ISSN 0029-3865

CBPF-NF-044/90

ON THE SELF-DUALITY CONDITION IN CHERN-SIMONS SYSTEMS

by

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Abstract

It is shown that in the Higgs and Chern-Simons-Higgs (without the Maxwell term) systems the 'self-duality' constraint on the scalar field (combined with the equations of motion) by itself leads to specific forms for the potential. Similar results are shown to hold also for the supersymmetric extensions of the theories written in terms of superfields. A 'supersymmetric self-duality' constraint on the matter superfield is proposed which contains the bosonic one and leads to specific forms of superpotentials without invoking arguments based on an explicit N=2 supersymmetry.

Key-words: Gauge theory; Supersymmetry; Chern-Simons systems.

1. Introduction

In (2+1) dimensional spacetime the possibility of including the Chern-Simons (CS) term[1] in the abelian Higgs model has drawn recently much interest. It was found[2] that in the Chern-Simons-Higgs (CSH) system, obtained by ignoring the Maxwell term, the energy functional obeys a Bogomol'nyi-type[3] lower bound for a special choice of the Higgs potential. The bound is achieved by fields satisfying a set of first order 'self-duality' equations. The (charged) vortex solutions and nontopological soliton solutions of these equations have also been discussed[4].

We show here that the self-duality ansatz for the scalar field taken along with the equations of motion lead by themselves, for the CSH system (and for the usual Higgs Lagrangian without the CS term), to the above-mentioned special choice of the Higgs potential. We consider also the supersymmetric extension of the CSH system using superfields. A supersymmetric 'self-duality' condition on the matter superfield in terms of the gauge covariant spinorial derivative is formulated. The equations of motion may again be solved to obtain a special choice for the superpotentials which contains the result of the purely bosonic system. No explicit use of N=2 supersymmetry is invoked[5].

2. Chern-Simons Higgs System

The Lagrangian for the bosonic Chern-Simons Higgs system with the Maxwell term added to it reads as follows

$$\mathcal{L} = -(\tilde{\mathcal{D}}^l a^*)(\mathcal{D}_l a) - V(|a|^2) - \frac{\kappa}{4}\epsilon^{lmn} v_l f_{mn} - \frac{1}{4} f_{lm} f^{lm}, \tag{1}$$

where $\mathcal{D}_m = \partial_m + iev_m$, $\tilde{\mathcal{D}}_m = \partial_m - iev_m$ and $m = 0, 1, 2$ are the spacetime indices. Our metric is $\eta_{mn} = diag(-1, 1, 1)$ with $\epsilon^{012} = 1$. The equations of motion are derived to be

$$\mathcal{D}^l \mathcal{D}_l a = V'(|a|^2) a, \tag{2}$$

and

$$-\partial_m f^{ml} + \frac{\kappa}{2}\epsilon^{lmn} f_{mn} = j^l, \tag{3}$$

Here $V'(|a|^2) = \partial V/\partial|a|^2$ and $j^l(v)$ is the conserved Noether current

$$j^l = ie(a^*\mathcal{D}^l a - a\tilde{\mathcal{D}}^l a^*), \tag{4a}$$

$$\partial_l j^l(v) = 0, \tag{4b}$$

For static configurations eq.(2) reduces to $(i, j = 1, 2)$

$$\mathcal{D}_i\mathcal{D}_i a = (V' - e^2{v_0}^2)a, \tag{5}$$

and we find from eq.(3) corresponding to $l = 0, 1$ and 2, respectively,

$$\partial_i\partial_i v_0 + \kappa f_{12} = 2e^2 v_0\,|a|^2\,, \tag{6}$$

$$\partial_2(f_{12} + \kappa v_0) = j_1, \tag{7}$$

$$\partial_1(f_{12} + \kappa v_0) = -j_2, \tag{8}$$

where we have adopted the gauge $\partial_l v^l = 0$.

On imposing the self-duality condition $\mathcal{D}_1 a = -i\mathcal{D}_2 a$, $\tilde{\mathcal{D}}_1 a^* = i\tilde{\mathcal{D}}_2 a^*$, eq.(5) reduces to

$$e^2{v_0}^2 + ef_{12} = V'(|a|^2), \tag{9}$$

where we use $\mathcal{D}_i\mathcal{D}_i a = -i[\mathcal{D}_1, \mathcal{D}_2]a = ef_{12}$ which follows from the self-duality ansatz. We also obtain $j_1 = e\partial_2|a|^2$ and $j_2 = -e\partial_1|a|^2$ so that *eq.*(4*b*) expressing current conservation holds. We then derive from eqns.(7) and (8)

$$f_{12} + \kappa v_0 = e(|a|^2 - C^2), \tag{10}$$

where C is a constant.

Consider now first the case when the CS term is absent ($\kappa = 0$). It follows from eq.(6) that we may set $v_0 = 0$. Eqs.(9) and (10) then lead to $V'(|a|^2) = e^2(|a|^2 - C^2)$ and consequently to the following specific form

$$V(|a|^2) = (e^2/2)(|a|^2 - C^2)^2, \tag{11}$$

for the potential apart from a convenient constant of integration. The existence of neutral vortex solutions was pointed out in the present case by Nielsen and Olesen[6] and a self-dual vortex solution may be constructed[7] explicitly.

We discuss next the case of the CSH system without the Maxwell term. Since the first term in eq.(6) is now absent we find

$$f_{12} = (2e^2/\kappa)v_0|a|^2 \tag{12}$$

A nonvanishing magnetic field is accompanied by a nonzero v_0 (and consequently an electric field) even for the static solutions. Eq.(10) is now replaced by

$$v_0 = (e/\kappa)(|a|^2 - C^2), \tag{13}$$

Eliminating v_0 and f_{12} from eq.(9) by using eqs.(12) and (13) we find $V' = (e^4/\kappa^2)$ $(|a|^2 - C^2)(3|a|^2 - C^2)$ which leads to the following specific potential which is of sixth degree in the scalar field

$$V(|a|^2) = \frac{e^4}{\kappa^2}|a|^2(|a|^2 - C^2)^2 \tag{14}$$

This result should be compared with the earlier case where only a fourth power of the scalar field was needed to obtain a time independent self-dual solution. Eqs.(11) and (14) remain unaltered even if we adopt the self-duality conditon with an opposite sign.

It is shown in ref. 2 that when the potential is given by eq.(14) we obtain a lower bound on the energy (of static vortex solutions[8]) saturated by the fields obeying the self-duality condition and with f_{12} as given by eqs.(12) and (13).

3. Supersymmetric Chern-Simons Higgs System

a) Gauge Superfield. Super Chern-Simons Action.

The gauge vector potential in the case of 2+1 spacetime dimensions is contained[9] in a Majorana spinor connection superfield

$$\Gamma^\alpha(x,\theta) = \chi^\alpha(x) + \bar{\theta}_\beta(\frac{1}{2}\epsilon^{\beta\alpha}v(x) + \gamma_l^{\beta\alpha}v^l(x)) + i\bar{\theta}\theta\eta^\alpha(x), \tag{15}$$

where $\eta^\alpha = \lambda^\alpha(x) - \frac{1}{2}(\gamma^l\partial_l\chi(x))^\alpha$. Here the Majorana 2-spinor field $\lambda(x)$ is the superpartner of the gauge field $v_l(x)$ while the spinor $\chi(x)$ and scalar $v(x)$ are auxiliary fields. We use a Majorana representation for gamma matrices with $(\gamma^{0\alpha}{}_\beta) = i\sigma_2$ and define $(\epsilon^{\alpha\beta}) = i\sigma_2$, $(\epsilon_{\alpha\beta}) = -i\sigma_2$ where $\alpha, \beta = 1,2$ are spinorial indices. A Majorana spinor then has real components. The spinors with lower index carry an upperbar for convenience with $\bar{\psi}_\alpha = \epsilon_{\alpha\beta}\psi^\beta$ and it is easily shown that $\bar{\psi}_\alpha\xi^\alpha \equiv \bar{\psi}\xi$ is Lorentz invariant.

The generator of $N = 1$ supersymmetry transformations, Q^α, is given by $iQ^\alpha = (\partial/\partial\bar{\theta}_\alpha) - i(\gamma^l\theta)^\alpha\partial_l$ while the covariant spinorial derivative is $D^\alpha = (\partial/\partial\bar{\theta}_\alpha) + i(\gamma^l\theta)^\alpha\partial_l$ and $\bar{D}_\alpha = \epsilon_{\alpha\beta}D^\beta$. They satisfy $\left\{\bar{D}_\alpha, D^\beta\right\} = -2i\gamma^{l\beta}{}_\alpha\partial_l$. The gauge transformation parameter is a real scalar superfield

$$\tilde{\Phi}(x,\theta) = \tilde{a}(x) + i\bar{\theta}\tilde{\psi}(x) + i\bar{\theta}\theta\tilde{f}(x), \tag{16}$$

with the real scalars $\tilde{a}(x)$,$\tilde{f}(x)$ and Majorana spinor $\tilde{\psi}(x)$ as component fields. The infinitesimal gauge transformation of the spinor superfield is $\delta\Gamma^\alpha = -iD^\alpha\tilde{\Phi}$ and we find $\delta v_l = -\partial_l\tilde{a}$, $\delta\lambda = 0$, $\delta\chi = \tilde{\psi}$ and $\delta v = -4\tilde{f}$.

The fact that $\lambda^\alpha = \frac{i}{2}\bar{D}_\beta D^\alpha\Gamma^\beta|_{\theta=0}$ and its gauge invariance property suggest defining the field strength superfield by

$$W^\alpha = \frac{i}{2}\bar{D}_\beta D^\alpha\Gamma^\beta. \tag{17}$$

Its gauge invariance follows from the identity $\bar{D}_\beta D^\alpha D^\beta = 0$. Explicitly

$$W^\alpha(x,\theta) = \lambda^\alpha(x) + \frac{1}{2}\bar{\theta}_\beta(\epsilon^{lmn}f_{lm})\gamma_n^{\beta\alpha} + \frac{i}{2}\bar{\theta}\theta(\gamma^l\partial_l\lambda)^\alpha, \tag{18}$$

where $f_{lm} = \partial_l v_m - \partial_m v_l$.

The normalization in eq.(17) is chosen such that the gauge superfield action

$$I_g = \frac{1}{8}\int d^3x d^2\theta\, \bar{W}_\alpha W^\alpha \equiv \frac{1}{8}\int d^3x\, \bar{D}D(\bar{W}_\alpha W^\alpha)|_{\theta=0} \tag{19}$$

(if the surface terms are ignored) gives rise to the standard action in terms of the component fields.

The bosonic CS term is found to be contained in $\bar{\Gamma}W = \bar{\Gamma}\gamma^l\partial_l\Gamma - \frac{i}{2}\bar{\Gamma}D\bar{D}\Gamma$ and the action for the super CS term is written as

$$I_{c.s.} = -\frac{\kappa}{8}\int d^3x d^2\theta \bar{\Gamma}W \equiv -\frac{\kappa}{8}\int d^3x\, \bar{D}D(\bar{\Gamma}W)|_{\theta=0}. \tag{20}$$

Its expression in terms of the component fields is easily obtained in the supersymmetric gauge $\bar{D}\Gamma = 0$ which corresponds to setting $v = 0$, $\partial_l v^l = 0$ and $\chi = \frac{1}{\Box}(\gamma^l\partial_l\lambda)$.

b) Gauge Covariant Coupling to Matter Superfield.

The matter superfield is a complex scalar superfield

$$\Phi(x,\theta) = a(x) + i\bar{\theta}\psi(x) + i\bar{\theta}\theta f(x). \tag{21}$$

Here $a(x)$ is a complex scalar, $\psi^\alpha(x)$ its complex superpartner and $f(x)$ an auxiliary complex scalar. Under an infinitesimal gauge transformation it transforms as $\delta\Phi = ie\tilde{\Phi}\Phi$ and hence the gauge covariant spinorial derivatives may be defined to be

$$\nabla^\alpha\Phi = (D^\alpha + e\Gamma^\alpha)\Phi, \qquad \tilde{\nabla}^\alpha\Phi^* = (D^\alpha - e\Gamma^\alpha)\Phi^*. \tag{22}$$

The following closure relation

$$\{\bar{\nabla}_\alpha, \nabla^\beta\} = -2i\gamma^{l\beta}{}_\alpha\nabla_l, \tag{23}$$

where $\nabla_l = (\partial_l + e\Gamma_l)$ and $\Gamma_l = \frac{i}{2}\bar{D}\gamma_l\Gamma$, is easily established. The Bianchi identities are satisfied due to the identity $\bar{D}W = 0$ and we have an irreducible representation[9]. The matter action with minimal coupling is

$$I_m = \int d^3x d^2\theta \; (\frac{1}{4}\bar{\tilde{\nabla}}_\alpha \Phi^* \nabla^\alpha \Phi + iV(|\Phi|^2)), \tag{24}$$

where V is the superpotential.

c) Supersymmetric Self-Duality Constraint. Specific Form of Superpotential.

From the total action we obtain the following equations of motion

$$\frac{1}{4}\bar{\nabla}_\alpha \nabla^\alpha \Phi(x,\theta) = iV'(|\Phi|^2)\Phi, \tag{25}$$

$$(\gamma^l \partial_l W)^\alpha - \kappa W^\alpha = e(\Phi^* \nabla^\alpha \Phi - \Phi \tilde{\nabla}^\alpha \Phi^*). \tag{26}$$

The conservation of Noether's current requires

$$\bar{D}_\alpha(\Phi^* \nabla^\alpha \Phi - \Phi \tilde{\nabla}^\alpha \Phi^*) = 0. \tag{27}$$

We adopt the supersymmetric gauge $\bar{D}\Gamma = 0$ and consider static configurations. The self-duality constraint on the matter superfield now takes the form

$$\nabla^\alpha \Phi = i(\gamma^0 \nabla)^\alpha \Phi, \qquad \tilde{\nabla}^\alpha \Phi^* = -i(\gamma^0 \tilde{\nabla})^\alpha \Phi^*. \tag{28}$$

Eq.(27) is seen to be satisfied if we use eq.(28) along with the identity $\bar{D}\gamma^l D = -2i\partial^l$. We derive from eqs.(25) and (28)

$$\Gamma^0 = \frac{2i}{e} V'(|\Phi|^2), \tag{29}$$

where $\Gamma^l = \frac{i}{2}\bar{D}\gamma^l \Gamma$ with $l = 0,1,2$ and the supersymmetric gauge corresponds to $\partial_l \Gamma^l = 0$. Finally, on multiplying eq.(26) by $(\bar{D}\gamma^m)_\alpha$ we obtain after a straightforward manipulation

$$-2\,\Box\,\Gamma^m + 2\kappa\epsilon^{mnl}\partial_n \Gamma_l = e(\eta^{0m}\bar{D}D + 2i\epsilon^{0ml}\partial_l)\,|\Phi|^2. \tag{30}$$

Ignoring the (super) Maxwell term and treating eq.(30) in a fashion similar to eq.(3) in Sec. 2 we derive

$$\kappa F_{12} = -\frac{e}{2}\bar{D}D|\Phi|^2, \tag{31}$$

$$\kappa \Gamma_0 = ie(|\Phi|^2 - C^2). \tag{32}$$

where $F_{12} = (\partial_1\Gamma_2 - \partial_2\Gamma_1)$, the indices 1, 2 here being the spatial indices. From eqs.(29) and (32) we derive immediately the specfic superpotential

$$V(|\Phi|^2) = -\frac{e^2}{4\kappa}(|\Phi|^2 - C^2)^2. \tag{33}$$

On the other hand for the case of vanishing κ the superpotential corresponding to the self-dual solutions is found to satisfy $V' = -(ie^2/4)(\bar{D}D/\Box)\,|\Phi|^2$ which leads to

$$V(|\Phi|^2) = -\frac{i}{4}(\frac{\bar{D}D}{\Box})\,\frac{e^2}{2}(|\Phi|^2 - C^2)^2. \tag{34}$$

In both cases the supersymmetric actions contain the results of the purely bosonic theory as is easily shown by integrating the superfield action over θ. The same is true of the supersymmetric self-duality condition when analysed in terms of the component fields. We obtain these results without the arguments[5] for invoking an explicit N=2 supersymmetry of the action.

Acknowledgements

One of the authors (P.P.S) acknowledges the hospitality of Ohio State University where this work began and of the Theory Division at CERN under the CERN-CNPq agreement where it was completed. The other (K.T.) thanks K. Higashijima and T. Yukawa for the hospitality at KEK. We are grateful to G. Veneziano, K. Higashijima and D. Kazakov for constructive discussions. This work was supported in part by the U.S. Department of Energy under Contract No. EY-76-C-02-1415*00.

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