E}(1-\cos\theta)\frac{\beta-1+\beta\cos\theta}{1-\beta\cos\theta} \\
H^{(a)}_{++;1-1}(\theta) &= H^{(a)}_{--;-11} = e^2\frac{m}{E}\,\frac{\beta\sin^2\theta}{1-\beta\cos\theta} \\
H^{(a)}_{--;1-1}(\theta) &= H^{(a)}_{++;-11} = e^2\frac{m}{E}\,\frac{\beta\sin^2\theta}{1-\beta\cos\theta}
\end{aligned}
\tag{2.1.2}
$$

where $\beta = p'/E$. The above 16 amplitudes are not all independent but satisfy the usual parity relationships, which, in general, for a process $A + B \to C + D$ in the xz plane, read [1.1]

$$H_{-\lambda_C-\lambda_D;-\lambda_A-\lambda_B}(\theta) = \eta(-1)^{\lambda-\mu}H_{\lambda_C\lambda_D;\lambda_A\lambda_B}(\theta) \tag{2.1.3}$$

where

$$\lambda = \lambda_A - \lambda_B,\ \mu = \lambda_C - \lambda_D,\ \eta = \frac{\eta_C\eta_D}{\eta_A\eta_B}(-1)^{s_A+s_B-s_C-s_D} \tag{2.1.4}$$

with η_i the intrinsic parity of particle i with spin s_i $(i = A, B, C, D)$.

The helicity amplitudes corresponding to the Feynman diagram (b) in Fig.2.1 can be derived from those of diagram (a): in fact the two diagrams only differ by the exchange of the initial photons. In general, under the exchange of two identical particles A and B of spin s in the $A + B \to C + D$ process, we have, always for $\varphi = 0$ [1.1]:

$$H_{\lambda_C\lambda_D;\lambda_A\lambda_B}(\theta) \to (-1)^{2s}e^{i\pi(\lambda_C-\lambda_D)}H_{\lambda_C\lambda_D;\lambda_B\lambda_A}(\pi-\theta) \tag{2.1.5}$$

and a similar relation holds in case of $C \to D$ (if they are identical particles).

Then we can immediately write

$$H^{(b)}_{\lambda_1'\lambda_2';\lambda_1\lambda_2}(\theta) = (-1)^{\lambda_1'-\lambda_2'} H^{(a)}_{\lambda_1'\lambda_2';\lambda_2\lambda_1}(\pi - \theta) \tag{2.1.6}$$

On summing $H^{(a)}$ and $H^{(b)}$ we obtain the full set of Feynman helicity amplitudes for the $\gamma\gamma \to e^+e^-$ process (at lowest order perturbative QED)

$$\begin{aligned}
&H_{\pm\mp;11}(\theta) = -H_{\mp\pm;-1-1}(\theta) = 0 \\
&H_{\pm\mp;1-1}(\theta) = -H_{\mp\pm;-11}(\theta) = -2e^2 \frac{\beta \sin\theta(1 \pm \cos\theta)}{1-\beta^2\cos^2\theta} \\
&H_{\pm\pm;11}(\theta) = H_{\mp\mp;-1-1}(\theta) = -2e^2 \frac{m}{E} \frac{\beta \pm 1}{1-\beta^2\cos^2\theta} \\
&H_{\pm\pm;1-1}(\theta) = H_{\mp\mp;-11}(\theta) = 2e^2 \frac{m}{E} \frac{\beta \sin^2\theta}{1-\beta^2\cos^2\theta}
\end{aligned} \tag{2.1.7}$$

2.2 Helicity amplitudes for $e^+e^- \to \gamma\gamma$

The helicity amplitudes for the process $e^+e^- \to \gamma\gamma$ can be obtained directly by exploiting the properties of the helicity amplitudes under a time-reversal transformation. Again, in general, the helicity amplitudes $H_{\lambda_C\lambda_D;\lambda_A\lambda_B}(\theta)$ for the process $A + B \to C + D$ and $H'_{\lambda_A\lambda_B;\lambda_C\lambda_D}(\theta)$ for the process $C + D \to A + B$ are related by [1.1]

$$H'_{\lambda_A\lambda_B;\lambda_C\lambda_D}(\theta) = (-1)^{2(s_B - s_D)}(-1)^{\lambda-\mu} H_{\lambda_C\lambda_D;\lambda_A\lambda_B}(\theta) \tag{2.2.1}$$

with λ and μ as in Eq.(2.1.4).

In our case (keeping the helicity indices as in Fig.2.1)

$$H_{\lambda_1\lambda_2;\lambda_1'\lambda_2'}(e^+e^- \to \gamma\gamma;\theta) = -(-1)^{(\lambda_1-\lambda_2)-(\lambda_1'-\lambda_2')} H_{\lambda_1'\lambda_2';\lambda_1\lambda_2}(\gamma\gamma \to e^+e^-;\theta) \tag{2.2.2}$$

2.3 Helicity amplitudes for the Compton scattering $\gamma e^- \to \gamma e^-$

We can still make explicit use of yet another property of the helicity amplitudes, the "crossing relations".

The amplitudes for the two reactions

$$
\begin{aligned}
A + B &\to C + D \qquad s\text{-channel} \\
\bar{D} + B &\to C + \bar{A} \qquad t\text{-channel}
\end{aligned}
$$

are related by the $t \to s$ crossing relation which, in the limit in which we neglect the masses of the particles compared to their energies, reads [1.1]

$$
\begin{aligned}
H_{\lambda_C \lambda_D; \lambda_A \lambda_B}(AB \to CD; \theta) = d^{s_C}_{\mu_C \lambda_C}(0) d^{s_D}_{\mu_D \lambda_D}(\pi) d^{s_A}_{\mu_{\bar{A}} \lambda_A}(\pi) d^{s_B}_{\mu_B \lambda_B}(0) \times \\
\times H_{\mu_C \mu_{\bar{A}}; \mu_D \mu_B}(\bar{D}B \to C\bar{A}; \theta)
\end{aligned}
\tag{2.3.1}
$$

We refer the reader to Ref.[1.1] for a complete treatment of the crossing relations.

When $A, C \equiv \gamma$ and $B, D \equiv e^-$ Eq.(2.3.1) allows us to write immediately the helicity amplitudes for the Compton scattering , $\gamma e^- \to \gamma e^-$, in terms of those for the process $e^+e^- \to \gamma\gamma$, which, in turns, are related by Eq.(2.2.2) to the helicity amplitudes for the two photon annihilation process $\gamma\gamma \to e^+e^-$, given in Eq.(2.1.7). The d matrices needed are shown in Appendix A.1. We then find, in the $m/E \to 0$, $\beta \to 1$ limit, that the only non zero center of mass helicity amplitudes for the Compton scattering $\gamma e^- \to \gamma e^-$ are

$$
\begin{aligned}
H_{1+;1+}(\theta) = H_{-1-;-1-}(\theta) &= \frac{2e^2}{\cos(\theta/2)} \\
H_{1-;1-}(\theta) = H_{-1+;-1+}(\theta) &= 2e^2 \cos(\theta/2)
\end{aligned}
\tag{2.3.2}
$$

Of course, the same results could have been obtained by computing the Feynman amplitudes corresponding to the sum of the Feynman diagrams of Fig.2.2.

2.4 Cross-sections

We have now explicitly computed the full sets of independent helicity amplitudes contributing to $\gamma\gamma \to e^+e^-, e^+e^- \to \gamma\gamma$ and $\gamma e^- \to \gamma e^-$ scatterings.

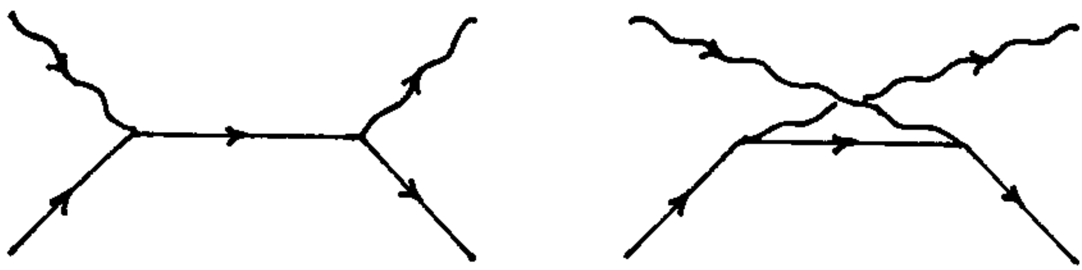

Fig.2.2 *Lowest order Feynman diagrams which contribute to Compton scattering,* $\gamma e^- \to \gamma e^-$

We have both applied the helicity techniques explained in Chapter 1 and recalled the symmetry properties of the helicity amplitudes under parity, time-reversal, crossing and identical particle exchange transformations, showing how to exploit them.

For any given process, once we know the full set of independent helicity amplitudes, *all* related physical observables can be computed. For example, for all of the above processes, the unpolarized center of mass cross-section is given by

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{32\pi(E_1+E_2)^2}\frac{p'}{p}\frac{1}{4}\sum_{\{\lambda\}}|H_{\{\lambda\}}(\theta)|^2 \qquad (2.4.1)$$

where we have used the same kinematical notations as in Appendix A.4.

In particular, in the case of Compton scattering, Eqs.(2.4.1) and (2.3.2) give the usual large energy c.m. result

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{4E^2}\frac{1+\cos^4(\theta/2)}{\cos^2(\theta/2)} \qquad (2.4.2)$$

Although we have considered in details here only QED processes, the same techniques apply, with only minor modifications, to the computation of processes involving quarks and gluons; these minor modifications are simply the extra colour factors and the additional gluon-gluon couplings, but the general procedure is not modified.

We are now well equipped to compute, at the tree level, any helicity amplitude for a process involving spin 1/2 Dirac fermions (quarks or leptons) and spin 1 particles (photons, gluons or massive ones). We can then turn to our discussion of many spin effects in hadronic reactions.

3 - Spin effects in hadronic exclusive reactions

Let us now consider hadronic interactions at high energy and large momentum transfer, that is in the kinematical region where one expects the constituents to play a direct active role in the process.

Exclusive hadronic interactions are usually described in terms of the interactions among constituents by the Brodsky-Farrar-Lepage (BFL) scheme [3.1], according to which the $A+B\to C+D$ high energy and large angle center of mass helicity scattering amplitudes are given by

$$\begin{aligned} H_{\lambda_C\lambda_D;\lambda_A\lambda_B}(\theta) = & \sum_{a,b,c,d;\lambda_a,\lambda_b,\lambda_c,\lambda_d} \prod_i \int [dx_i]\Psi^*_C(x_c)\Psi^*_D(x_d)\times \\ & \times \hat{H}_{\lambda_c\lambda_d;\lambda_a\lambda_b}(x_a,x_b,x_c,x_d;\theta)\Psi_A(x_a)\Psi_B(x_b) \end{aligned} \tag{3.1}$$

Each hadron I is seen as a collection of n_I constituents, each carrying a fraction x_{ij} of its four-momentum, so that

$$[dx_i] = \prod_{j=1}^{n_I} dx_{ij}\delta(1-\sum_j x_{ij}) \tag{3.2}$$

($i=a,b,c,d$ when $I=A,B,C,D$, respectively; by x_i we simply denote the whole set of x_{ij} four-momentum fractions)

In Eq.(3.1) $\hat{H}_{\lambda_c\lambda_d;\lambda_a\lambda_b}(\theta)$ is the helicity amplitude describing the "elementary" constituent interaction, $a+b\to c+d$. The $\Psi_I(x_i)$ are the spin, momentum and colour hadronic wave functions in terms of the constituents.

The BFL scheme, Eq.(3.1), with quarks and gluons as constituents, is supposed to be correct in the large momentum transfer limit, $Q^2\to\infty$, where the strong coupling constant is small, $\alpha_s(Q^2)\to 0$, and all masses can be neglected.

In such a case the elementary $a+b \to c+d$ process can be computed in perturbative QCD, and the leading hadronic configurations are those with a minimum number of constituents, in most cases the valence quarks only (see the comments after Eq.(3.1.2)). We have not explicitly written here the Q^2 dependence of the hadronic wave functions coming from QCD evolution [3.1]. For a rigorous, general treatment of exclusive reactions in the framework of perturbative QCD, leading to the scheme of Eq.(3.1), see also Refs.[3.2,3].

Eq.(3.1) relates the $A+B \to C+D$ scattering amplitude to the elementary ones; of course, all the different elementary amplitudes contributing to the same final states, must be taken into account and summed, as explicitly shown in Eq.(3.1). It is then clear that it is crucial to know the elementary helicity amplitudes, *including* their relative phases. Delicate interference effects might in fact take place between amplitudes with relative phases.

3.1 Dimensional counting rules and helicity selection rule

The actual computation of the scattering amplitudes for a physical process $A+B \to C+D$ according to Eq.(3.1), has been carried out in some simple cases like $\gamma\gamma \to \pi\pi, p\bar{p}$ [3.4,5], $\gamma p \to \gamma p$ [3.6], $\gamma^* N \to N$ (nucleon electromagnetic form factors) [3.7] and some heavy meson decays $J/\Psi \to p\bar{p}$, $\chi_{0,2} \to \pi\pi, \rho\rho$ [3.5,8,9].

For other processes, like proton-proton elastic scattering, instead, such a task might become prohibitive, due to the enormous number of elementary diagrams which have to be summed [3.5,10]. However, some definite conclusions can already be drawn from the scheme of Eq.(3.1).

Let us consider, for example, a typical diagram describing the elementary interaction contributing to, say, meson-baryon elastic scattering, Fig.3.1.

By simple dimensional arguments it is easy to see that the fixed angle (θ) and large energy ($\sqrt{s}$) c.m. elastic cross section

$$\frac{d\sigma}{dt} \propto \frac{1}{s^2} \left| \sum_{\{\lambda\}} \hat{H}_{\{\lambda\}} \right|^2 \qquad (3.1.1)$$

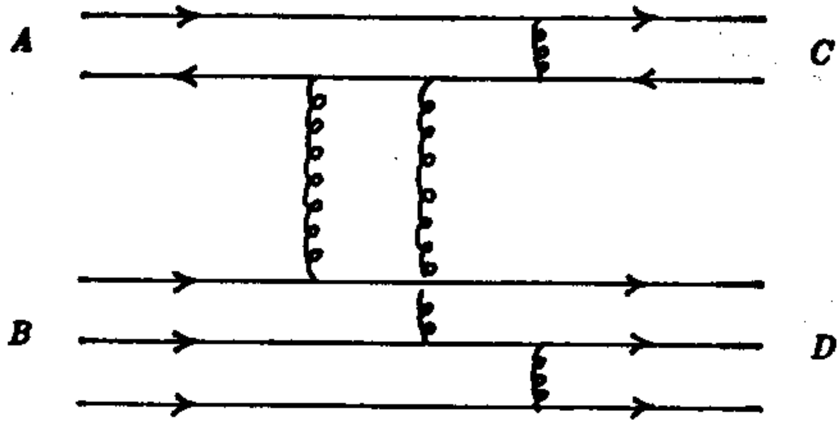

Fig.3.1 *One of the many Feynman diagrams contributing to the elementary processes involved in meson-baryon elastic scattering*

will behave, in general, like

$$\frac{d\sigma}{dt} \sim f(\theta)s^{2-n} \tag{3.1.2}$$

where n is the total number of elementary constituents taking part in the elementary interactions, $n = n_A + n_B + n_C + n_D$. Eq.(3.1.2) reproduces the so called "dimensional counting rules" [3.11]. We have neglected here, because they are irrelevant for the subsequent discussions, the extra logarithmic s dependence coming from the strong coupling constant and the hadronic wave function QCD evolution [3.1]. Eq.(3.1.2) also justifies our previous statement that only the configurations with a minimum number of constituents do contribute: each extra quark or gluon would in fact add a factor s^{-2} to the r.h.s. of Eq.(3.1.2).

There is another simple and remarkable consequence of Eq.(3.1) concerning its helicity structure. Each fermion line in the elementary Feynman diagrams (see, e.g., Fig.3.1) contributes a factor like

$$\bar{u}_{\lambda_{q'}}(q')\gamma^\alpha ... \gamma^\beta u_{\lambda_q}(q) \equiv \Gamma^{\alpha...\beta} \tag{3.1.3}$$

with the product of an *odd* number of γ matrices between two fermion spinors. Remembering the expressions of the helicity projection operators, see Eq.(1.1.13), one has, in the $m_q/\sqrt{s} \to 0$ limit ($\gamma_5\gamma^\alpha = -\gamma^\alpha\gamma_5$)

$$\begin{aligned}\Gamma^{\alpha...\beta} &= \bar{u}_{\lambda_{q'}} \frac{1-\lambda_{q'}\gamma_5}{2}\gamma^\alpha ... \gamma^\beta \frac{1+\lambda_q\gamma_5}{2} u_{\lambda_q} \\ &= \bar{u}_{\lambda_{q'}} \frac{1}{4}(1-\lambda_{q'}\gamma_5)(1+\lambda_q\gamma_5)\gamma^\alpha ... \gamma^\beta u_{\lambda_q} \\ &= \delta_{\lambda_q \lambda_{q'}} \Gamma^{\alpha...\beta}\end{aligned} \tag{3.1.4}$$

that is, *helicity is conserved* along each fermion line (the same conclusion, of course, still holds true when the u spinors are replaced by v antispinors). If, furthermore, one assumes that the hadron helicity equals the sum of the constituent helicities (as it is natural for constituents all moving parallel to the parent hadron), then for each $A + B \to C + D$ exclusive reaction the sum of the initial helicities equals the sum of the final ones [3.1]:

$$\lambda_A + \lambda_B = \lambda_C + \lambda_D \tag{3.1.5}$$

(we cannot deduce $\lambda_A = \lambda_C, \lambda_B = \lambda_D$, due to diagrams where, say, a quark initially in particle A ends up in D, and one in B ends up in C).

The "helicity conservation rule", Eq.(3.1.5), might only be broken by terms proportional to m_q/E_q (which allow helicity flips) and terms proportional to the intrinsic $k_\perp$ of the quarks inside the hadrons (in which case it is not obvious any more that the sum of the constituent helicities equals the hadron helicity).

Eq.(3.1.2) and the dimensional counting rules are in good agreement with the experimental data [3.12]. Eq.(3.1.5), the helicity conservation rule, instead, seems to be a source of constant troubles when comparing its theoretical consequences with the existing experimental information on spin effects in hadronic exclusive reactions. We now turn to a detailed analysis of these cases.

Proton-proton elastic scattering

Let us consider first proton-proton elastic scattering, $pp \to pp$. In such a case we have, for any center of mass scattering angle θ, five independent helicity amplitudes [1.1]

$$\begin{aligned}
\Phi_1(\theta) &\equiv H^S_{++;++}(\theta) \\
\Phi_2(\theta) &\equiv H^S_{++;--}(\theta) \\
\Phi_3(\theta) &\equiv H^S_{+-;+-}(\theta) \\
\Phi_4(\theta) &\equiv H^S_{+-;-+}(\theta) \\
\Phi_5(\theta) &\equiv H^S_{++;+-}(\theta)
\end{aligned} \tag{3.2.1}$$

All other amplitudes (for a total of 16) can be obtained from Eqs.(3.2.1) by using the parity relations (2.1.3), the time reversal ones (2.2.1) and the identical particle exchange property (2.1.5); the apex S in Eqs.(3.2.1) reminds in fact that all amplitudes have already been correctly symmetrized in order to take into account the possible exchange of the identical particles both in the initial and the final states (what we did, in the case of $\gamma\gamma \to e^+e^-$ annihilation process, by summing diagrams (a) and (b) of Fig.2.1). This leads to the relations

$$\begin{aligned} H^S_{++;+-}(\theta) &= -H^S_{++;-+}(\theta) \\ H^S_{+-;+-}(\theta) &= -H^S_{+-;-+}(\pi-\theta) \\ H^S_{++;+-}(\theta) &= -H^S_{++;+-}(\pi-\theta) \end{aligned} \tag{3.2.2}$$

which imply

$$\begin{aligned} \Phi_3(\pi/2) &= -\Phi_4(\pi/2) \\ \Phi_5(\pi/2) &= 0 \end{aligned} \tag{3.2.3}$$

As we said, the knowledge of the five independent helicity amplitudes Φ_i ($i = 1,..,5$) allows to compute any observable quantity, in pp elastic scattering. Let us first define the spin observables for which we have some experimental data. The proton analyzing power A is given by

$$A \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \tag{3.2.4}$$

where $d\sigma^{\uparrow(\downarrow)}$ is the cross section for the elastic scattering of an unpolarized proton off a proton polarized perpendicularly to the scattering plane; if (xz) is the scattering plane, then $\uparrow$ ($\downarrow$) means parallel (antiparallel) to $\hat{y}$. The final proton spins are not observed. By time reversal, such a quantity is the same as the analogous one in which the observed spin is that of the scattered proton, that is the proton polarizing power or, simply, the scattered proton polarization.

The double spin asymmetry A_{NN} is defined by

$$A_{NN} \equiv \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} \tag{3.2.5}$$

where, now, both initial protons are polarized in the direction normal (N) to the scattering plane, as explained for A.

Analogously to A_{NN}, one can define the double spin asymmetries A_{SS} and A_{LL} which only differ from A_{NN} by the direction along which the initial protons are polarized: the S direction is the $\hat{x}$ direction, always choosing (xz) as the scattering plane, and the L direction is along $\hat{z}$. The analogous of A, Eq.(3.2.4), with the spin in the S or L direction, is zero by parity invariance.

The expressions of A, A_{NN}, A_{SS} and A_{LL} in terms of the helicity amplitudes (3.2.1) are [1.1]

$$A = X^{-1}\mathrm{Im}[\Phi_5^*(\Phi_4 - \Phi_1 - \Phi_2 - \Phi_3)] \tag{3.2.6a}$$

$$A_{NN} = X^{-1}\mathrm{Re}[\Phi_1\Phi_2^* - \Phi_3\Phi_4^* + 2|\Phi_5|^2] \tag{3.2.6b}$$

$$A_{SS} = X^{-1}\mathrm{Re}[\Phi_1\Phi_2^* + \Phi_3\Phi_4^*] \tag{3.2.6c}$$

$$A_{LL} = X^{-1}\frac{1}{2}[|\Phi_3|^2 + |\Phi_4|^2 - |\Phi_1|^2 - |\Phi_2|^2] \tag{3.2.6d}$$

where

$$X = \frac{1}{2}[|\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2] \tag{3.2.7}$$

From Eqs.(3.2.3), (3.2.6) and (3.2.7) we have, at $\theta = \pi/2$,

$$A_{NN}(\pi/2) - A_{SS}(\pi/2) - A_{LL}(\pi/2) = 1 \tag{3.2.8}$$

Let us now go back to our Eq.(3.1.5), the helicity conservation rule found to be valid in the perturbative QCD, BFL scheme for exclusive reactions. It immediately gives

$$\Phi_2(\theta) = \Phi_5(\theta) = 0 \tag{3.2.8}$$

which implies

$$A(\theta) = 0 \tag{3.2.9a}$$

$$A_{NN}(\theta) = -A_{SS}(\theta) \tag{3.2.9b}$$

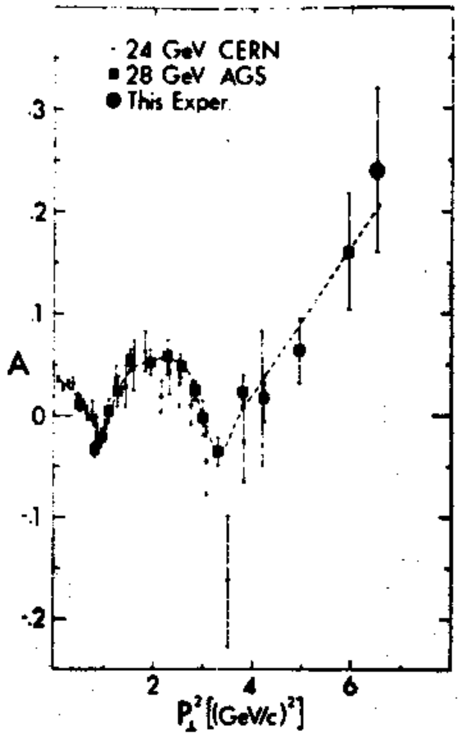

Fig.3.2 *The proton analyzing power A as a function of $p_\perp^2$, from Ref.[3.13]*

and

$$2A_{NN}(\pi/2) - A_{LL}(\pi/2) = 1 \tag{3.2.10}$$

How do these results, Eqs.(3.2.9,10), compare with experiment? The measured proton analyzing power is plotted as a function of $p_\perp^2$ [3.13] in Fig.3.2. It, very surprisingly, shows a large increase of A at large values of $p_\perp^2$, a trend opposite to what one would expect according to Eq.(3.2.9a), which should be valid exactly at large values of Q^2.

Analogously we plot in Fig.3.3 the measured values of A_{NN} and A_{LL} vs. $p_\perp$, at $\theta = \pi/2$ [3.14]. The large energy ($p_{Lab} = 11.75\,GeV/c$) results read

$$\begin{aligned} A_{NN}(\pi/2) &= 0.58 \pm 0.04 \\ A_{LL}(\pi/2) &= -0.12 \pm 0.16 \end{aligned} \tag{3.2.11}$$

from which

$$2A_{NN}(\pi/2) - A_{LL}(\pi/2) = 1.28 \pm 0.18 \tag{3.2.12}$$

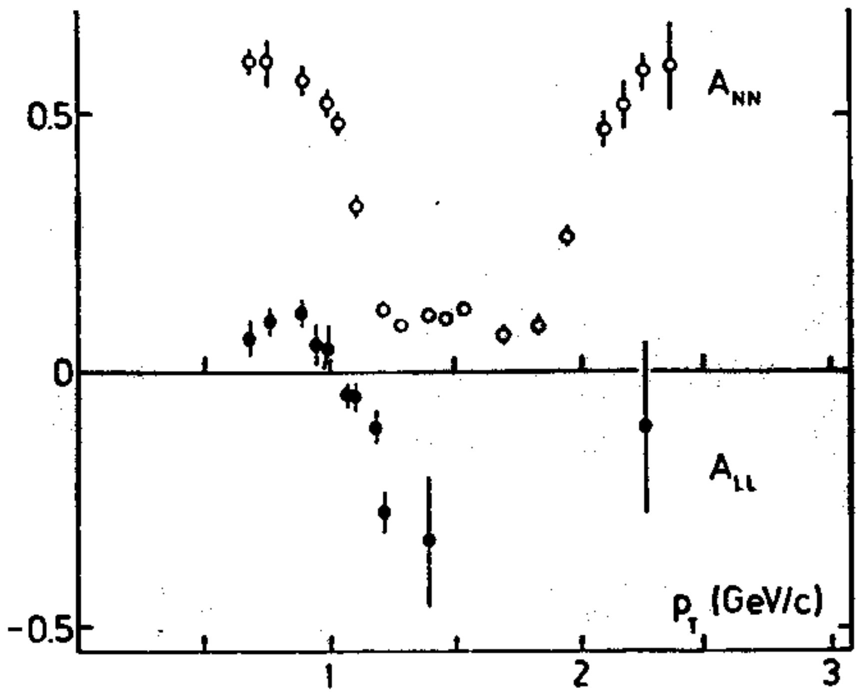

Fig.3.3 *The proton double spin asymmetries $A_{NN}(\circ)$ and $A_{LL}(\bullet)$ as a function of $p_\perp$*

to be compared with Eq.(3.2.10).

Again, we see that, although the experimental errors are still very large, the theoretical consequences of Eq.(3.1.5) are not in good agreement with the experimental information we have.

Before discussing the possible reasons for the apparent failure of the helicity sum rule let us consider some more cases in which such a failure shows up. We only stress, for the moment, that getting a non zero value for the analyzing power A, Eq.(3.2.6a), is highly non trivial: not only the single helicity flip amplitude Φ_5 must be different from zero, but it also must have a non zero phase relatively to, at least, one of the other amplitudes.

3.3 ρ polarization in $\pi p \to \rho p$ reactions

Another challenging piece of experimental information comes from the $\pi^- p \to \rho^- p$ process, with the measurement of the final ρ^- vector particle helicity density matrix elements.

Let us recall that the helicity density matrix of a particle C, produced in the $A + B \to C + D$ reaction is given, in the case of unpolarized initial A and B particles, by [1.1]

$$\rho_{\lambda_C \lambda'_C}(C) = \frac{1}{N} \sum_{\lambda_A, \lambda_B, \lambda_D} H_{\lambda_C \lambda_D; \lambda_A \lambda_B} H^*_{\lambda'_C, \lambda_D; \lambda_A, \lambda_B} \tag{3.3.1}$$

with $N = \sum_{\{\lambda\}} |H_{\{\lambda\}}|^2$, so that $Tr \rho = 1$.

Such helicity density matrix is just the ordinary, non relativistic spin density matrix for particle C, if we observe C in its helicity rest frame (that is the frame which, seen from the c.m. system, has its z-axis moving along the direction of particle C with its same speed; see, e.g., Ref.[1.1] for a precise definition). The helicity density matrix, Eq.(3.3.1), then tells us how particle C, produced via the $A + B \to C + D$ reaction described by the corresponding helicity amplitudes $H_{\lambda_C \lambda_D; \lambda_A \lambda_B}$, is polarized. If particle C decays, its decay angular distribution will reflect its spin orientation, and will then depend on the elements of $\rho(C)$.

In the $\pi^- p \to \rho^- p$ interaction we have

$$\rho_{\lambda \lambda'}(\rho^-) = \frac{1}{N} \sum_{\lambda_{p'}, \lambda_p} H_{\lambda \lambda_{p'}; \lambda_p} H^*_{\lambda', \lambda_{p'}; \lambda_p} \tag{3.3.2}$$

and this helicity density matrix can be studied through the angular distribution of the π^- as it emerges from the decay of the $\rho^-, \rho^- \to \pi^- \pi^0$, in its helicity rest frame:

$$\begin{aligned} W(\cos\Theta, \Phi) = \frac{3}{4\pi} [&\rho_{00} \cos^2\Theta + (\rho_{11} - \rho_{1-1}) \sin^2\Theta \cos^2\Phi + \\ &+ (\rho_{11} + \rho_{1-1}) \sin^2\Theta \sin^2\Phi - \sqrt{2} (\mathrm{Re}\rho_{10}) \sin 2\Theta \cos\Phi] \end{aligned} \tag{3.3.3}$$

or, by integration over $d\Phi$ and $d(\cos\Theta)$

$$\begin{aligned} W(\cos\Theta) &= \frac{3}{2} [\rho_{00} + (\rho_{11} - \rho_{00}) \sin^2\Theta] \\ W(\Phi) &= \frac{1}{2\pi} [1 - 2\rho_{1-1} + 4\rho_{1-1} \sin^2\Theta] \end{aligned} \tag{3.3.4}$$

The reported values, for the production of ρ^- at $\theta = \pi/2$ in the c.m. system, are [3.15]

$$\begin{aligned} \rho_{11} &= 0.44 \pm 0.15 \\ \rho_{00} &= 0.12 \pm 0.30 \\ \rho_{1-1} &= 0.32 \pm 0.10 \\ Re\rho_{10} &= -0.01 \pm 0.05 \end{aligned} \tag{3.3.5}$$

The surprising result is the sizeable, non zero value of the non diagonal element ρ_{1-1}. In fact the helicity conservation rule, Eq.(3.1.5), reads in this case $\lambda_\rho + \lambda_{p'} = \lambda_p$, always implying, see Eq.(3.3.2)

$$\rho_{\lambda\lambda'} = 0 \qquad \lambda \neq \lambda' \tag{3.3.6}$$

3.4 η_c decay into proton-antiproton, $\eta_c \to p\bar{p}$

Another simple and drastic example of the failure of the helicity conservation rule comes from the η_c decay into proton-antiproton, decay which has been observed and measured [3.16].

Let us consider a typical Feynman diagram contributing to the elementary interactions underlying the c.m. $\eta_c \to p\bar{p}$ decay, Fig.3.4.

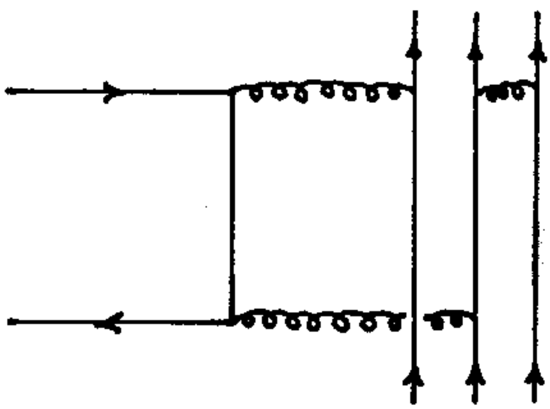

Fig.3.4 *One of the Feynman diagrams contributing to a heavy meson decay into baryon-antibaryon*

Analogously to the argument of Section 3.1, each final quark line contributes a factor

$$\bar{u}_{\lambda_q}(q)\gamma^{\alpha}...\gamma^{\beta}v_{\lambda_{\bar{q}}}(\bar{q}) \equiv \overline{\Gamma}^{\alpha...\beta} \tag{3.4.1}$$

with an *odd* number of Dirac gamma matrices. Then, from Eqs.(1.1.13), one sees that, in the large energy limit ($E_q \gg m_q$)

$$\begin{aligned}\overline{\Gamma}^{\alpha...\beta} &= \bar{u}_{\lambda_q}\frac{1-\lambda_q\gamma_5}{2}\gamma^{\alpha}...\gamma^{\beta}\frac{1-\lambda_{\bar{q}}\gamma_5}{2}v_{\lambda_{\bar{q}}} \\ &= \delta_{\lambda_{\bar{q}},-\lambda_q}\overline{\Gamma}^{\alpha...\beta}\end{aligned} \tag{3.4.2}$$

that is, quarks and antiquarks emitted in pairs via gluons (or photons) always have, when neglecting their masses, opposite helicities.

We then conclude, again assuming the final proton ($\bar{p}$) helicity to be the sum of the quark ($\bar{q}$) helicities, that diagrams like those of Fig.3.4 would give

$$\lambda_p = -\lambda_{\bar{p}} \tag{3.4.3}$$

On the other hand the quantum number of the decaying η_c, $J^{PC} = 0^{-+}$, and the obvious requests of parity and total angular momentum conservation force the final $p\bar{p}$ system to be in a $L = S = 0$ state; this contradicts Eq.(3.4.3) which implies, for the $p\bar{p}$ system, a spin component along the final c.m. proton direction $S_{\hat{p}} = 2\lambda_p$. The $\eta_c \to p\bar{p}$ decay is then forbidden in the BFL scheme, contrary to experimental observation.

3.5 Discussion and comments

We have then to face the problem that, far from being actually able to compute all exclusive hadronic processes in the realm of perturbative QCD, already at the simple, immediate conclusion of Eq.(3.1.5), the helicity selection rule, the experimental indications are not at all encouraging. On the other hand, one must not forget that the similar, general conclusion of Eq.(3.1.2), the dimensional counting rules, is well satisfied by experiment; also, some cross sections for very simple exclusive reactions have been successfully computed [3.4-9]. What could be the reason for all that?

It might well be that, simply, the BFL perturbative QCD scheme of Eq.(3.1) is not adequate to describe hadronic exclusive processes; the factorization between the elementary "hard" interactions and the "soft" hadronization processes, hidden in the wave functions, might not be justified [3.17]. Or, more optimistically, the Q^2 and energy ranges involved in the experimental processes feasable at the moment are still too small and do not allow yet a meaningful comparison with the theoretical predictions, strictly valid only when $Q^2 \to \infty$ [3.18].

If such is the case, most spin effects, like the proton polarization in pp elastic scattering and the non diagonal helicity density matrix elements of ρ's produced in $\pi p \to \rho p$, should disappear, could we perform the same experiments at higher energies. In general spin effects should give results in agreement with Eq.(3.1.5).

The actual Q^2 values involved in the processes described in the previous Sections are of the order of few $(GeV)^2$, up to $\sim 10(GeV)^2$. Even if for the same values of Q^2 in the deep inelastic scattering we do probe the nucleon internal structure, in the more complicated case of exclusive processes, where scattered constituents have to recombine into the observed hadrons, these values of Q^2 might not be large enough as to really observe the multibody free constituents elementary interactions.

Non perturbative, higher twist, mass and/or intrinsic $k_\perp$ effects might still be non negligible. We must not forget, for example, that one of the most intriguing experimental result, the proton polarization in pp elastic scattering (see Fig.3.2), gives (at $p_\perp^2 = 6.5\,(GeV/c)^2$) the value [3.13]

$$A = 0.24 \pm 0.08 \tag{3.5.1}$$

Such a value, at the energy at which the experiment has been performed, $\sqrt{s} \simeq \sqrt{54}$, is still compatible with the value

$$m_p/\sqrt{s} \simeq 0.13 \tag{3.5.2}$$

That is, it might still be a genuine (proton) mass effect, which will disappear in the true large energy limit $m_p/\sqrt{s} \to 0$.

Let us finally notice that the main source of problems, towards a satisfactory QCD description of exclusive processes, is the helicity selection rule, Eq.(3.1.5), rather than the dimensional counting rules, Eq.(3.1.2). This, once more, emphasizes the specialness of spin effects. The reason is clear from a look at Eqs.(3.2.6,7): while unpolarized experiments only test a theory at the $\sum_i |H_i|^2$ level, spin measurements are sensible to quantities like $H_i H_j^*$ $(i \neq j)$, involving not only moduli but also *relative phases*. This makes spin measurements a much more severe test of any theory than simple unpolarized cross section measurements.

3.6 Alternatives and modifications to the BFL scheme

Although it is outside the scope of this paper - that of presenting some technical tools and some unsolved problems whose treatment and (hopefully) solution requires using those tools - we will briefly mention here some attempts of escaping the difficulties encountered in this chapter.

A different perturbative QCD approach to the description of elastic scatterings was introduced in Ref.[3.19]. It leads to a power law behaviour of the fixed large angle cross sections $d\sigma/dt$ different from that given in Eq.(3.1.2). Still, it suffers the same disease as the BFL scheme, namely the helicity selection rule.

Two models based essentially on a different way of combining the quark spins to obtain the hadron spins are discussed in Refs.[3.20-23]. The first [3.20,21], the end point model, in which the whole contribution to the scattering comes from leading $(x \to 1)$ quarks, only succeeds in describing, among the spin effects, the *pp* double spin asymmetry A_{NN}. The second [3.22,23] considers partonic configurations in which one of the quarks (the scattered, leading one) recombines with the other quarks (the spectators), at a large angle with it, to form the observed hadrons: in such a recombination each quark conserves its spin, so that the hadron helicity is not at all the sum of the constituent helicities. Such a model describes well the double pp spin asymmetries $A_{NN}(\pi/2)$ and $A_{LL}(\pi/2)$ [3.14] and the non diagonal values of $\rho(\rho^-)$ [3.15]. Preliminary results indicate, however, that it does not predict correctly the analogous value of $\rho(\rho^+)$ in $\pi^+ p \to \rho^+ p$ reactions [3.24].

Also in the model of Ref.[3.25], not a QCD perturbative model, which in general we do not discuss here, the spin rather than the helicity is conserved in the hadronization mechanism.

None of the above models can explain the large value of the proton analyzing power A, Eqs.(3.2.4) and (3.2.6a), in that none of them can reproduce amplitudes with relative complex phases.

Finally, a generalization of the BFL scheme which introduces the presence of diquarks, bound states of two quarks, as constituents, has been used in Refs.[3.26-28]. Such a scheme allows the description of the $\eta_c \to p\bar{p}$ decay [3.28] and, in principle [3.26], should be able to obtain, for pp elastic scattering, a non zero single helicity flip amplitude, Φ_5, and relative phases. A complete calculation is, however, extremely complicated, due to the large number of diagrams contributing and to the (unknown) couplings of two or more gluons to diquarks.

Needless to say, many unsolved spin problems arise also in hadronic inclusive interactions. We do not review them here as their description in perturbative QCD does not necessarily require summing elementary amplitudes, but rather elementary cross sections. Moreover, they have been discussed in the literature much more extensively than the exclusive reactions [1.3], [3.29]. We only stress that, even in inclusive cases, a correct, full treatment of the problem at the amplitude level, might allow a deeper insight into the elementary interaction properties [3.30].

As the only exception we will dedicate the next Chapter to a discussion of the deep inelastic scattering of polarized muons off polarized protons, $\mu p \to \mu X$. In fact, very recent experimental data on such a process have, once more, caused great surprise and drastically changed our usual picture of polarized nucleons in terms of polarized constituents. We then feel that, dealing with spin problems, we cannot avoid discussing this last argument, the so called "nucleon spin crisis".

The spin problem in the proton

The deep inelastic scattering (DIS) of leptons on nucleons, at large momentum transfer Q^2. probes the internal structure of protons and neutrons. The unpolarized cross section for such a process, $lN \to lX$, can be written, at the one photon exchange order, in terms of two structure functions:

$$\frac{d\sigma}{d\Omega dE'} = \frac{4\alpha^2(E')^2}{Q^4}(2W_1 \sin^2\frac{\theta}{2} + W_2 \cos^2\frac{\theta}{2}) \tag{4.1}$$

Analogously, in case of longitudinally polarized leptons (↑) on longitudinally polarized nucleons (↑↓), one has

$$\frac{d\sigma^{\uparrow\downarrow}}{d\Omega dE'} - \frac{d\sigma^{\uparrow\uparrow}}{d\Omega dE'} = \frac{4\alpha^2 E'}{Q^2 E}[(E + E'\cos\theta)m_N G_1 - Q^2 G_2] \tag{4.2}$$

The unpolarized structure functions $W_{1,2}(Q^2,\nu)$ together with the polarized ones $G_{1,2}(Q^2,\nu)$ describe the nucleon internal structure and it is well known that they can be expressed in terms of quark distribution functions. All variables which appear in the above Eqs.(4.1,2) and which will be used in this Chapter are the usual ones in DIS [4.1].

Recently the EMC group measured the proton polarized structure functions [4.2,3]; such data, also confirmed by SLAC data [4.4], are the ones which caused in the last two years a great amount of theoretical activity, in a variety of attempts aimed at explaining the somewhat unexpected results.

4.1 The experimental data

Let us briefly recall what the EMC measurements are. In the DIS of longitudinally polarized muons on longitudinally polarized protons the EMC measure the asymmetry A,

$$A \equiv \frac{d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\downarrow} + d\sigma^{\uparrow\uparrow}} \tag{4.1.1}$$

which, using Eqs.(4.1,2) can be expressed as

$$A = \frac{Q^2}{2EE'} \cdot \frac{(E + E'\cos\theta)MG_1 + q^2 G_2}{2W_1 \sin^2(\theta/2) + W_2 \cos^2(\theta/2)} \tag{4.1.2}$$

In the Bjorken limit $(\nu, Q^2 \to \infty, Q^2/(2m_N\nu) = x)$ one has

$$\begin{aligned} M^2\nu G_1(Q^2,\nu) &= g_1(x) \\ M\nu^2 G_2(Q^2,\nu) &= g_2(x) \end{aligned} \tag{4.1.3}$$

where we have not indicated, in $g_{1,2}$, the usual QCD $\ln Q^2$ scale breaking.

From the experimental measurements and Eqs.(4.1.2,3) we now try to get some information on g_1, g_2. We show here a more direct analysis [4.5] than that actually followed by the EMC, although completely equivalent.

Using (4.1.2) and the expression for the unpolarized differential cross-section, Eq.(4.1), we define [4.5]

$$\mathcal{G}(x, Q^2, E) \equiv \frac{M\nu Q^2 E}{2\alpha^2 E'(E + E'\cos\theta)} \cdot \frac{d^2\sigma}{d\Omega dE'} \cdot A \tag{4.1.4}$$

$$= g_1 - \frac{2xM}{E + E'\cos\theta}\, g_2$$

$$\simeq g_1 - \frac{xM}{E - Q^2/4Mx}\, g_2 \tag{4.1.5}$$

The RHS of (4.1.4) can be constructed directly from experiment.

In principle g_2 can be separated in (4.1.5) by a study of the E dependence of $\mathcal{G}$. However, for the EMC data, the errors do not permit a statistically significant result. Following the approximation used by the EMC, the values of $g_1(x)$ are extracted neglecting the term in g_2 in (4.1.5); it has been shown that such a neglect leads to insignificant errors [4.5].

By measuring $g_1(x)$ for different values of $x\,(0.015 \le x \le 0.466)$, combining their results with previous values of $g_1(x)$ given by SLAC [4.4], and extrapolating to the values of x not covered by the experiment(*), the EMC give the final result [4.3]

$$\Gamma_p \equiv \int_0^1 g_1(x)dx = 0.126 \pm 0.010 \pm 0.015 \tag{4.1.6}$$

(*) Some criticism on the extrapolation to small x values, answered in Ref. [4.3], has been raised in Ref. [4.6].

4.2 What is the proton spin problem?

Let us now see why the experimental result (4.1.6) is (or looks) surprising.

In the framework of the Operator Product Expansion (OPE), and including perturbative QCD corrections, Γ_p can be written as [4.7]

$$\Gamma_p = \int_0^1 g_1(x)dx = \frac{1}{12}\left\{\left[1-\frac{\alpha_s}{\pi}\right]\left[a_3+\frac{1}{\sqrt{3}}a_8\right]+2\sqrt{\frac{2}{3}}\left[1-\frac{\alpha_s}{3\pi}\right]a_0\right\} \quad (4.2.1)$$

where the a_j are related to the matrix elements of the quark $SU(3)$ axial vector currents taken between proton states with covariant spin vector S^μ:

$$<P,S|A_j^\mu|P,S> = 2Ma_jS^\mu \quad (4.2.2)$$

with

$$A_j^\mu \equiv \overline{\Psi}\gamma^\mu\gamma_5(\lambda_j/2)\Psi \quad (4.2.3)$$

wherein the λ_j ($j = 1,..,8$) are the usual Gell-Mann $SU(3)$ matrices and λ_0 is proportional to the unit matrix.

By explicitely inserting λ_3, λ_8 and λ_0 into Eqs.(4.2.3) and (4.2.2) we have

$$a_3 = \Delta u' - \Delta d' \quad (4.2.4)$$

$$a_8 = \frac{1}{\sqrt{3}}(\Delta u' + \Delta d' - 2\Delta s') \quad (4.2.5)$$

$$a_0 = \sqrt{\frac{2}{3}}(\Delta u' + \Delta d' + \Delta s') \quad (4.2.6)$$

where

$$<P,S|\bar{q}\gamma^\mu\gamma_5 q|P,S> = 2M\Delta q' S^\mu \quad (4.2.7)$$

($q = u, d, s$; throughout the paper by q we always mean quarks and antiquarks of flavour q).

Some of the matrix elements (4.2.4-6) are known from β decays. From the neutron β decay one has [4.8]

$$a_3 = g_A = 1.254 \pm 0.006 \quad (4.2.8)$$

Furthermore, if $SU(3)_F$ is a good symmetry of the strong interactions then all the β-decays of the members of the baryon octet can be expressed in terms of just two parameters F and D, and one then has [4.9]

$$a_8 = \frac{1}{\sqrt{3}}(3F - D) \quad ; \quad a_3 = F + D \tag{4.2.9}$$

It is then clear that the EMC measurement of Γ_p, Eq.(4.2.1), together with Eqs.(4.2.8-9) and (4.2.4-6) allows the determination of the separate values of the $\Delta q'$, Eq.(4.2.7). By using $F/D = 0.631$ [4.10] and $F + D = g_A$, one gets [4.3]

$$\begin{aligned}\Delta u' &= 0.782 \pm 0.032 \pm 0.046\\ \Delta d' &= -0.472 \pm 0.032 \pm 0.046\\ \Delta s' &= -0.190 \pm 0.032 \pm 0.046\end{aligned} \tag{4.2.10}$$

In the naive parton model, where the quarks are supposed to be *free, non interacting* constituents, the quark operators are treated as free operators. In such a case

$$\Delta q' = \Delta q = \int dx\ [q_+(x) + \bar{q}_+(x) - q_-(x) - \bar{q}_-(x)] \tag{4.2.11}$$

where $q_\pm(\bar{q}_\pm)$ are the number densities of quarks (antiquarks) with helicities $\pm\frac{1}{2}$ in a proton with helicity $+\frac{1}{2}$. We have then that, inside a proton with $S_z = \frac{1}{2}$, the total amount of spin carried by the quarks is, from Eqs. (4.2.6) and (4.2.10-11)

$$S_z^{quarks} = \frac{1}{2}\sum_q \Delta q \tag{4.2.12}$$

$$= \frac{1}{2}\sum_q \Delta q' = \frac{1}{2}\sqrt{\frac{3}{2}}a_0 = 0.060 \pm 0.047 \pm 0.069 \tag{4.2.13}$$

This is the surprising result: the total amount of spin carried by the quarks, far from being $\frac{1}{2}$ as we would expect in the naive parton model, is only a tiny fraction of the proton spin. Hence the proton spin crisis.

The result (4.2.13) seems to be very reliable. Some criticism may be cast on the numerical choices of F and D, which are still debated in the literature.

However, different values [4.11,12] lead again to the same result, $\sum_q \Delta q' \simeq 0$ [4.3,13]. Even an analysis of the experimental data which makes use of Eq.(4.2.8), only based on isospin invariance, but does not use Eq.(4.2.9), allowing for large $SU(6)$ violations, again finds $\sum_q \Delta q'$ to be compatible with zero [4.14].

4.3 The gluon contribution to Γ_p

It appears from Eqs.(4.2.12-13) that the total amount of spin carried by the quarks is almost zero. Is such a conclusion really unavoidable?

The point is that quarks are not free, at least for finite values of Q^2, and they can interact with other constituents, like the gluons. In other words, one should add to the naive parton model prescription

$$d\sigma^{\mu p \to \mu X} = \sum_q q(x) d\sigma^{\mu q \to \mu q} \tag{4.3.1}$$

also higher order (in α_s) corrections, like the gluon contribution

$$d\sigma_g^{\mu p \to \mu X} = g(x) d\sigma^{\mu g \to \mu q \bar{q}} \tag{4.3.2}$$

If it happens, by some miracle, that the α_s of the higher order elementary interaction is canceled by a term proportional to α_s^{-1}, coming from the QCD evolution of the distribution function, then we get a correction which is of $O(1)$ rather than $O(\alpha_s)$. This is indeed the case for the gluon contribution to Γ_p [4.15-18].

By computing the contribution of the elementary $\gamma^* g \to q\bar{q}$ scattering to Γ_p we have [4.16-18] (*)

$$\Gamma_p(gluon) = -\frac{\alpha_s(Q^2)}{4\pi} f < e^2 > \Delta g \tag{4.3.3}$$

(*) The QCD corrections already given in Eq.(4.2.1), different from the gluon contribution computed here, should also apply to Γ_p(gluon). We neglect them here as they are irrelevant for the subsequent discussion.

where $< e^2 >= \frac{1}{f}\sum_q e_q^2, f$ is the number of flavours (here $f = 3, < e^2 >= \frac{2}{9}$) and

$$\Delta g = \int dx[g_+(x) - g_-(x)] = S_z^g \tag{4.3.4}$$

The same result, obtained above in the framework of the parton model, can be derived in the OPE formalism. Let us start by writing the matrix element of the $SU(3)$ flavour singlet axial vector current which corresponds to (4.2.13)

$$\sum_q 2M\Delta q' S^\mu = 2M(\Delta u' + \Delta d' + \Delta s')S^\mu = \sum_q < P, S|\bar{q}\gamma^\mu\gamma_5 q|P, S > \tag{4.3.5}$$

As it was first noticed in Ref. [4.15], because of the anomaly [4.19,20] associated with the current

$$J_5^\mu = \sum_q \bar{q}\gamma^\mu\gamma_5 q\,, \quad \partial_\mu J_5^\mu = \frac{\alpha_s f}{8\pi}\sum_{i=1}^{8}\epsilon_{\mu\nu\rho\sigma}F_i^{\mu\nu}F_i^{\rho\sigma}, \tag{4.3.6}$$

Eq.(4.3.5) does not measure quark spins alone, but it gets an extra contribution from the anomaly triangle diagram. This extra contribution can be interpreted as the gluon contribution and one has [4.16,17]

$$< P, S|\bar{q}\gamma^\mu\gamma_5 q|P, S >= 2MS^\mu(\Delta q - \frac{\alpha_s}{2\pi}\Delta g) \tag{4.3.7}$$

thus leading to the identification (Eq.(4.2.7) and (4.3.7))

$$\Delta q' = \Delta q - \frac{\alpha_s}{2\pi}\Delta g = \frac{1}{2}S_z^q - \frac{\alpha_s}{2\pi}S_z^g \tag{4.3.8}$$

Eq.(4.3.8), combined with Eqs.(4.2.4-6) and used in Eq.(4.2.1), gives the gluon contribution to Γ_p shown in Eq.(4.3.3).

It is then clear that what the EMC measure, the matrix element between proton states of the flavour singlet quark axial vector current, does not represent the total spin carried by the quarks, but rather the combination $\sum_q(S_z^q - \frac{\alpha_s}{4\pi}S_z^g)$. The experimental results (4.2.10) can be written as

$$\begin{aligned} \Delta u &= 0.782 \pm 0.032 \pm 0.046 + \frac{\alpha_s}{2\pi}\Delta g \\ \Delta d &= -0.472 \pm 0.032 \pm 0.046 + \frac{\alpha_s}{2\pi}\Delta g \\ \Delta s &= -0.190 \pm 0.032 \pm 0.046 + \frac{\alpha_s}{2\pi}\Delta g \end{aligned} \tag{4.3.9}$$

and

$$\sum_q = 0.120 \pm 0.094 \pm 0.138 + 3\frac{\alpha_s}{2\pi}\Delta g \tag{4.3.10}$$

The point is now: how big can the higher order term, $(\alpha_s/2\pi)\Delta g$, be? From the Altarelli-Parisi QCD evolution equations [4.21], one can obtain [4.22-24] that the large Q^2 behaviours of Δq and Δg are, to leading order

$$\begin{aligned} \Delta q(Q^2) &= \Delta q(Q_0^2) \\ \Delta g(Q^2) &= \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}\left[\Delta g(Q_0^2) + \frac{4}{9}\sum_q \Delta q\right] \end{aligned} \tag{4.3.11}$$

Notice that Δq does not evolve with Q^2 and that $\Delta g(Q^2)$ will grow with Q^2 even if the starting evolution value $\Delta g(Q_0^2)$ is zero, provided $\sum_q \Delta q \neq 0$. This simply reflects the fact that polarized quarks emit polarized gluons.

The second important remark concerning Eq.(4.3.11) is that $\alpha_s(Q^2)\Delta g(Q^2)$ is *not* $O(\alpha_s)$ but $O(1)$. This is the miracle we were hoping for at the beginning of this Section and it makes the correction terms $(\alpha_s/2\pi)\Delta g$ not negligible a priori, as a higher order one.

Let us also notice that, from the obvious sum rule

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \Delta g + L_z \tag{4.3.12}$$

where L_z is the orbital angular momentum contribution to the spin of the proton, and from Eqs.(4.3.11) we have

$$\frac{\partial}{\partial Q^2}\Delta g = -\frac{\partial}{\partial Q^2}L_z\,, \tag{4.3.13}$$

that is the gluon contribution to the proton spin must be compensated by orbital angular momentum [4.23-25]

When analysing the experimental results, Eqs.(4.3.9-10), one has now an extra degree of freedom, the amount of spin carried by the gluons. It is then possible, for example, to find values of Δg such that $\Delta s = 0$ [4.26] or such that $\sum_q \Delta q \simeq 1$.

In the latter, extreme case, one finds that, at the EMC experiment average Q^2 value:

$$\Delta g(Q^2 \simeq 10\, GeV^2) \simeq 7 \tag{4.3.14}$$

Polarized gluon distribution functions $g_+(x)$ and $g_-(x)$ can be found in the literature [4.22,27].

This solution of the spin problem, which assigns to the gluons a large amount of spin so that one gets nearer the naive expectation $\sum_q \Delta q \simeq 1$, has been criticized on various grounds.

It has been noted that the values needed for Δg are too big and cannot be obtained using the gluon distribution functions accepted as reasonable prior to the EMC measurement [4.28]; the distribution functions given in the literature [4.22,27] get most of their contribution to Γ_p from the small x region not probed by the EMC experiment[4.29]; also, the $[g_+(x) - g_-(x)]$ contribution to $g_1(x)$, rather than Γ_p, appears to have the wrong x dependance [4.28].

We leave this problem open and turn to discuss the non perturbative contributions to Γ_p. Let us only add that demanding $\sum_q \Delta q \simeq 1$ might be unrealistic. In fact, assuming $\Delta s = 0$, from the experimental value of a_8 we have[4.11]

$$\sqrt{3}a_8 = \Delta u' + \Delta d' - 2\Delta s' = \Delta u + \Delta d \simeq 0.6 \div 0.7 \tag{4.3.15}$$

A value, at $Q^2 \simeq 10\, GeV^2$, of $\Delta g \simeq 4$ would give $\sum_q \simeq 0.6$ and $\Delta s \simeq 0$. Such a value of Δg, when extrapolated by the Altarelli-Parisi evolution equations to values of $Q^2 \simeq 1 GeV^2$, implies a value of $L_z \simeq 1.5$, which is not in contradiction with the experimental indications about the average $k_\perp$ value of the quarks in a nucleon [4.5,26]. Also, taking into account higher twist $(1/Q^2)$ corrections, leads to an asymptotic value of Γ_p, to be used in partonic formulae, somewhat higher than the EMC result (4.1.6), [4.3.11]. This, in turn, requires smaller values of Δg.

4.4 Non perturbative interpretations of the EMC data

We discuss in this Section some other attempts of interpreting the EMC measurement and solving the spin problem, not based on perturbative QCD. We start from the non perturbative contribution to $\Delta q'$ discussed in Refs.[4.31,32]

Let us say again that the flavour singlet axial vector current

$$J_5^\mu = \sum_q \bar{q}\gamma^\mu \gamma_5 q \qquad (4.4.1)$$

is not conserved due to the anomaly

$$\partial_\mu J_5^\mu = \frac{\alpha_s f}{8\pi} \sum_{i=1}^{8} \epsilon_{\mu\nu\rho\sigma} F_i^{\mu\nu} F_i^{\rho\sigma}, \qquad (4.4.2)$$

where $F^{\mu\nu}$ is the gluon field strength.

As a consequence the expectation value of the associated axial charge operator

$$< Q_5 > \equiv < P,S|Q_5|P,S > = \sum_q < P,S|\bar{q}\gamma^0\gamma_5 q|P,S > \frac{1}{2MS^0} = \sum_q \Delta q' \quad (4.4.3)$$

is not conserved. Notice that in the case of free quarks ($\alpha_s = 0$) Eq.(4.4.3) would give $\Delta q = \Delta q'$ and the R.H.S. of Eq.(4.4.2) is zero. If quarks are not free, however, by quantizing the fermion fields in the presence of background fields to which they are coupled, one gets [4.31,33]

$$Q_5 = Q_5^{op} + Q_5^{an} \qquad (4.4.4)$$

where

$$< P,S|Q_5^{op}|P,S > \equiv < Q_5^{op} > = \sum_q \Delta q \qquad (4.4.5)$$

The second term in Eq.(4.4.4), the anomalous contribution, can be computed explicitely if the quarks are coupled minimally to the gluon potential A^μ [4.31,32]

$$Q_5^{an} = f\frac{g^2}{4\pi^2}\int d^3x\, Tr\, \epsilon^{ijk}\left(A_i\partial_j A_k + \frac{2}{3} g A_i A_j A_k\right) \qquad (4.4.6)$$

Taking as background fields hard free gluons one has

$$< Q_5^{an} >= f < g|\frac{\alpha_s}{\pi}\int d^3x\, Tr\,\epsilon^{ijk}\,(A_i\partial_j A_k)\,|g >= -f\frac{\alpha_s}{2\pi}\Delta g \qquad (4.4.7)$$

thus recovering the perturbative QCD result, Eq.(4.3.7).

However, there might be also non perturbative, confining, soft gluons inside a proton. By decomposing the gluon background field in the sum of a hard part and a soft one, one obtains an additional contribution to $< Q_5^{an} >$, that is one has [4.32]

$$\Delta q' = \Delta q - \frac{\alpha_s}{2\pi}\Delta g - \frac{1}{f} < Q_{cl}^{an} > \qquad (4.4.8)$$

The soft gluon contribution, $< Q_{cl}^{an} >$, could, in principle, be measured through the sum rule (4.3.12), which, using Eq.(4.4.8), reads

$$\frac{1}{2}\sum_q \Delta q' + \Delta g + L_z = \frac{1}{2} - \frac{3\alpha_s}{4\pi}\Delta g - \frac{1}{2} < Q_{cl}^{an} > \qquad (4.4.9)$$

provided we could measure Δg *and* L_z. Explicit calculations of the non perturbative contribution to $< Q_5^{an} >$ can be attempted in the case of a classical instanton field [4.31].

A somewhat similar conclusion, that a measurement of a_0, due to the anomaly, cannot be simply related to a combination of quark and gluon spins, is reached in Ref. [4.34], where a different interpretation of $\sum_q \Delta q'$ is proposed.

A suitable extension of the Goldberger-Treiman relation gives [4.34]

$$\sum_q \Delta q' = \left(\frac{F_\pi}{2M}\right) G(\eta_1^{(O)} NN)\sqrt{6} \qquad (4.4.10)$$

where $G(\eta_1^{(O)} NN)$ is the coupling to the nucleons of $\eta_1^{(O)}$, the flavour singlet Nambu-Goldstone boson associated with the OZI-conserving limit of massless QCD. Eq.(4.4.10) holds for the measured value of $\sum_q \Delta q'$ and includes OZI-violations for $G(\eta_1^{(O)} NN)$.

From the knowledge of the OZI-conserving coupling $G^{(O)}(\eta_1^{(O)} NN)$ and the experimental result (4.2.13), one then simply deduces that $G(\eta_1^{(O)} NN)$ must have a large OZI-violating contribution. There is no reason to talk about a spin crisis.

4.5 Alternative models

Let us finally consider the interpretation of the EMC data in the framework of some other models. We start from the Skyrme model.

In Skyrme models there are no quarks and baryons correspond to solitons of an effective chiral lagrangian which is written in terms of meson fields. One can, however, introduce an effective operator that corresponds to $\sum_q \bar{q}\gamma^\mu\gamma_5 q = J_5^\mu$. Detailed numerical results for the expectation value of this operator between proton states depend on what version of the Skyrme model is chosen. Such results can be summarized by [4.35-37]

$$\sum_q \Delta q' \sim \frac{1}{N_c} \quad \Delta g \simeq 0 \quad L_z \simeq \frac{1}{2} \tag{4.5.1}$$

in the large N_c limit.

In such models the spin is mostly due to orbital angular momentum and the experimental result (4.2.13) comes as no surprise. Some criticism has been raised in Ref. [4.38], arguing that, when properly incorporating the physics of the $U(1)$ anomaly in the model, one should get for $\sum_q \Delta q'$ a result of order unity in a $\frac{1}{N_c}$ expansion.

To avoid the extreme cases of the pure Skyrme model ($N_c \to \infty, \Delta q = \Delta g = 0, L_z = 1/2$) and the naive quark model ($\Delta u + \Delta d = 1, \Delta s = \Delta g = L_z = 0$), some authors use hybrid chiral bag models, which interpolate between the two [4.13, 4.39-40]. In general, by adjusting their parameters, they can reproduce the EMC results.

The main concern about the Skyrme or skyrmion-like models is how much we can trust the large N_c limit results. For example, it has been noted [4.11] that the Skyrme model predicts also, at leading order in N_c

$$\frac{M_{\eta'}}{M} \sim N_c^{-3/2} \quad g_A \sim N_c \tag{4.5.2}$$

far from the experimental values.

Let us conclude by mentioning that in the framework of the Massive Quark Model and Quantum Geometro Dynamics (MQM/QGD) a prediction for the spin asymmetry A, Eq.(4.1.1), was given some time ago [4.41] and it turns out to be in good agreement with the EMC results [4.42]. The MQM/QGD model, however, also predicts a strong violation of the Bjorken sum rule [4.8], widely believed to be correct. A test of this sum rule will be crucial in discriminating between different models.

4.6 Comments and conclusions

Undoubtedly the polarized EMC experiment has prompted a great variety of theoretical activity with a renewed interest in the structure of the nucleon; once more, a polarization experiment, by testing at a much subtler level the existing theories, has led to unexpected and non trivial problems.

We have learned, in the framework of perturbative QCD, that the gluons carry spin (what we already knew) and that their contribution to Γ_p is not a higher order correction as it might appear (what we did not know).

The gluon solution to the "spin problem" might not be a complete one; the amount of spin carried by the gluons, in order to explain alone the EMC data, appears to be unconfortably large. Non perturbative effects have also to be taken in consideration. Both the gluon and the non perturbative contributions are related to the existence of the axial anomaly, thus linking phenomenological results with deep field theory properties.

More experimental information is needed. The amount of spin carried by the gluons should be measured, in order to distinguish between perturbative QCD solutions, which always imply a certain amount of gluon polarization, and alternative models, like the skyrmion ones, which have $\Delta g = 0$. Such a measurement, although difficult, is not impossible in principle (gluon fusion, direct γ production, etc. [4.17,27,43-46]).

The EMC experiment has to be redone, to confirm the existing data; it has also to be done on neutron targets, in order to test the fundamental Bjorken sum rule; proposals already have been presented at CERN and HERA. Let us finally remind that independent information on the same matrix elements measured by the EMC can be obtained in low energy elastic νp scattering [4.11].

5 - General conclusions

The main aim of this paper was that of supplying the basic technique necessary to tackle some difficulties in describing exclusive hadronic interactions in the realm of perturbative QCD; more than on the conceptual foundations of the underlying theory we have concentrated our attention on its phenomenological applications.

We have seen how spin effects can represent a further crucial test of many models, even when they agree with experiment in predicting unpolarized cross sections or, in general, quantities not involving subtle quantum mechanical effects, like interference between different amplitudes.

We have then presented a comprehensive analysis of all the exclusive reactions for which spin data are available and discussed how they compare with the simplest predictions of quark perturbative QCD models; this leads, with almost no exceptions, to serious problems. The nature of the vector gluon couplings to quarks, combined to the simple hadronic configurations with collinear constituents, give, at the moment, predictions in strong disagreement with the experimental information.

If such a disagreement should persist at much higher energies, when all non perturbative and higher order effects should really be negligible, then many of our ideas about hadronic interactions in terms of constituents, via perturbative QCD and hadronic wave functions, should be reconsidered.

That our understanding of hadrons and their interactions, at the constituent level, is far from being completely and definitely clear, has resulted also from our discussion of the proton spin problem in deep inelastic scattering. Much work, both theoretical and experimental, has still to be done.

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Appendix A

We collect here for convenience and completeness some useful formulae and relations which are not fully displaid in the text; we also define the conventions used throughout the paper and make some comments.

A.1 Rotation matrices

Let us denote by $r(\alpha,\beta,\gamma)$ the finite rotation of a physical system with respect to fixed (x,y,z) coordinate axes, where (α,β,γ) are the standard Euler angles. Such a rotation can be achieved by

$$r(\alpha,\beta,\gamma) = r_z(\alpha)r_y(\beta)r_z(\gamma) \qquad (A.1.1)$$

corresponding to the rotation of the physical system by γ around the z-axis, β around the y-axis and α around the z-axis, always with respect to the fixed (x,y,z) coordinate system. The same final result can be reached by the different sequence of rotations

$$r(\alpha,\beta,\gamma) = r_{z''}(\gamma)r_{y'}(\beta)r_z(\alpha) \qquad (A.1.2)$$

corresponding to a rotation by an angle α around the z-axis, by β around the y'-axis (the position of the y-axis after the first rotation) and by γ around the new z''-axis. Obviously, the last rotation does not alter the direction achieved after the first two rotations by a vector originally lying along the z-axis and, if we are only interested in this new direction, the γ rotation is a matter of convention. We will choose, for a rotation changing the z-direction into a direction defined by the polar angles (θ,φ),

$$\alpha=\varphi \qquad \beta=\theta \qquad \gamma=0 \tag{A.1.3}$$

following the convention adopted in Refs.[1.1,2].

To each rotation $r(\alpha,\beta,\gamma)$ there corresponds a unitary operator $U[r]$

$$U[r(\alpha,\beta,\gamma)]=e^{-i\alpha J_z}e^{-i\beta J_y}e^{-i\gamma J_z} \tag{A.1.4}$$

which acts on the states $|jm>$ of spin j and z-component m to give

$$U[r(\alpha,\beta,\gamma)]|jm>=\sum_{m'}D^j_{m'm}(\alpha,\beta,\gamma)|jm'> \tag{A.1.5}$$

where D^j is the standard rotation matrix [A.1].

$$\begin{aligned} D^j_{m'm}(r)\equiv D^j_{m'm}(\alpha,\beta,\gamma)&=<jm'|U[r(\alpha,\beta,\gamma)]|jm> \\ &=e^{-im'\alpha}d^j_{m'm}(\beta)e^{-im\gamma} \end{aligned} \tag{A.1.6}$$

and

$$d^j_{m'm}(\beta)=<jm'|e^{-i\beta J_y}|jm> \tag{A.1.7}$$

We give here the explicit expressions of $D^j_{m'm}$ for $j=1/2$ and $j=1$, for rotations specified by the Euler angles given in Eq.(A.1.3)

$$D^{1/2}(\varphi,\theta,0)\equiv\mathcal{D}^{1/2}(\theta,\varphi)=\begin{pmatrix} e^{-i\varphi/2}\cos\frac{\theta}{2} & e^{-i\varphi/2}\sin\frac{\theta}{2} \\ e^{i\varphi/2}\sin\frac{\theta}{2} & e^{i\varphi/2}\cos\frac{\theta}{2}\end{pmatrix} \tag{A.1.8}$$

$$D^1(\varphi,\theta,0)\equiv\mathcal{D}^1(\theta,\varphi)=\begin{pmatrix} e^{-i\varphi}\cos^2\frac{\theta}{2} & -e^{-i\varphi}\frac{1}{\sqrt{2}}\sin\theta & e^{-i\varphi}\sin^2\frac{\theta}{2} \\ \frac{1}{\sqrt{2}}\sin\theta & \cos\theta & -\frac{1}{\sqrt{2}}\sin\theta \\ e^{i\varphi}\sin^2\frac{\theta}{2} & e^{i\varphi}\frac{1}{\sqrt{2}}\sin\theta & e^{i\varphi}\cos^2\frac{\theta}{2}\end{pmatrix} \tag{A.1.9}$$

A.2 Canonical and helicity spinors

From the usual expression for the canonical spinors

$$u_{s_z}(p)=\frac{\not p+m}{N}\begin{pmatrix}\chi_{s_z}\\ 0\end{pmatrix} \tag{A.2.1}$$

and Eqs.(1.1.7) and (1.1.4) one has the relationship between canonical and helicity Dirac particle spinors

$$u_\lambda(p) = \sum_{s_z} \mathcal{D}^{1/2}_{s_z\lambda} u_{s_z}(p) \tag{A.2.2}$$

Analogously one has

$$v_{s_z}(p) = i\gamma_2 u^*_{s_z}(p) = \frac{\not{p} - m}{N} \begin{pmatrix} 0 \\ -2s_z\chi_{-s_z} \end{pmatrix} \tag{A.2.3}$$

and

$$v_\lambda(p) = \sum_{s_z} \mathcal{D}^{1/2}_{s_z\lambda} v_{s_z}(p) \tag{A.2.4}$$

Let us notice here that the canonical spinors at rest, when properly using the charge conjugation operator (1.1.8), are given, from Eqs.(A.2.1,3), by

$$\begin{array}{ll} u_{\frac{1}{2}}(m,\vec{0}) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} & u_{-\frac{1}{2}}(m,\vec{0}) = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \\ v_{\frac{1}{2}}(m,\vec{0}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} & v_{-\frac{1}{2}}(m,\vec{0}) = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} \end{array} \tag{A.2.5}$$

The (-) sign in $v_{-\frac{1}{2}}$ is usually dropped in the literature: it is actually irrelevant when computing most observables but it becomes crucial when dealing with more subtle quantities involving relative phases or charge conjugation properties.

A.3 Canonical and helicity polarization vectors

The relations (A.2.2) and (A.2.4) between canonical and helicity spinors hold in general between canonical $|js_z>$ and helicity $|j\lambda>$ spin states. In the case of massive spin 1 particles we have

$$\epsilon^\mu_\lambda(p) = \sum_{s_z} \mathcal{D}^1_{s_z\lambda}(\theta,\varphi)\epsilon^\mu_{s_z}(p) \tag{A.3.1}$$

We have not explicitly written in the text the canonical polarization vectors $\epsilon^{\mu}_{s_z}(p)$. They can be obtained either by inversion of Eq.(A.3.1)

$$\epsilon^{\mu}_{s_z}(p) = \sum_{\lambda} \mathcal{D}^{1*}_{s_z\lambda}(\theta,\varphi)\epsilon^{\mu}_{\lambda}(p) \tag{A.3.2}$$

or by directly applying a Lorentz boost along the direction $\hat{p}(\theta,\varphi)$ to the polarization vectors at rest (1.2.1,2):

$$\epsilon^{\mu}_{s_z}(p) = \Lambda^{\mu}{}_{\nu}\epsilon^{\nu}_{s_z}(m,\vec{0}) \tag{A.3.3}$$

where

$$\Lambda^{\mu}{}_{\nu} = \begin{pmatrix} \gamma & \beta\gamma s_\theta c_\varphi & \beta\gamma s_\theta s_\varphi & \beta\gamma c_\theta \\ \beta\gamma s_\theta c_\varphi & 1+s_\theta^2 c_\varphi^2(\gamma-1) & s_\theta^2 c_\varphi s_\varphi(\gamma-1) & s_\theta c_\theta c_\varphi(\gamma-1) \\ \beta\gamma s_\theta s_\varphi & s_\theta^2 c_\varphi s_\varphi(\gamma-1) & 1+s_\theta^2 s_\varphi^2(\gamma-1) & s_\theta c_\theta s_\varphi(\gamma-1) \\ \beta\gamma c_\theta & s_\theta c_\theta c_\varphi(\gamma-1) & s_\theta c_\theta s_\varphi(\gamma-1) & 1+c_\theta^2(\gamma-1) \end{pmatrix} \tag{A.3.4}$$

with $\gamma = E/m, \beta\gamma = p/m, s(c)_\theta = \sin(\cos)\theta$ and $s(c)_\varphi = \sin(\cos)\varphi$.

A.4 Particular cases of helicity spinors and polarization vectors

Let us consider the most common case of a $2 \to 2$ scattering process involving spin 1/2 and spin 1 particles only, in the center of mass frame:

$$p_1(E_1,\vec{p};\lambda_1) + p_2(E_2,-\vec{p};\lambda_2) \to p_1'(E_1',\vec{p}\,';\lambda_1') + p_2'(E_2',-\vec{p}\,';\lambda_2') \tag{A.4.1}$$

Let us choose xz as the scattering plane, so that the polar angles of the four particles are given by $\hat{p}_1(0,0)$, $\hat{p}_2(\pi,\pi)$, $\hat{p}_1'(\theta,\varphi)$ and $\hat{p}_2'(\pi-\theta,\varphi+\pi)$. The corresponding helicity spinors are then given by, from Eqs.(1.1.2-4), (1.1.10) and (A.1.8):

$$u_{\lambda_1}(p_1) = \frac{1}{N_1}\begin{pmatrix} E_1+m_1 \\ 2p\lambda_1 \end{pmatrix}\chi_{\lambda_1}$$
$$u_{\lambda_2}(p_2) = \frac{i}{N_2}\begin{pmatrix} E_2+m_2 \\ 2p\lambda_2 \end{pmatrix}\chi_{-\lambda_2}$$

$$
\begin{aligned}
u_{\lambda_1'}(p_1') &= \frac{1}{N_1'}\begin{pmatrix} E_1'+m_1' \\ 2p'\lambda_1' \end{pmatrix} \mathcal{D}^{1/2}(\theta,0)\chi_{\lambda_1'} \\
u_{\lambda_2'}(p_2') &= \frac{i}{N_2'}\begin{pmatrix} E_2'+m_2' \\ 2p'\lambda_2' \end{pmatrix} \mathcal{D}^{1/2}(\theta,0)\chi_{-\lambda_2'} \\
v_{\lambda_1}(p_1) &= \frac{1}{N_1}\begin{pmatrix} -p \\ 2\lambda_1(E_1+m_1) \end{pmatrix} \chi_{-\lambda_1} \\
v_{\lambda_2}(p_2) &= \frac{i}{N_2}\begin{pmatrix} -p \\ 2\lambda_2(E_2+m_2) \end{pmatrix} \chi_{\lambda_2} \\
v_{\lambda_1'}(p_1') &= \frac{1}{N_1'}\begin{pmatrix} -p' \\ 2\lambda_1'(E_1'+m_1') \end{pmatrix} \mathcal{D}^{1/2}(\theta,0)\chi_{-\lambda_1'} \\
v_{\lambda_2'}(p_2') &= \frac{i}{N_2'}\begin{pmatrix} -p' \\ 2\lambda_2'(E_2'+m_2') \end{pmatrix} \mathcal{D}^{1/2}(\theta,0)\chi_{\lambda_2'}
\end{aligned}
\qquad (A.4.1)
$$

where $p = |\vec{p}|, p' = |\vec{p}'|$ and the N's are the normalization factors $N_i = \sqrt{m_i + E_i}$ (if we normalize as $\bar{u}u = 2m, \bar{v}v = -2m$). We have also used the relationship

$$
\mathcal{D}^{1/2}(\pi-\theta,\pi+\varphi)\chi_\lambda = i\mathcal{D}^{1/2}(\theta,\varphi)\chi_{-\lambda} \qquad (A.4.2)
$$

Analogously, the helicity polarization vectors are given by, from Eqs.(1.2.3-4) and (A.1.9)

$$
\begin{aligned}
\epsilon^\mu_{\lambda_1=\pm1}(p_1) &= -\frac{\lambda_1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ i\lambda_1 \\ 0 \end{pmatrix} & \epsilon^\mu_{\lambda_1=0}(p_1) &= \frac{1}{m_1}\begin{pmatrix} p \\ 0 \\ 0 \\ E_1 \end{pmatrix} \\
\epsilon^\mu_{\lambda_2=\pm1}(p_2) &= -\frac{\lambda_2}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ -i\lambda_2 \\ 0 \end{pmatrix} & \epsilon^\mu_{\lambda_2=0}(p_2) &= \frac{1}{m_2}\begin{pmatrix} p \\ 0 \\ 0 \\ E_2 \end{pmatrix} \\
\epsilon^\mu_{\lambda_1'=\pm1}(p_1') &= -\frac{\lambda_1'}{\sqrt{2}}\begin{pmatrix} 0 \\ \cos\theta \\ i\lambda_1' \\ -\sin\theta \end{pmatrix} & \epsilon^\mu_{\lambda_1'=0}(p_1') &= \frac{1}{m_1'}\begin{pmatrix} p' \\ E_1'\sin\theta \\ 0 \\ E_1'\cos\theta \end{pmatrix} \\
\epsilon^\mu_{\lambda_2'=\pm1}(p_2') &= -\frac{\lambda_2'}{\sqrt{2}}\begin{pmatrix} 0 \\ \cos\theta \\ -i\lambda_2' \\ -\sin\theta \end{pmatrix} & \epsilon^\mu_{\lambda_2'=0}(p_2') &= \frac{1}{m_2'}\begin{pmatrix} p' \\ -E_2'\sin\theta \\ 0 \\ -E_2'\cos\theta \end{pmatrix}
\end{aligned}
\qquad (A.4.3)
$$

For massless spin 1 particles the $\lambda = 0$ helicity polarization vectors are absent and the others are unchanged.

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