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Intrinsic Magnetic Torque at Low Magnetic Induction

by

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Abstract

Using anisotropic London theory we obtain the intrinsic magnetic torque for extreme type II uniaxial superconductors for any value of the magnetic induction. We consider the vortex lines straight and take into account the contribution of the supercurrents flowing inside the vortex core within the London theory. We show that the interline and intraline free energies give opposite torque contributions, the first drives the magnetic induction parallel to the superconductor's axis of symmetry and the second orthogonal to it. At high magnetic induction our torque expression generalizes V. Kogan's formula since it has no free parameters other than the anisotropy $\gamma = m_1/m_3$ and the Ginzburg-Landau parameter κ. At low magnetic induction we propose a way to observe vortex chains effects in the total torque based on the fact that London theory is linear and the energy to make a single vortex line in space is independent of the magnetic induction.

Key-words: Superconductors; Vortex; Torque.

I. INTRODUCTION

Below T_c a superconductor experiences a torque whenever its magnetization is not aligned to the external magnetic field. Flux pinning and shape effects are the main reasons for this effect[2]. However anisotropy can also produce a magnetization not oriented along the external field and this is known as the intrinsic torque[1].

For temperatures close to T_c and fields $H >> H_{c1}$ the torque τ has been measured[5] in samples of $Y_1Ba_2Cu_3O_{7-\delta}$ and $Bi_2Sr_2Cu_2O_{8+\delta}$ as a function of the angle θ between the external field H and the axis of symmetry $\hat{z}$.

In the range of temperature and field considered in these experiments, collecting the data through increasing or decreasing the angle θ gives essentially the same results. This total reversibility excludes pinning as the source of torque. θ is the angle between the axis of symmetry ($\hat{z}$) and the vortex lines. Shape effects were also disregarded as the source of torque in these compounds according to D.E. Farrel et al.[2]. Such experiments were in fact measuring a new effect, the intrinsic torque whose only source is the anisotropy of the copper-oxide materials.

At sufficiently high temperatures where reversibility exists[4], the magnetic torque data fits remarkably well the formula first derived by V.G. Kogan[1]. His result was obtained in the context of anisotropic London theory for the mixed state.

In this paper we also calculate the intrinsic magnetic torque in the context of anisotropic London theory. Our analysis generalizes V. Kogan's results in many aspects. His formula only applies to sufficiently high values of the magnetic induction B where the vortex line density is high whereas ours is valid for all range of B where anisotropic London theory holds. Therefore the external field is assumed to be far off the critical fields, $H_{c1} < H < H_{c2}$. In Kogan's formula, a short distance cutoff removes the interactions on a range smaller than the coherence length, we do not remove them, and take into account the electromagnetic contribution of the vortex core to the torque. Our model has no free parameters describing properties of the vortex state such as the vortex lattice arrangement (η) or the upper critical field (H_{c2}). In fact we determine these parameters as functions of the anisotropy γ and the Ginzburg-Landau parameter κ. Differences of the order $1/\kappa$ between the magnetic induction and the external field are ignored since κ is assumed to be very large in this work.

In this paper we show that there are two opposite contributions to the intrinsic torque stemming from the *interline* and the *intraline* energies. We also show here that the dependence on θ of each of these two contributions is remarkably distinct and we propose ways to independently extract their effects from the total magnetic torque. The *interline* vortex energy is due to the interaction among the distinct vortex lines in space. The *intraline* vortex energy is the one necessary to make the independent vortex lines in space and so only considers the interaction among the segments of the same line. In our approximation we treat the vortex core under the Gaussian extension of London theory[9] that ignores fluctuations of the order parameter, but considers the contributions to the torque coming from the electromagnetic interactions within the coherence length region.

The reason why the interline and the intraline vortex interactions give distinct and opposite contributions to the magnetic torque is easy to understand. The intrinsic magnetic torque rotates the anisotropic superconductor towards the angle θ corresponding to the energy minimum. For a certain vortex line density B, one searches for the angle θ that renders the lowest vortex lattice energy. The supercurrent flow costs less energy if confined to the crystal's plane of low mass, orthogonal to the axis of symmetry. Because of anisotropy, such supercurrents can create a magnetization leaning towards the axis of symmetry instead of the applied field, thus resulting on a magnetic torque. This intuitive argument is enough to determined the lowest energy lattice at least in the limit of large vortex line density. Since supercurrents flow preferably on the least mass plane (CuO planes), among all the vortex lattices, the well known triangular lattice at $\theta = 0°$ has the lowest energy.

However the picture developed so far only concerns phenomena occuring outside the vortex core namely, the interline part of the free energy. The intraline contribution is also important in torque considerations because the energy to make straight vortex lines in space depends on θ. Phenomena taking place within the vortex line volume, defined by the core area and the line length L, are associated to the intraline energy. This volume is smaller for lines perpendicular to the axis of symmetry ($L\pi\xi_1\xi_3$) than parallel to it ($L\pi\xi_1^2$). Provided that the energy density does not change significantly as a function of θ, one expects that to make non-interacting vortex lines in space is energetically easier at $\theta = 90^o$ then at $\theta = 0^o$. In summary the interline and intraline energies are opposite and this is immediately seen when comparing them at the two limit angles, $\theta = 0^o$ and $\theta = 90^o$, for fixed vortex density (B). The intraline energy is larger at $\theta = 0^o$ than at $\theta = 90^o$ whereas the interline energy is just the opposite. Interline and intraline free energy contribute in opposite ways to the torque, the former pushes the superconductor to $\theta = 0^o$ and the latter to $\theta = 90^o$. In this paper we give a detailed description of such competing contributions to the magnetic torque and show that their differences should be noticeable mainly at extremely high and small values of B. The torque grows with the density of vortex lines and to discuss intraline and interline effects we find more suitable the study of the curve $\tau(\theta, H)/H$ versus θ. For instance at high magnetic field $\tau(\theta, H)/H$ decreases for increasing H and we see this as a consequence of the competition between interline and intraline effects. A more quantitative descrition of such effect is also given in this paper.

At low magnetic field, a new effect was predicted some time ago, the onset of the so-called vortex chains[6,7], whose origin is on the attraction between vortex lines separated by a distance of the order of the penetration length and located on the plane of symmetry, this one defined by $\hat{z}$ and $\vec{B}$[20,16,21]. The chain state was experimentally observed at low temperature ($4K$) and low magnetic field ($25G$) in untwinned $YBCO$ single crystals by magnetic decoration experiment[7]. We calculate in this paper the intrinsic torque at the limit of the chain state. Remarkably the chain state occuring at some intermediate angle θ has lower energy than the triangular lattice ($\theta = 0^\circ$) and is the absolute interline energy minimum[14]. However at low B, the lattice energy is very weak and the total free energy is totally dominated by the intraline energy resulting that $\tau(\theta, H)/H$ undergoes no significant change as a function of the applied field. In summary the interline contribution to the total free energy can not be directly detected in the total torque at low B. Taking advantage of the linearity of London theory, We suggest here a way to extract the intraline effects from the total torque. Basically we propose that the magnetic torque difference be analyzed, that is $\tau(\theta, H_1)/H_1 - \tau(\theta, H_2)/H_2$ versus θ, where H_1 and H_2 are close in values. We claim that the intraline energy does not contribute to this difference and only interline effects do it. This curve is obtained here using this paper's formulation of the London theory.

We take the assumption that vortex lines are straight at any value of the vortex density and are always arranged on a periodic array, with just one vortex line per unit cell. This work is restricted to the large κ limit and corrections of the order $1/\kappa$ between the magnetic induction and the external field[13] are not considered. The effects due to the superconductor shape are not treated, thus the dimagnetization factor vanishes so that the external magnetic field is the thermodynamic field $\vec{H}$.

This paper is partitioned as follows. Section 2 describes the major considerations about anisotropic London theory, intrinsic torque and the definition of the reduced unit system. Section 3 presents the intraline free energy and a formula that fits the gaussian model reasonably well for large values of κ. The intraline contribution to the intrinsic torque is analyzed in this Section. Section 4 quickly reviews the interline vortex energy, describes the search for the optimal lattice cell parameters for known θ and B and gives the interline contribution to the torque. In Section 5 our results for the intrinsic torque at high and low B are discussed and the major conclusions of this paper drawn.

II. THE FREE ENERGY AND THE INTRINSIC TORQUE IN ANISOTROPIC LONDON THEORY

In London theory,

$$F_{total} = \frac{1}{8\pi} \int dv \; \{\lambda^2_{jk} \vec{J}_j \vec{J}_k + \vec{h} \cdot \vec{h}\} \qquad (1)$$

the supercurrent kinetic energy is also determined by the local magnetic field $\vec{h}(\vec{r})$ through Ampère's law, $\vec{\nabla} \wedge \vec{h} = 4\pi \vec{J}/c$. For a superconductor with uniaxial anisotropy along the $\hat{z}$ axis, the penetration length tensor is

$$\lambda = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_1 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \tag{2}$$

in the crystal's frame of reference.

For the mixed state of type II superconductors London theory determines the distribution of vortex lines in space. The free energy of a system of vortex lines becomes

$$F = \frac{\Phi_0^2}{8\pi\lambda_1^2} \sum_{i,j} \oint \oint d\vec{l}_i \cdot [\mathbf{G}(\vec{r}_i - \vec{r'}_j)] \cdot d\vec{l'}_j, \tag{3}$$

where i and j label the vortex lines, $d\vec{l}_i(d\vec{l'}_j)$ is the i-th (j-th) vortex line length element located at $\vec{r}_i(\vec{r'}_j)$ and $\mathbf{G}(\vec{r})$ is a 3×3 matrix whose elements are obtained from London's equations. Recently expressions for G in the crystal's axes have been derived both on a non-diagonal[24,10,11] and on a diagonal form[12,23] and they are equivalent, assuming that vortices form either closed loops or infinite lines. The diagonal expression for G is

$$\mathbf{G}(\vec{r}) = \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \begin{pmatrix} g_1(\vec{k}) & 0 & 0 \\ 0 & g_1(\vec{k}) & 0 \\ 0 & 0 & g_3(\vec{k}) \end{pmatrix} \tag{4}$$

$$g_1(\vec{k}) = \frac{\lambda_1^2 |f(\vec{k})|^2}{\lambda_3^2 k_\parallel^2 + \lambda_1^2 k_3^2 + 1} \qquad g_3(\vec{k}) = g_1(\vec{k}) \frac{\lambda_3^2 \vec{k}^2 + 1}{\lambda_1^2 \vec{k}^2 + 1} \tag{5}$$

In the above equations $\vec{k}_\parallel$ is the component of $\vec{k}$ orthogonal to the axis of symmetry.

In order to obtain the energy of a single vortex line in space in the context of London theory, the function $f(\vec{k})$ is introduced. It determines the local magnetic field inside the core and controls the divergences that exist in London theory for coreless vortices. We analyze here the Gaussian model[9], where $|f(\vec{k})|^2 = \exp(-k_\parallel^2\xi_1^2 - k_3^2\xi_3^2)$, added of the Ginzburg-Landau relation, $\lambda_1\xi_1 = \lambda_3\xi_3$, which holds once the anisotropic Ginzburg-Landau mass dependence is assumed for the penetration length, $\lambda_i \propto \sqrt{m_i}$, and for the anisotropic coherence length, $\xi_i \propto 1/\sqrt{m_i}$. We define the parameter $\gamma = m_1/m_3 < 1$ that determines the anisotropy in London theory. All the numerical results in this paper are obtained for a fixed value of anisotropy, namely $\gamma = 0.02$ which is the typical anisotropy of $YBa_2Cu_3O_7$. This anisotropy parameter can be obtained in several measurements of magnetic properties[17-19]. The Ginzburg-Landau parameter characterizing the type II superconductor is κ, defined with respect to the the average penetration length, $\lambda_{av} = (\lambda_1^2\lambda_3)^{1/3}$, and the average coherence length, $\xi_{av} = (\xi_1^2\xi_3)^{1/3}$ [22]. In many expressions in this paper we use the Ginzburg-Landau parameter along the uniaxial direction, κ_z,

$$\kappa = \frac{\lambda_{av}}{\xi_{av}} = \frac{\Phi_0}{2\pi\sqrt{2}H_c\xi_{avg}^2}, \qquad \kappa_z = \frac{\lambda_3}{\xi_3} = \frac{\kappa}{\gamma^{2/3}} \tag{6}$$

The above equation also defines the critical field H_c in terms of κ. Consider the London free energy of N straight vortices along the direction $\hat{Z} = \cos\theta\hat{z} + \sin\theta\hat{x}$, the uniaxial symmetry given by $\hat{z}$. The two other unit vectors, $\hat{x}$ and $\hat{y}$, correspond to the the crystal's plane of low mass (CuO planes), orthogonal to the to the axis of symmetry. London's free energy, Eq.(1), is linear and decomposes into the sum of two parts,

$$F_{total} = F_{intra} + F_{inter}, \qquad F_{intra} = NF_{line}, \qquad F_{inter} = \frac{\Phi_0}{8\pi}\sum_{ij}{}' \hat{Z}\cdot\vec{h}(\vec{r}_i - \vec{r}_j) \tag{7}$$

where F_{line} is the energy required to make one single line of length L in space, and $\vec{r}_i$ becomes just the position of the vortices on the plane orthogonal to the line direction $\hat{Z}$.

Vortex lines are oriented along $\hat{Z}$ and the magnetic flux is quantized, thus giving

$$\vec{B} = \frac{1}{Vol}\int dv\vec{h} = \frac{\Phi_0}{L_1L_2\sin\phi}\hat{Z} \tag{8}$$

The vortices are taken to form a regular array with one vortex per unit cell. L_1 is the unit cell length along the $(\hat{Z},\,\hat{z})$ plane and L_2 is the other unit cell length that makes an angle ϕ with L_1. The free energy per volume is $f = F/Vol$, and the volume is, according to the previous discussion, $Vol = N\,L\,(L_1L_2\sin\phi)$.

We shall work here in reduced units, such that fields are expressed in units of the critical field, $\sqrt{2}H_c$ and the free energy per volume is in units of $H_c^2/4\pi$. In these reduced units, the thermodynamic field is obtained from the free energy per volume by $\vec{H} = (1/2)\partial f/\partial\vec{B}$. The flux quantization condition becomes in reduced units,

$$\frac{L_1}{\lambda_3} = \sqrt{\frac{2\pi\rho}{B\kappa_z \sin\phi}} \tag{9}$$

We take that the free energy for a system of vortex lines must have the following shape,

$$f = \frac{B}{\kappa_z}[\epsilon(\kappa_z,\gamma,\theta) + V(B\kappa_z,\gamma,\theta,\rho,\phi) - B\kappa_z] + B^2, \tag{10}$$

This expression follows from a previous derived formula for the interline energy of coreless vortices plus the Gaussian model for vortex lines. The multiplicative B term indicates that the free energy per volume should be proportional to the density of vortex lines. The intraline contribution, $B\epsilon/\kappa_z$, does not depend on the unit cell parameters, ρ and ϕ, nor on the vortex line density, B. This is expected since the self-energy of the vortex lines should depend on none of these parameters. However the interline term BV/κ_z, which describes the interaction among vortex lines, must depend on the density, B, and on the arrangement of vortex lines in space, described by ρ and ϕ.

The above Eq.(10) is the starting point of our torque considerations. The magnetic torque, $\vec{\tau} = \vec{M} \wedge \vec{H}$, becomes in reduced units, $\vec{\tau} = \vec{B} \wedge \partial f/\partial \vec{B}$. Here we consider a superconductor with no dimagnetization tensor, and the external field is obtained from $\vec{B} = \vec{H} + 4\pi\vec{M}$. The Helmholtz free energy is determined by the density of vortex lines B, and the angle θ, and in terms of these variables the torque becomes

$$\vec{\tau} = -\frac{B}{\kappa_z}[\frac{\partial\epsilon}{\partial\theta} + \frac{\partial V}{\partial\theta}]\hat{y} \tag{11}$$

The intrinsic torque must be expressed as a function of the experimentally accessible variables that is, the external magnetic field and its angle with the uniaxial axis of symmetry. In the limit of extremely high values of κ, when the magnetization density $\vec{M}$ is sufficiently small, one can approximate in Eq.(11), B by H and the angle between $\vec{H}$ and $\hat{z}$ by θ. Further corrections to this limit will be considered elsewhere[13].

Many conclusions can be drawn from torque experiments by noticing that the intraline and interline energies contribute additively to the torque in this approximation of large κ values. This is the starting point of many of our considerations here. On the next two Sections we study separately the intraline and interline contributions to the free energy.

III. THE INTRALINE FREE ENERGY

In this Section, the vortex lines do not interact, therefore the magnetic induction B plays no role other than determine the vortex line density. The energy required to make a single vortex line in space considers the interaction among its segments. London theory is able to determine this energy once a model describing the currents circulating within the vortex core is assumed. Variations of the order parameter inside the core are not considered in the context of London theory. The size of the core is determined by the coherence lengths, which are anisotropic and obey the anisotropic Ginzburg-Landau theory condition, $\lambda_1\xi_1 = \lambda_3\xi_3$. Consider straight vortex lines not aligned to the crystal's axes. The intraline torque drives the system to its lowest intraline energy configuration. Thus for constant B, one searches for the angle θ of minimum energy. The energy of N non-interacting vortex lines is $F_{intra} = (N\ L\ \epsilon)(\Phi_0/4\pi\lambda_{av})^2$ and in reduced units becomes, $F_{intra}/(VolH_c^2/4\pi) = (B/\kappa_z)\ \epsilon$ where

$$\epsilon = \frac{1}{2\pi\sqrt{\Gamma(\theta)}}\Big(\frac{\lambda_{av}}{\xi_1}\Big)^2 \int dq \int dk_2\ [g_1(q,k_2)\sin^2\theta + g_3(q,k_2)\cos^2\theta] \tag{12}$$

where

$$g_1(q,k_2) = \frac{e^{-q^2-k_2^2}}{\left(\lambda_3/\xi_1\right)^2\left(q^2+k_2^2\right)+1}, \tag{13}$$

$$g_3(q,k_2) = g_1(q,k_2)\,\frac{\left(\lambda_3/\xi_1\right)^2\left(q^2/\Gamma(\theta)+k_2^2\right)+1}{\left(\lambda_1/\xi_1\right)^2\left(q^2/\Gamma(\theta)+k_2^2\right)+1} \tag{14}$$

and we define

$$\Gamma(\theta) = \sqrt{\cos^2\theta + \gamma\sin^2\theta} \tag{15}$$

At the limit of very large κ_z, we found an analytical expression that gives a close estimate to ϵ.

$$\epsilon_{approx} = \frac{\Gamma(\theta)}{\gamma}\ln\Big[\frac{\kappa_z\gamma}{\Gamma(\theta)}\Big] \tag{16}$$

We have computed the intraline energy and torque as a function of the angle θ for fixed anisotropy ($\gamma = 0.02$) and high Ginzburg-Landau parameter ($\kappa_z = 1000$), using both expressions Eq.(12) and Eq.(16). Fig.(1) shows our results indicating that the above formula, Eq.(16), (dash line) gives a good description of the Gaussian model in the limit of extreme type II superconductors (high κ value).

An immediate conclusion obtained from Fig.(1) is that to make isolated vortex lines perpendicular to the axis of symmetry is less costlythan to make them parallel. At fixed density B, and varying θ, the intraline energy decreases for increasing angle as shown in Fig.(1). The configurations $\theta = 0^o$ and $\theta = 90^o$ are torque free since they correspond to local extremes of the intraline energy. However the former is unstable whereas the latter is stable. If displaced from an equilibrium configuration, the torque drives the non-interacting vortex lines to the minimum energy configuration at $\theta = 90^o$. The outcome of this Section is very clear according to Eq.(11). The intraline torque contribution is always positive and null at the extreme angles.

On the next Section we discuss the interline energy and show, that its contribution to the torque is opposite in signal to the one studied here. Therefore the total torque results from the competition between these two effects.

IV. THE INTERLINE FREE ENERGY

The interaction among distinct vortex lines gives a contribution to the free energy which is completely determined by the local magnetic field of a single vortex in space, $\vec{h}(\vec{r})$. The magnetic field at position $\vec{r}$ due to an array of vortices, $\vec{L}(n_1, n_2) = n_1 L_1 \hat{e}_1 + n_2 L_2 \hat{e}_2$, $\hat{e}_1 \cdot \hat{e}_2 = \cos\phi$ where n_1 and n_2 are any integers, is $\vec{H}(\vec{r}) = \sum_{n_1,n_2} \vec{h}[\vec{r} - \vec{L}(n_1, n_2)]$. Then the total magnetic field at $\vec{r} = 0$, excluding the field of the vortex located at the origin is $\vec{H}' = \lim_{\vec{r}\to 0}[\vec{H}(\vec{r}) - \vec{h}(\vec{r})]$. This field $\vec{H}'$ describes the magnetic field felt by each vortex due to the presence of all others. Any vortex on the lattice gives identical contribution to the free energy, and to determine the interline free energy defined in Eq.(7), it is enough to multiply the field felt by a single vortex by the total number of vortex lines. Thus one obtains that $F_{inter} = N\Phi_0\hat{Z}\cdot\vec{H}'/8\pi$, which in reduce units becomes $f_{inter} = F_{inter}/(VolH_c^2/4\pi) = (B/\kappa_z)V$ For the sake of completeness we present the fast convergent series expansion for V obtained in Ref. 14,

$$V = \frac{\Gamma(\theta)}{\gamma}\tilde{V}_0 - \frac{1-\gamma}{\gamma}\cos^2\theta\int_0^1 \frac{du}{\sqrt{c_1(u)c_2(u)}}\tilde{V}_1(u) +$$

$$+\frac{|\cos\theta|}{\gamma}\ln\Big(\frac{\Gamma(\theta)+|\cos\theta|}{\sqrt{\gamma}(1+|\cos\theta|)}\Big) \qquad (17)$$

$$\tilde{V}_0 = \frac{1}{2\mu_o\tanh(\mu_o\sigma/2)} + 2\sum_{m,s=1}^{\infty}\cos(m\chi s)\frac{e^{-\sqrt{\mu_o^2+m^2}\sigma s}}{\sqrt{\mu_o^2+m^2}} +$$

$$+\sum_{m=1}^{\infty}[\frac{1}{\sqrt{\mu_o^2+m^2}} - \frac{1}{m}] + ln(\mu_o/2) + c_e \qquad (18)$$

$$\tilde{V}_1(u) = \frac{1}{4\mu_o}\left(\frac{1}{\tanh(\mu_o\sigma/2)} + \frac{\mu_o\sigma/2}{\sinh^2(\mu_o\sigma/2)}\right) +$$

$$+{\mu_o}^2\sum_{m,s=1}^{\infty}\cos(m\chi s)\frac{e^{-\sqrt{\mu_o^2+m^2}\sigma s}}{\mu_o^2+m^2}[\frac{1}{\sqrt{\mu_o^2+m^2}} + \sigma s] + \frac{{\mu_o}^2}{2}\sum_{m=1}^{\infty}\frac{1}{(\mu_o^2+m^2)^{\frac{3}{2}}} \qquad (19)$$

Both functions $\tilde{V}_0$ and $\tilde{V}_1$, depend on $\sigma = \sqrt{c_1(u)/c_2(u)}2\pi\sin\phi/\rho$, $\chi = 2\pi\cos\phi/\rho$, and $\mu_o = \sqrt{\rho/(2\pi\sin\phi\kappa_z B)}/\sqrt{c_1(u)}$ where $c_1(u) = \cos^2\theta + \gamma\sin^2\theta - \cos^2\theta(1-\gamma)u$, and $c_2(u) = 1-(1-\gamma)u$. For $\tilde{V}_0$, which is not being integrated, σ, χ and μ_o are functions of $c_1(0) = \cos^2\theta + \gamma\sin^2\theta$ and $c_2(0) = 1$. $c_e = 0.5772\ldots$ is Euler's constant. The anisotropic superconductor is characterized by κ and γ, and to find the interline free energy for a certain line density B and θ, the search for the optimal lattice arrangement (ρ, θ) has to be carried out first. In this paper we dont't do the utter search for the optimal lattice parameters. Our search is approximate and consists in determining the minimal interline free energy only within the set of rectangular unit cells ($\phi = 90°$). In this set, the search for the optimal lattice reduces to finding the ratio ρ between the unit cell sides, L_1 and L_2. For instance for $\theta = 0^o$, our search is limited to find the square lattice as the optimal unit cell instead of the the triangular lattice, known to give the vortex configuration of minimum energy. The reason for this approximation is twofold. The search for the optimal unit cell angle ϕ becomes increasingly difficult at low magnetic induction. The plane defined by the magnetic induction B and the axis of symmetry is a direction of "easy" elastic deformations where the shear moduli C_{66} drops sharply as a function of θ, $C_{66}(\theta = 90^o)/C_{66}(\theta = 0^o) = \gamma^{3/2}$[26]. At the limit of low magnetic induction the onset of vortex chains freezes the line separation along this plane, L_1, and the interchain separation, through the flux quantization condition, $L_2 \sin\phi = \Phi_0/(BL_1)$[14]. Hence the interline free energy has essentially the same value for all lattices characterized by ϕ and L_2 as long as these two variables satisfy the flux quantization and the vortex chain requirement. The other reason for not carrying the complete search for the optimal lattice parameters is that we found that distinct judicious choices of the unit cell angle ϕ matter very litlle because the free energy and the torque are more sensitive to changes on θ than ϕ. We took the two choices, $\phi = 90^o$, known to be a local energy maximum for any θ, and $\phi = \tan^{-1}(\sqrt{3}/\Gamma(\theta))$ the unit cell angle for high magnetic induction[25] and searched for the optimal ρ in each of these two cases. The result is that no significant differences in the free energy and in the torque versus θ plots were found. Hence our interline energy is of the form,

$$f_{inter} = \frac{B}{\kappa_z}[V(B\kappa_z, \gamma, \theta, \rho_{optimum}, \phi = 90^o) - B\kappa_z] + B^2 \tag{20}$$

The behavior of the interline free energy with θ has been previously obtained[14]. The lower and upper angles $\theta = 0^o$ and $\theta = 90^o$ correspond to energy equilibrium configurations as shown in Fig.(2) for three values of B. $\theta = 90^o$ is the maximum energy configuration for any B. Surprisingly $\theta = 90^o$ is the absolute minimum only for sufficiently high values of B although it is always a local minimum. For low magnetic induction, the vortex state coalesces into chains that eventually surpasses the $\theta = 0^o$ triangular lattice as the absolute interline energy minimum[14,15].

The attraction between vortex lines along the special plane defined by the magnetic induction and the axis of symmetry follows from the properties of $\vec{h}(\vec{r})$, the local magnetic field of a single vortex line in space[20,16,21]. Fig.(3) depicts this distance of minimum repulsion among the vortex lines as a function of θ. In this Figure we also show that the lattice states considered in Fig.(2) are indeed approaching the chain limit for decreasing B. The unit cell parameter along the plane of symmetry, L_1/λ_3, for each of these states, is displayed there.

The interline torque is shown in Fig.(4) and reflects the wealthy structure of the interline free energy previously discussed. At high B, the torque is negative because the lattice configuration of minimum energy is at $\theta = 0$ and this energy grows monotonically to $\theta = 90^o$. Notice that the resulting torque contribution is opposite in sign to the one found in the previous Section. Remarkably for low B the interline torque vanishes at angles other than 0^o and 90^o and this is due to the onset of the chains.

On the next Section the intraline and interline contributions are considered together, and we discuss a method to see the tiny interline effects on the torque at low magnetic induction.

V. CONCLUSION

The anisotropic superconductor experiences a torque if displaced of its equilibrium angle. Therefore to understand the intrinsic magnetic torque one should analyze the free energy and look for its equilibrium configurations. We study here a free energy for vortex lines in the context of London theory which is the sum of the independent intraline e interline contributions, Eq.(7). The vortices are assumed straight, the intraline energy is the energy of a single line times the number of lines. We consider the gaussian model describing the electromagnetic interaction within the core. The interline energy is obtained from the interaction among distinct coreless vortex lines. In the previous two sections we have examined separately each contribution, here we discuss them together.

At the limit of extremely large vortex density B, the total free energy considered here is in leading order, the same one found in Ref.(25) added to the intraline energy. Therefore it acquires the same form of Kogan's[5] expression,

$$f = B^2 + \frac{B\Gamma(\theta)}{2\kappa_z\gamma}\log[\frac{H_{c2}(\theta,\gamma)\eta(\theta,\gamma)}{B\Gamma(\theta)}], \tag{21}$$

however we have no free parameters other than γ and κ.

$$H_{c2}(\theta,\gamma) = \frac{\kappa_z\gamma}{\Gamma(\theta)}, \quad \eta(\theta,\gamma) = \frac{\gamma e^{2c_e} A_2^2(\theta,\rho,\gamma)}{4\Gamma(\theta)\sigma A_1^2(\theta,\rho,\gamma)} \tag{22}$$

The auxiliary functions are

$$A_1(\theta,\rho,\gamma) = \prod_{s=1}^{\infty}[1 - 2\exp(-\sigma\, s)\cos(\chi\, s) + \exp(-2\sigma\, s)]$$

and

$$A_2(\theta,\rho,\gamma) = [\frac{\Gamma(\theta) + |\cos\theta|}{\sqrt{\gamma}(1 + |\cos\theta|)}]^{\frac{|\cos\theta|}{\Gamma(\theta)}}$$

The above functions depend on $\sigma = 2\pi\Gamma(\theta)\sin\phi/\rho$, $\chi = 2\pi\cos\phi/\rho$ and the Euler constant c_e. The lattice parameters that extremize this free energy[25] are given by $\phi = \tan^{-1}(\sqrt{3}/\Gamma(\theta))$ and $\rho = 2\cos\phi$. Through Lindemann's criterion, the free energy has been used to predict[3] the angular dependence of the melting temperature[8]. Similar considerations concerning the above expression will be seen elsewhere[13].

The total free energy has its absolute maximum and minimum for all the B range at $\theta = 0^\circ$ and $\theta = 90^\circ$ respectively, as shown in Fig.(5). These angles also correspond to equilibrium energy configurations and this was found independently for the intraline and interline energies, according to Figs.(1) and (2). The intraline free energy is always the dominant contribution to the free energy. At low vortex density their interaction is weak and we find that the interesting features found in Section III for the interline energy are completely eclipsed by the intraline effects. When the vortex lines are closer to each other at high B, the interline effects are noticeable on the total free energy. This easily explains some features found in Fig.(5), like the B dependence of the maximum and minimum values of the total energy at $\theta = 0^\circ$ and $\theta = 90^\circ$. It follows from the above considerations that the maximum (minimum) energy decreases (increases) for increasing B. Consequently $\tau(\theta)/H$ must decrease for increasing H as shown in Fig.(6). At high fields, the vortex lattice of less energy is at $\theta = 0^o$ and this produces a negative torque not large enough to overcome the intraline contribution to the torque. At low fields $\tau(\theta)/H$ is essentially due to non-interacting vortex lines. The torque has a positive sign, drives the system to its energy minimum at $\theta^o = 90$, where vortex lines are less costly to make. In order to see the interesting interline effects in the magnetic torque, first one has to eliminate the dominant intraline contribution to it. This is possible, based on the fact that London theory is linear and the single vortex line energy is independent of the vortex density (B). For this reason we study $\tau(\theta, H_1)/H_1 - \tau(\theta, H_2)/H_2$, which carries no intraline energy contributions. Fig.(7) displays this torque difference for very low values of the fields as a function of θ. The rich structure found in these curves reflects the interesting interline effects studied in Section IV, namely the onset of the vortex attraction along the symmetry plane.

FIGURES

FIG. 1. Comparison of the intraline free energy and the torque for the Gaussian model, Eq.(12) (continuous line), and the approximated formula, Eq.(16) (dashed line).

FIG. 2. The Interline free energy removed of the background field, is shown for different values of $B\kappa_s$. For $B\kappa_s = 5$, the angle close to $\theta = 60^o$ is the absolute minimum and characterizes the onset of the vortex chains.

FIG. 3. The vortex distance along the symmetry plane, defined by the magnetic induction and the axis of symmetry, is displayed for the $B\kappa_s$ values studied in Fig.(2). For comparison, the single vortex magnetic field minimum along this plane of symmetry is also shown here (dashed line).

FIG. 4. The interline torque is shown here for the same values of $B\kappa_s$ studied in Fig.(2)

FIG. 5. The total free energy including the intraline and interline contributions and removed of the background field is shown here for an extremely low and high value of $B\kappa_s$.

FIG. 6. The total torque including the intraline and interline contributions is displayed for the values of $B\kappa_s$ considered in Fig.(5).

FIG. 7. The difference between the total torque is depicted for very low magnetic fields near the onset of vortex chains.

fig.1

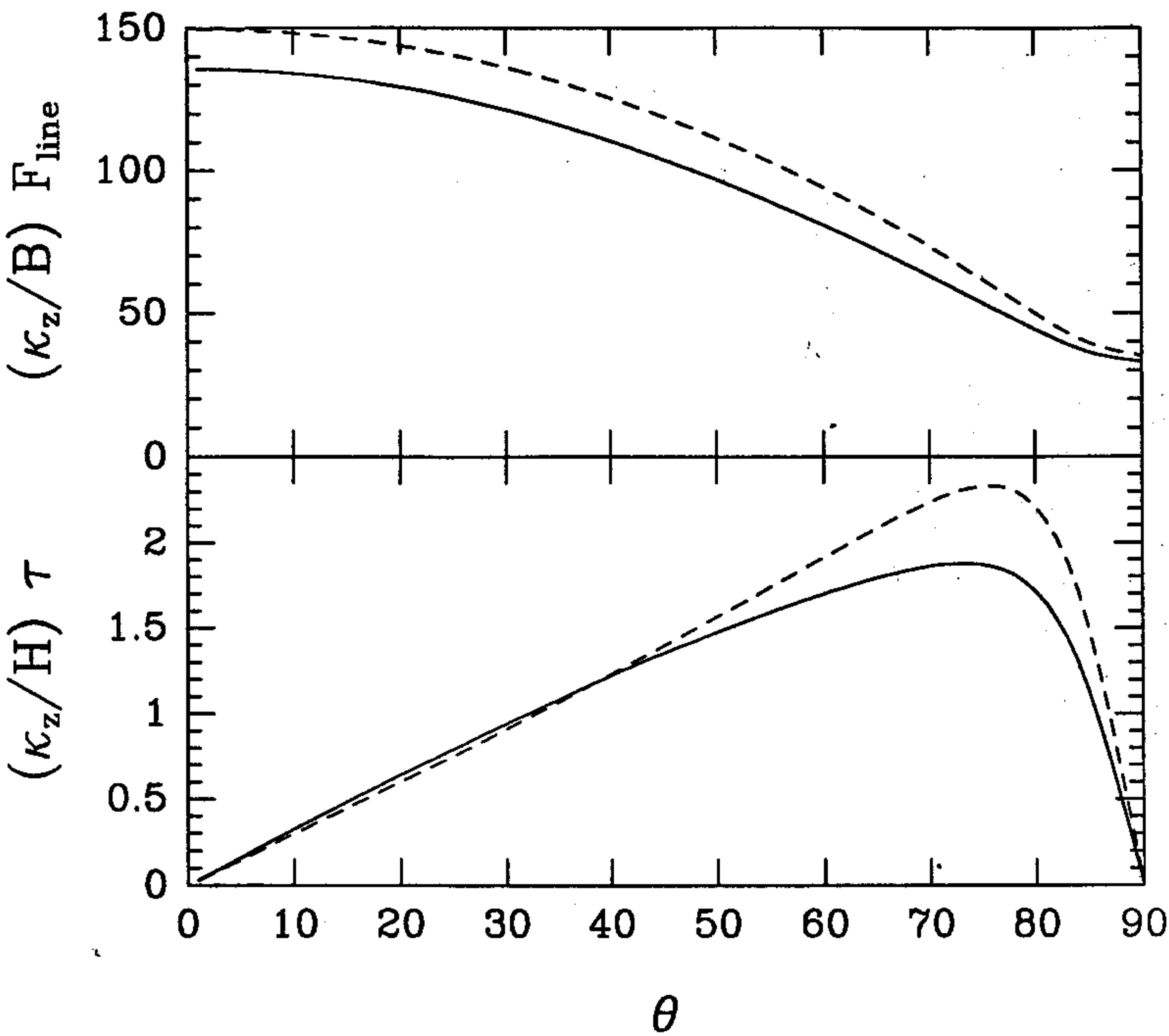

150
100
50
0
2
1.5
1
0.5
0
0
10
20
30
40
50
60
70
80
90
$(\kappa_z/B)\ F_{line}$
$(\kappa_z/H)\ \tau$
θ

fig.2

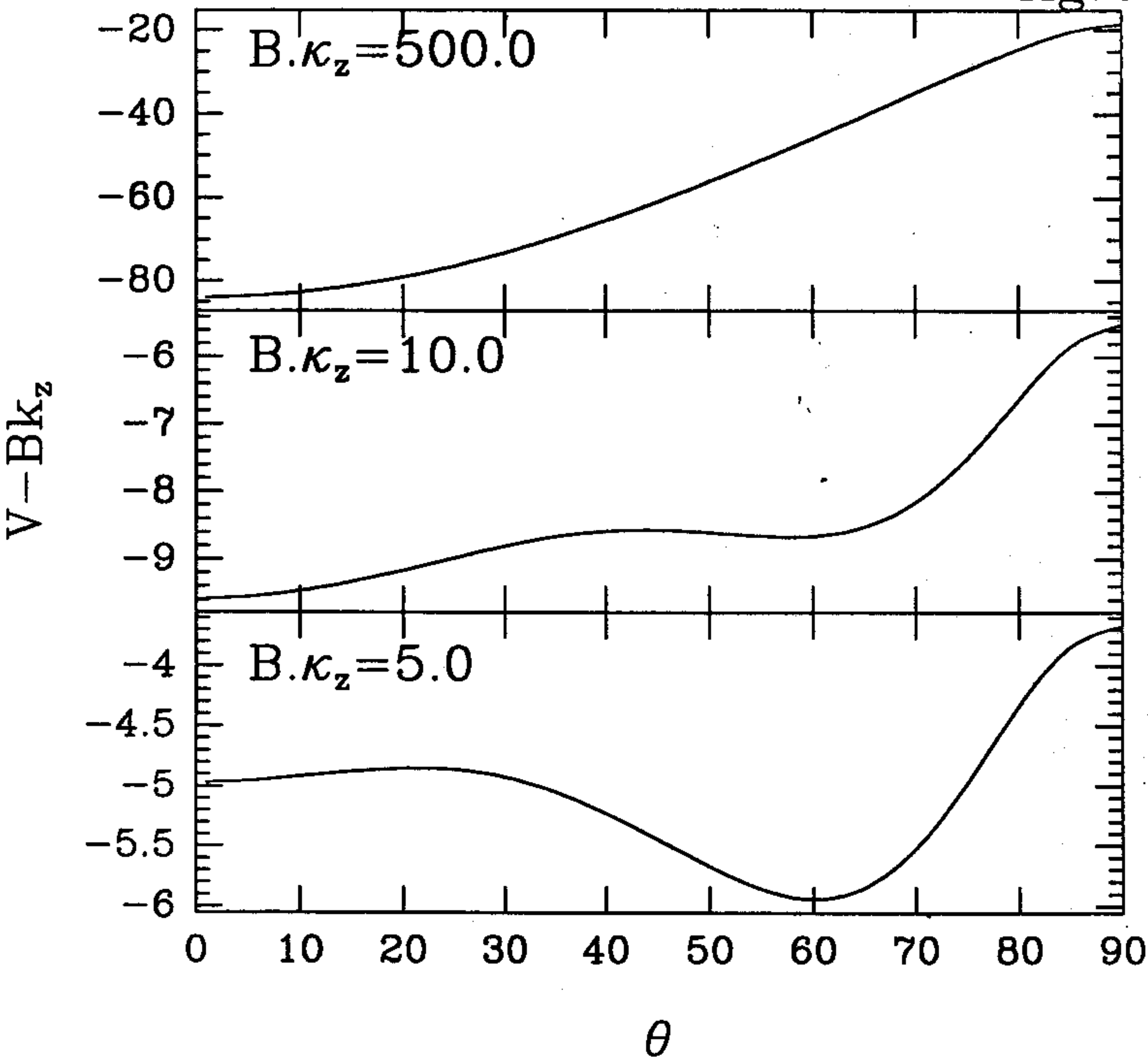

B.κz=500.0
B.κz=10.0
B.κz=5.0
V−Bkz
θ
−20
−40
−60
−80
−6
−7
−8
−9
−4
−4.5
−5
−5.5
−6
0
10
20
30
40
50
60
70
80
90

fig.3

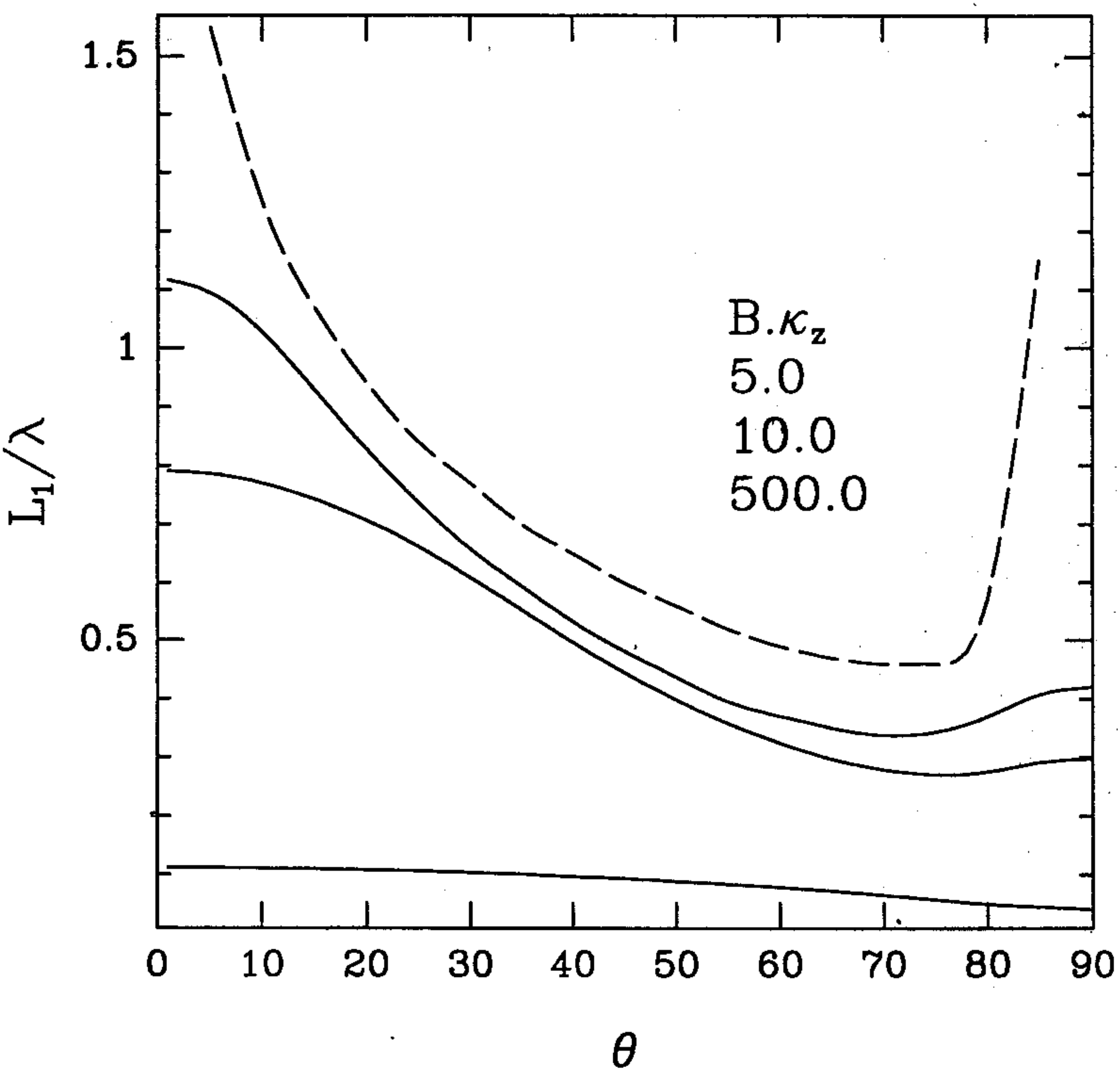

1.5
1
0.5
L_1/λ
$B.\kappa_z$
5.0
10.0
500.0
0
10
20
30
40
50
60
70
80
90
θ

fig.4

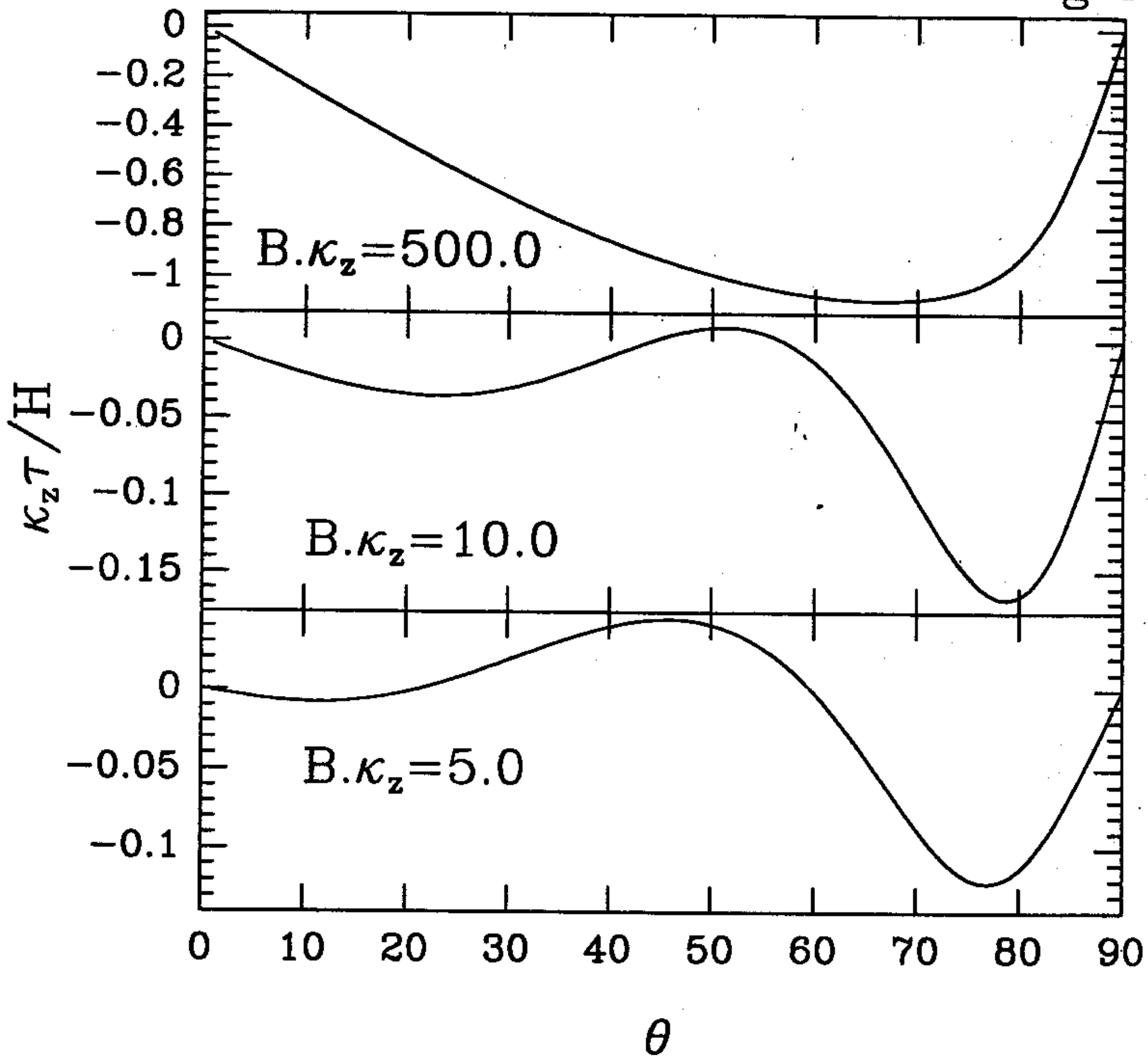

0
-0.2
-0.4
-0.6
-0.8
-1
B.κz=500.0
0
-0.05
-0.1
-0.15
B.κz=10.0
0
-0.05
-0.1
B.κz=5.0
κz τ/H
0 10 20 30 40 50 60 70 80 90
θ

fig.5

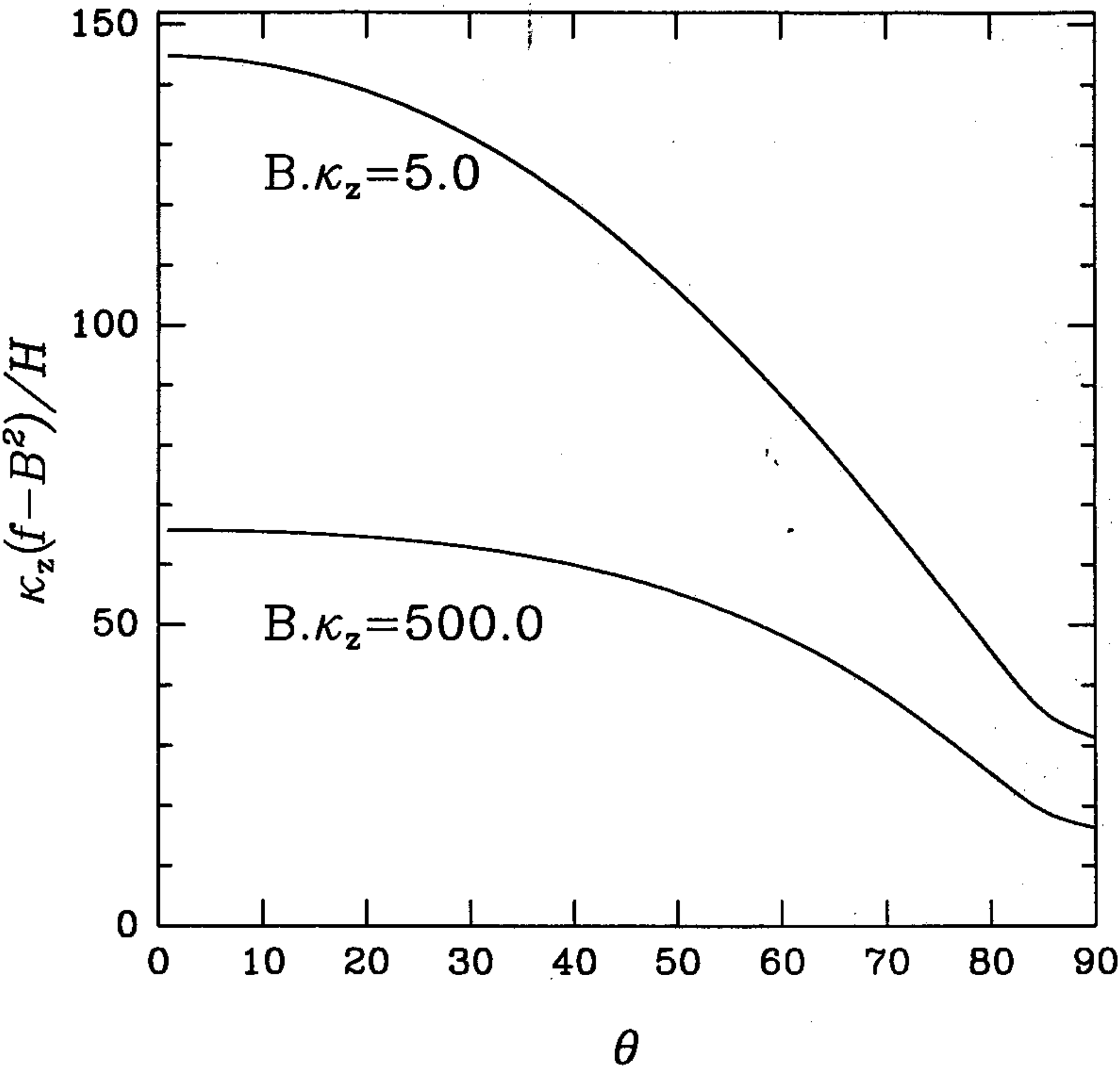

150
100
50
0
0
10
20
30
40
50
60
70
80
90
$\kappa_z(f-B^2)/H$
θ
B.κ_z=5.0
B.κ_z=500.0

fig.6

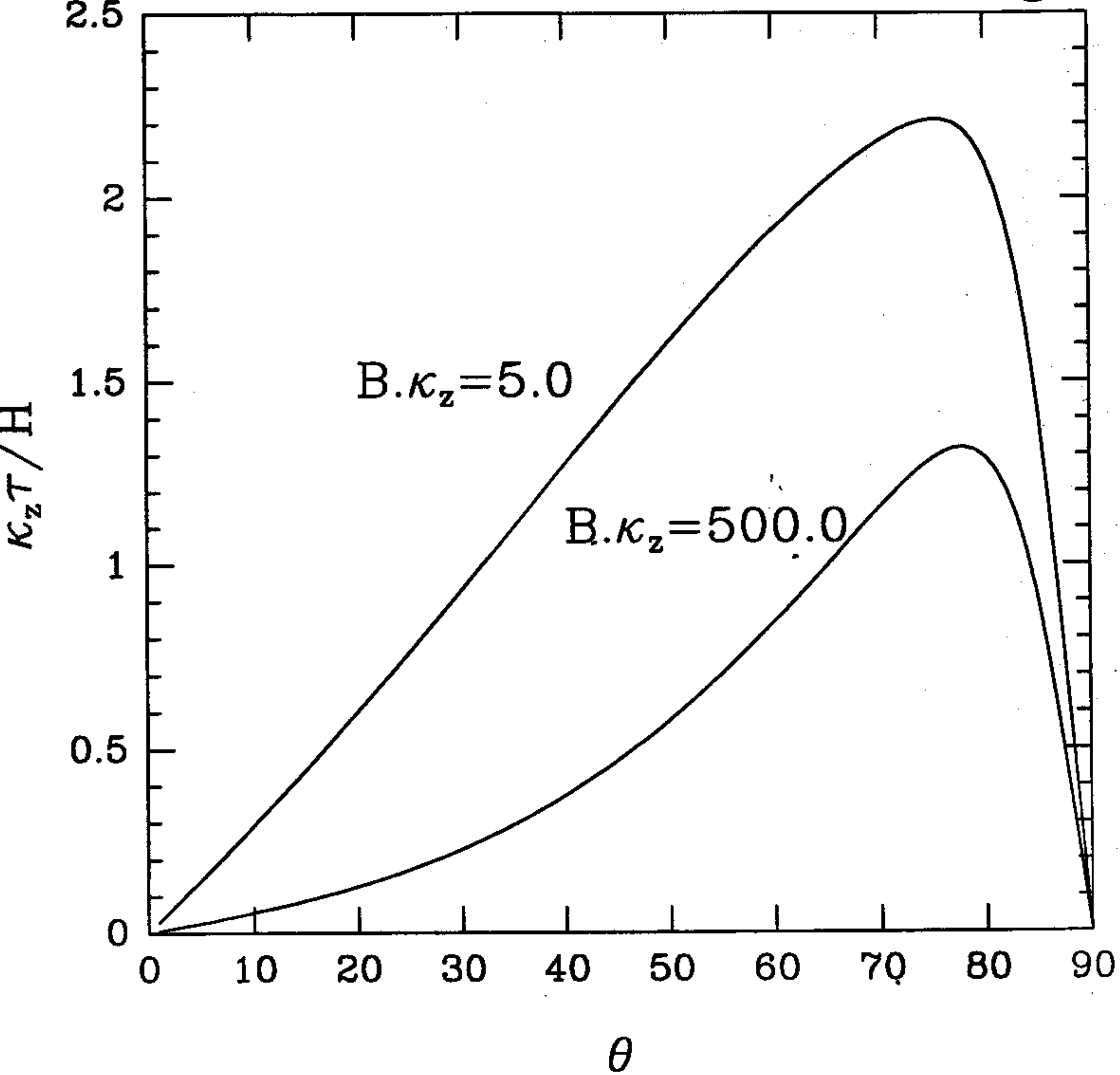
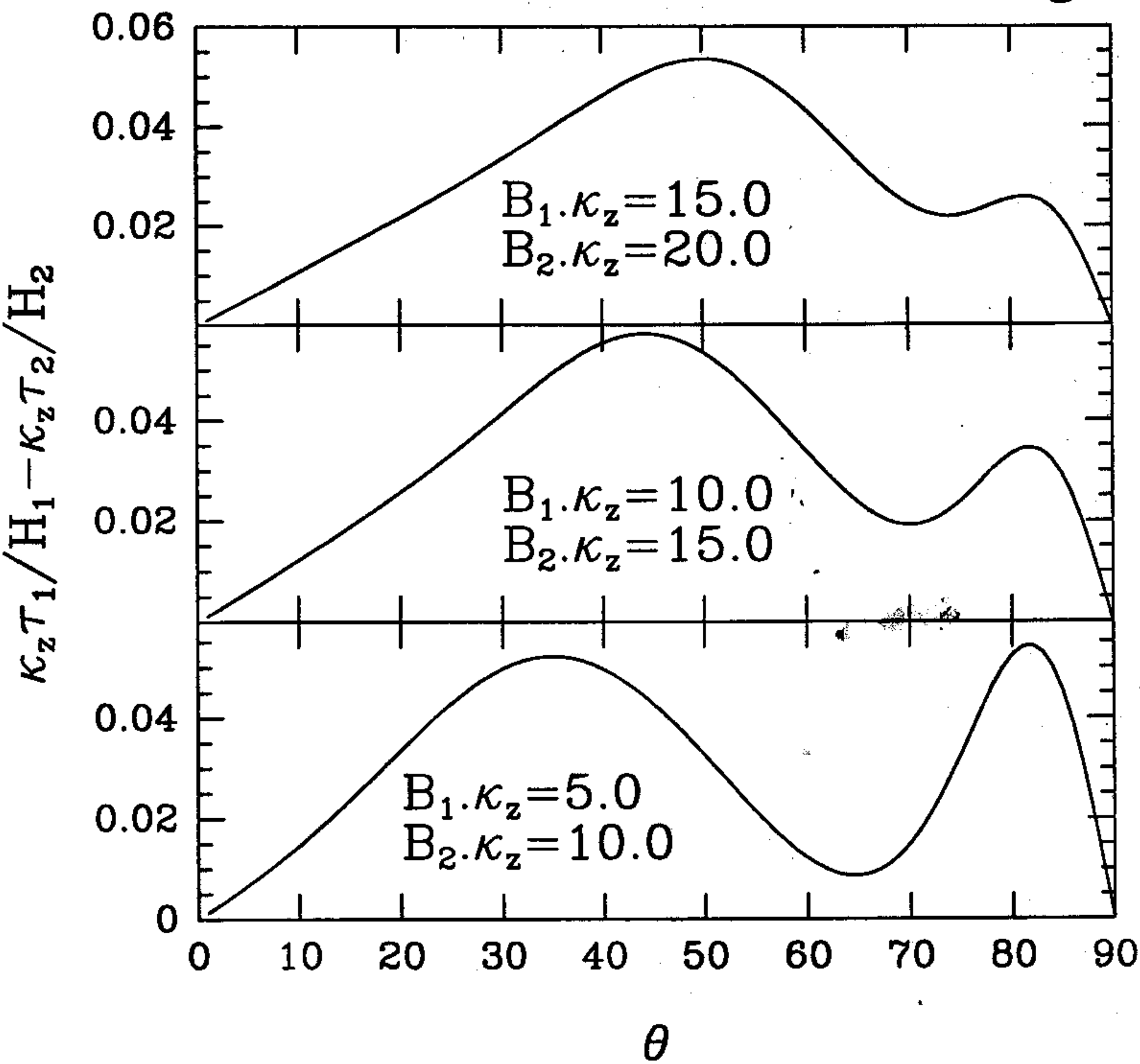

2.5
2
1.5
1
0.5
0
$\kappa_z \tau / H$
$B.\kappa_z=5.0$
$B.\kappa_z=500.0$
0
10
20
30
40
50
60
70
80
90
θ

fig.7


0.06
0.04
0.02
0.04
0.02
0.04
0.02
0
$B_1.\kappa_z=15.0$
$B_2.\kappa_z=20.0$
$B_1.\kappa_z=10.0$
$B_2.\kappa_z=15.0$
$B_1.\kappa_z=5.0$
$B_2.\kappa_z=10.0$
$\kappa_z\tau_1/H_1-\kappa_z\tau_2/H_2$
0
10
20
30
40
50
60
70
80
90
θ

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