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NUCLEAR FRAGMENTATION BY NUCLEATION APPROACH

by

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ABSTRACT

The nucleation model is used to simulate nuclear fragmentation processes. The "critical" value of the effective interaction radius is shown to vary linearly with the expansion factor α. The calculated mass and charge distributions are compared with some experimental data.

Key-words: Nuclear fragmentation; Nucleation.

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1. INTRODUCTION

Nuclear fragmentation is expected to yield important clues for the understanding of the nuclear equation of state [1-2]. In fact, the cluster formation process is a direct consequence of how the total available excitation energy is shared by all the constituents of the nuclear system, and this mechanism should have strong correlations with the fundamental nucleon-nucleon interaction. In particular, there is a suggestion that nuclear fragmentation may be a manifestation of some critical phenomenon in the hot nuclear matter[3].

Hence, much effort has been focused on nuclear fragmentation, in last years. As a matter of fact, many inclusive data have been produced in nuclear emulsion experiments, in high-energy-proton-induced reactions and in nucleus-nucleus collisions at intermediate energies. More recently, exclusive (4π) data have been reported also[4-5]. On the other hand, several theoretical approaches have been proposed. The most ambitious includes a full description of the process, from the very beginning of the reaction untill the breakup of the interacting system into many pieces. This time evolution of the whole process is very hard to treat and, in this case, use is made mainly of transport theories, such as BUU or VUU equations[6], or of molecular dynamics techniques[7]. Other approaches resort to a radical approximation and focus only in the latest expanding stage of the process, ignoring the details of the initial dynamics of the reaction. These non-dynamical treatments of the nuclear fragmentation seem to succeed in reproducing the main features of the inclusive experimental results. Of special interest, due to its particular simplicity, there are the percolation models, which uses only two basic ingredients, namely, lattice structure and (site- and/or bond-) probability concentration[8-10].

Recently, the nuclear fragment formation was investigated in terms of a stochastic nucleation model [11] or aggregation model [12], which essentially are similar to the percolation models, in the sense that they also are based completely on the laws of probability. The main difference is that, in this case, each constituent is assumed to be involved by an effective interaction sphere, whose radius R_{int},

for simplicity, is taken as constant and the same for all elements of the system. This latter assumption is equivalent to say that the nuclear interaction is charge independent. From the geometric point of view, the effective interaction radius is the analogous of the lattice size in percolation calculations and its use yields several advantages in comparison with the lattice size, namely: a) the nucleons are not constrained to ocupy fixed orientations as in a rigid lattice and the nucleon interdistance may assume any value, provided be larger than the nuclear hard-core radius R_c; b) the effective interaction radius is immediately identified as an effective range of the fundamental nucleon-nucleon interaction. In this sense, R_{int} carries the whole physics of the process, turning out the unique protagonist of the game. This latter feature of R_{int} allows us to say that, from the probabilistic point of view, it is completely analogous to the probability parameter of percolation models.

In Refs.[11-12], R_{int} is treated as a free parameter, with no relation to the volume (or density) of the expanding system just before the breakup. Its value has been assumed as around the nuclear force range or the *rms* proton radius. However, in this paper, we show how to relate R_{int} to the breakup volume. Especifically, since percolation and nucleation models are based on the same concept of connectedness, we assume the same prescription, suggested in Ref.[13] within the context of percolation calculations, for selecting the "critical" R_{int} which hence can be associated to nuclear fragmentation processes. This "critical" R_{int} is shown to depend on the breakup volume. Furthermore, as an example, we apply the model to calculate the mass and the charge distributions of a system with A_0 nucleons, comparing some results with experimental data.

In the following, the nucleation model is presented in Sect.2 and the results and discussion in Sect.3. Concluding remarks are addressed in Sect.4.

2. THE MODEL

The nucleation model, as mentioned in the Introduction, assumes each constituent of the nuclear system be involved by an effective interaction sphere of radius R_{int}, so that one nucleon will interact with another only when the interdistance is less or equal to $2R_{int}$. In this case, one nucleon simply sticks on another and then the nucleons are said connected. Of course, this sharp cut–off approximation is equivalent in attributing an effective range for the combined short-range nuclear and long-range Coulomb interactions.

We simulate the fragment formation by using the Monte Carlo method. Firstly, we choose randomly the positions $\mathbf{r}_i$ of all nucleons of the nuclear compound system inside a sphere of radius R, corresponding to the volume of the expanding system right before the breakup into many pieces (We have taken care in avoiding a nucleon to be placed with relation to another in a position with interdistance less than $R_c = 0.4fm$). In order to account for nuclear surface effects, we have randomly selected the magnitude of each $\mathbf{r}_i$ according to a trapezoidal distribution for the nuclear density. Secondly, a simple algorithm is used to identify clusters, which are defined in a standard way as a subset of connected nucleons such that there is a continuous path linking every nucleon of the cluster but there is no path connecting nucleons located in different clusters. It should be mentioned, however, that this definition does not prevent the appearance of quite exotic clusters, such as filamented or hole-endowed or very ramified configurations. Or, still worse, completely pathological structures, with a disconnected topology like two interlaced rings. Obviously, all of these clusters can not be identified with nuclear fragments, at least with the nuclei which are detected experimentally. Instead, they should be associated with excited primordial fragments and, in this case, one has to allow them to evaporate or to fragment.

As we are concerned with the role of R_{int} in nuclear fragmentation and with inclusive data only, we believe that the exotic contributions will be smeared out and will not change qualitatively our results. Therefore, in this paper, we will pay no attention to this relevant question, althought many recipes are available to remedy this drawback, e.g., restructuration scheme [12], evaporation [13] and compactness restraints [8].

In the following, let us consider the simulation of the breakup of a system formed by A_o nucleons, with Z_o protons and $N_o = A_o - Z_o$ neutrons. We assume the initial system, due to the compression heating, to expand from the initial radius R_o to the final value R, such that $R = \alpha R_o$, where α is the expansion factor. In fact, α represents the parametrization of our ignorance about the collisional stage of the process and, in this work, is regarded as a parameter. As it will be shown, it is the unique parameter of the model, and can be taken, as estimated by different models, to be approximately between 1.26 and 1.71, corresponding to a number density between 1/2 and 1/5 of the normal nuclear density ($n_0 = 1.53fm^{-3}$). It should be noted that α plays the same role than the lattice size in percolation calculations.

With relation to the cluster charges, model calculations so far have assumed symmetrical fragments. This approximation may be fine, if one is restricted to calculate mass distributions. However, if charge distributions are also to be computed,

the isospin degrees of freedom should be introduced. In a very naïve manner, we assume the N_o/P_o ratio to be almost invariant under the clusterization process. Especifically, for each cluster with A nucleons, we select randomly Z protons, such that the N/P ratio be approximately equal to N_o/P_o. For very light clusters, however, we impose $N/P = 1$ for $A = 2$ or 4, and equal 2 for $A = 3$. For isolated nucleons, the charge is simply attributed by chance. We impose also a strict charge conservation for the whole partition, *i.e.*,

$$\sum_{i=1}^{M} Z_i = Z_o, \tag{1}$$

where M is the multiplicity of the partition. It should be mentioned, however, that in the nucleation model, like in percolative models, energy conservation is not taken into account.

After each sampling, the clusters are stored and the procedure is repeated untill the statistics is considered satisfactory.

Of course, the results are quite dependent upon the interaction radius R_{int}, for a fixed value of the expansion factor α. In fact, one is easily convinced that for small R_{int}, most of the constituents will appear isolated or aggregate themselves in very small clusters, producing a large total multiplicity, while for large R_{int} we will have large clusters, with low total multiplicity. In this simple picture, the total multiplicity is expected to decrease monotonically with relation to R_{int}, from a maximum equal to A_0 (if hard-core effect is neglected) to 1, when all the nucleons aggregates into a single cluster. However, this behavior is dramatically changed if we take the small fragments out, plotting only the multiplicity of clusters with $A \geq 4$. As a matter of fact, this multiplicity, denoted by $\langle M \rangle$, now displays a bump, decreasing to 0 for $R_{int} \to 0$ and going to 1 for $R_{int} \to R$. Then, we propose that the value of R_{int}, which can simulate appropriately the nuclear fragmentation results, at least in what inclusive data of mass and charge distributions are concerned, is simply those which extremizes the curve $\langle M \rangle$ *versus* R_{int} for fixed α, *i.e.*,

$$\left.\frac{\partial \langle M \rangle}{\partial R_{int}}\right|_{\alpha} = 0. \tag{2}$$

The R_{int}, found by this manner, is denoted by R_{int}^{cr} and translates approximately the empirical fact that the intermediate fragments in nuclear fragmentation processes are produced copiously. It should be mentioned that this procedure, applied in the context of percolation calculations [13], has given quite satisfactory results.

Finally, it is interesting to note that the nucleation model can be related to the steady-state limit of a dynamical process, called coagulation, which involves an

ensemble of aggregates, where the time evolution of the cluster concentrations is described by a reversible (generalized) Smoluchowski rate equation [14].

3. RESULTS AND DISCUSSION

Let us first consider two expanded systems with $(A_0, Z_0) = (87, 37)$ and $(326, 133)$, which are suited for studying the $p + Kr$ and the $Xe + Ag$ reactions, respectively. In Fig.1, we display $\langle M \rangle$ as function of R_{int} for two arbitrary values of α. It is seen that the qualitative behavior of $\langle M \rangle$, such as mentioned in Sect.2, is confirmed at all and that $\langle M \rangle$ is not changed by α, which merely provoke a shift of the whole curve. In particular, the peak occurs at about $R_{int} = 1.2(1.4)fm$ for $\alpha = 1.44(1.71)$ in the case of $(87, 37)$, and at about $R_{int} = 1.1(1.3)fm$ for $\alpha = 1.44(1.71)$ in the case of $(326, 133)$. As said in the Introduction, the "critical" R_{int}^{cr}, in this nucleation picture, plays a similar role than the "critical" probability p_c in the percolation description.

It should be pointed out two relevant features of R_{int}^{cr}. First, R_{int}^{cr} seems to be sensitive to the finite size effects. In fact, it shows significant dependence on the size of the system, decreasing its value with increasing A_0 for a fixed α. Of course, it is expected that for $A_0 \to \infty$, R_{int}^{cr} will stabilize in some point close to the value corresponding to $(326, 133)$. Second, R_{int}^{cr} is strongly dependent upon the expansion factor α. In order to see this point better, we plot in Fig.2 R_{int}^{cr} against α, in the interval we are interested in. It is clearly seen that R_{int}^{cr} changes linearly with α, in both systems. In a very good approximation, R_{int}^{cr} can be fitted as

$$\begin{aligned} R_{int}^{cr} &= 0.8\alpha \qquad for \quad A_0 = 87 \\ &= 0.7\alpha + 0.1 \qquad for \quad A_0 = 326, \end{aligned} \tag{3}$$

In addition, R_{int}^{cr} is of order of the nuclear interaction range, as it should be. It is interesting to note that the linear dependence of R_{int}^{cr} on α suggests that, in the expanding nuclear system, the distance between any two nucleons is altered in the same way as the radius of the system, *i.e.*, the system performs a uniform expansion. This hypothesis, first speculated by Hirsch *et al.*[15], seem to be supported by this simple calculations.

The inclusive experimental fragment-mass data follow a power-law, *i.e.*, $Y(A) \propto A^{-\tau}$, where $Y(A)$ is the yield of clusters with A nucleons and τ, the apparent

exponent[3,15]. In Fig.3, we plot the apparent exponent τ as function of R_{int} for the two above nuclear systems and two particular choice of α. It is seen that, just in a manner opposite to $\langle M \rangle$, the curves now display a well, with minimum at location which is larger than the corresponding R_{int}^{cr}. In Table 1 we display the "critical" R_{int}^{cr} and the τ and $\langle M \rangle$ calculated at $R_{int} = R_{int}^{cr}$, for different values of α.

Table I: "Critical" Values

	(87, 37)			(326, 133)		
α	R_{int}^{cr}	τ	$\langle M \rangle$	R_{int}^{cr}	τ	$\langle M \rangle$
1.00	0.8	2.68	5.5	0.8	2.14	21.0
1.26	1.0	2.54	5.8	1.0	2.13	21.4
1.44	1.2	2.09	5.9	1.1	2.28	22.1
1.59	1.3	2.13	5.9	1.2	2.33	22.5
1.71	1.4	2.11	6.0	1.3	2.28	22.5
2.00	1.6	2.17	6.1	1.5	2.28	22.8

From Table 1, we also can see that $\langle M \rangle$ is almost independent of α and that $\langle M \rangle(326)/326 \simeq \langle M \rangle(87)/87$. This leads to $\langle M \rangle(A_0) \simeq 0.07 A_0$, which is true at least in the range $87 \leq A_0 \leq 326$. For the sake of comparison, in the case of $A_0 = 197$ and 233, for instance, the above relation yields $\langle M \rangle = 13.8$ and 16.3, while the actual calculation gives 13.6 and 15.9, respectively. Another interesting point is that for smaller A_0, τ is more sensitive to α, especially for small α.

Although no scheme has been introduced for treating the exotic fragments, we believe that it is usefull to compare the results with experimental data, in order to gain some insight in the qualitative behavior of the nucleation model. Along with this reasoning line, we maintain, as much as possible, the probabilistic character of the model, so that no physical ingredients, *e.g.* temperature, impact parameter deformation etc., has been taken into account.

In the following, we assume $\alpha = 1.71$. We depict in Fig.4 the mass yield, altogether with the results from $p + Kr$ reactions of Hirsch *et al.*[15]. It is seen that the nucleation model overestimates the experimental data, although it obeys the same qualitative behavior. On the other hand, the calculation yields $\tau = 2.11$ (*cf* Table 1), while the experiments give 2.3[16] and the percolation in three dimensional lattices, 2.2[17].

Next, the charge distributions are plotted and compared with the data from Au reactions in nuclear emulsions of Waddington and Freier[18] (Fig.5) and with

the results from $Kr + Ag$ and $Xe + Ag$ reactions of Phair *et al.*[5] (Figs.6 and 7, respectively). Except for the $Xe+Ag$ reaction, our results show a quite surprisingly good accordance with the data, if we have in mind that the whole computation was based only on a single parameter, namely, the value of α. For the $Xe+Ag$ reaction, however, the nucleation model underestimates the experimental data. It should note that the percolation model also is unable to reproduce this large experimental intermediate fragment yields[5]. In this case, probably, the inclusion of other degrees of freedom will reduce the discrepancies.

Concerning to the statistics of this simulation, the number of runs for $A_0 = 87, 197, 233$ and 326 were 2000, 1000, 1000 and 500, respectively.

Finally, it should be reminded that the scenarios for emulsion experiments, proton-induced and nucleus–nucleus collisions do not need be the same. In other words, α may be different for different scenario. This means that in the nucleation model still there is room for incorporating this and other physical aspects of the process.

IV. CONCLUDING REMARKS

In this work, we have used a quite simple clusterization model for simulating nuclear fragmentation processes, which includes somewhat the isospin degrees of freedom. It has been shown that R_{int}^{cr} varies linearly with the expansion factor α and this may be a indication that the system undergoes a uniforme expansion before breaking into many portions. In our scheme, α is a free parameter – and the unique one – related to the collisional stage of the reaction. The average multiplicity of heavy fragments ($A \geq 4$) is shown to be proportional to the number of constituents of the nuclear system (around 7%). The calculated mass and charge distributions display the same qualitative behavior than the inclusive experimental data, while the apparent exponent τ reproduces the experimental value, within a reasonable approximation.

We also have stressed the similarities between the nucleation model and the percolation models. In particular, the "critical" R_{int}^{cr} is much more amenable to a physical interpretation than the percolation "critical" probability, in the sense that the former is, in a some extent, related to an effective nucleon-nucleon cross section. As a matter of fact, the present computation has shown that it is of the order of the nuclear interaction range. Especifically, R_{int}^{cr} is between 0.8 and 1.6 fm

for $1 \leq \alpha \leq 2$.

In summary, the nucleation model may be very usefull in studying nuclear fragmentation processes. In particular, it can be used with advantage whenever one wishes to incorporate some physical input, such as the evaporation of primordial excited clusters, or the deformation of primordial configurations. Hence, this model is especially suited for dealing with the above-mentioned exotic clusters, and in this case the calculation is expected to yield better accordance with the experiments.

FIGURE CAPTIONS

Fig.1:

Multiplicity $\langle M \rangle$ *versus* R_{int} for $A_0 = 87$ and 326, and $\alpha = 1.44$ (dashed lines) and 1.71 (solid lines).

Fig.2:

R_{int}^{cr} *versus* α for $A_0 = 87$ (dashed lines) and 326 (solid lines).

Fig.3:

Apparent exponent τ against R_{int} for $A_0 = 87$ and 326, and $\alpha = 1.44$ (dashed lines) and 1.71 (solid lines).

Fig.4:

Fragment mass distribution for $A_0 = 87$. Squares denote data from Hirsch *et al.* and dots, calculated results (normalized to $A = 1$).

Fig.5:

Fragment charge distribution for $^{197}_{79}Au$. Squares denote data from Waddington and Freier and dots, calculated results (normalized to $Z = 1$).

Fig.6:

Fragment charge distribution for $A_0 = 233$. Squares denote data from Phair *et al.* and dots, calculated results (normalized to $Z = 1$).

Fig.7:

Fragment charge distribution for $A_0 = 326$. Squares denote data from Phair *et al.* and dots, calculated results (normalized to $Z = 1$).

FIGURE 1:

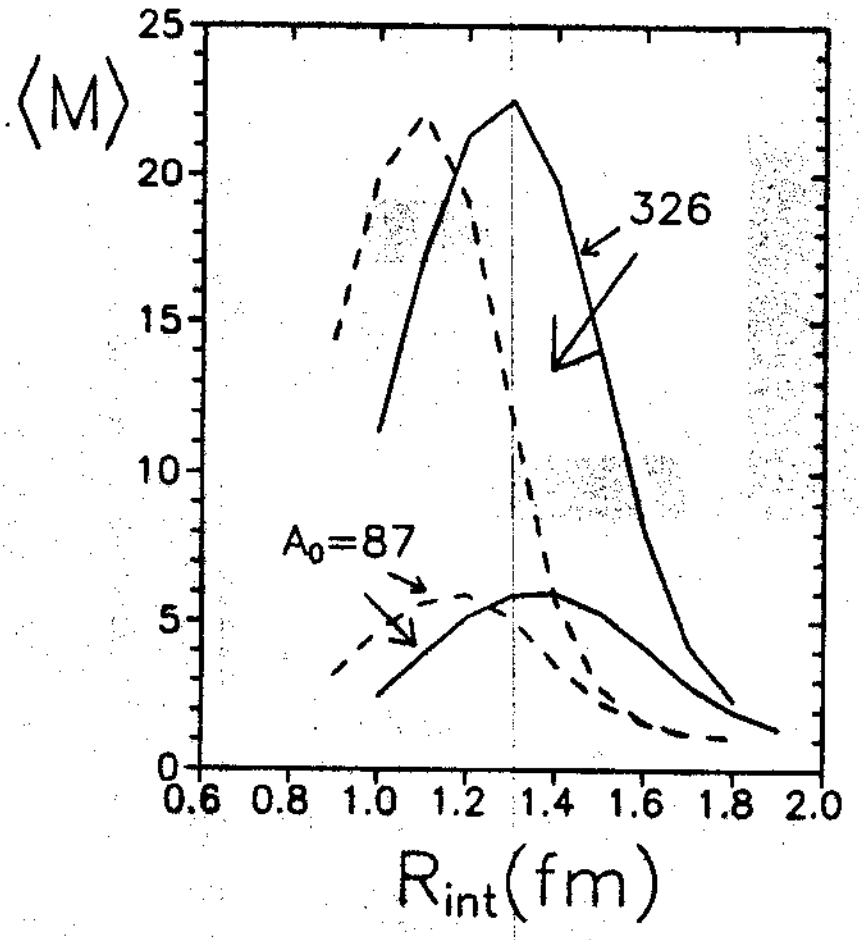
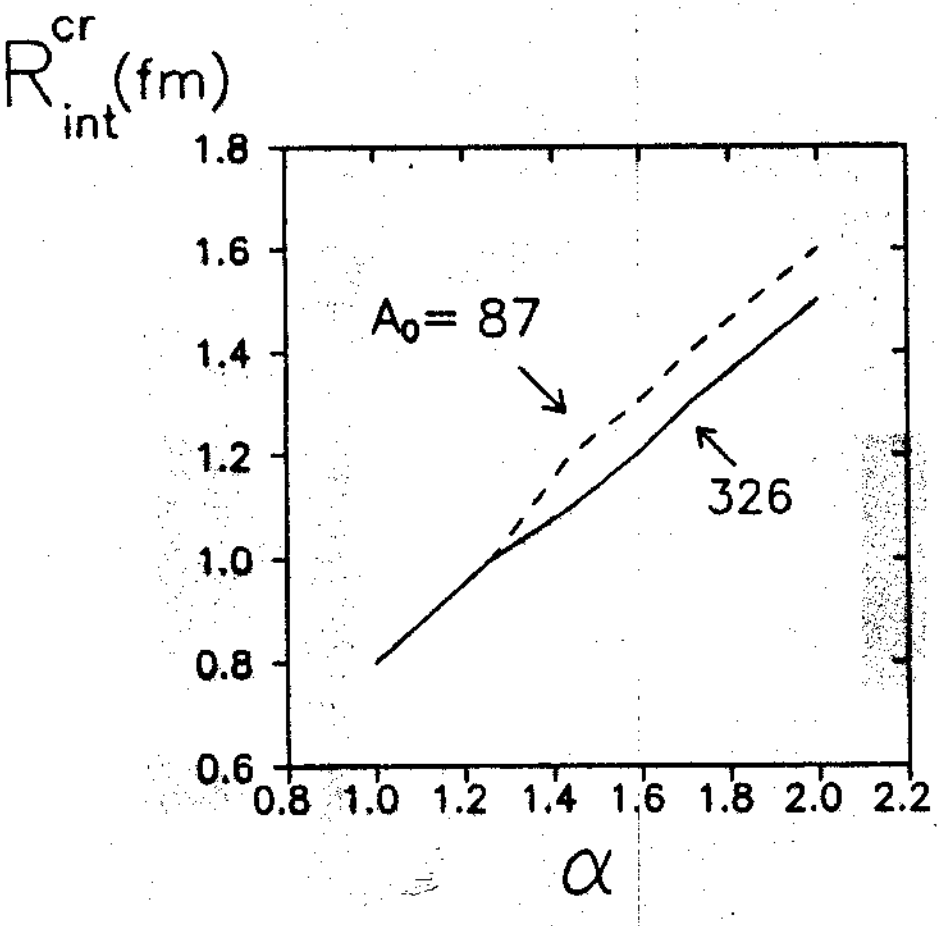


FIGURE 2:



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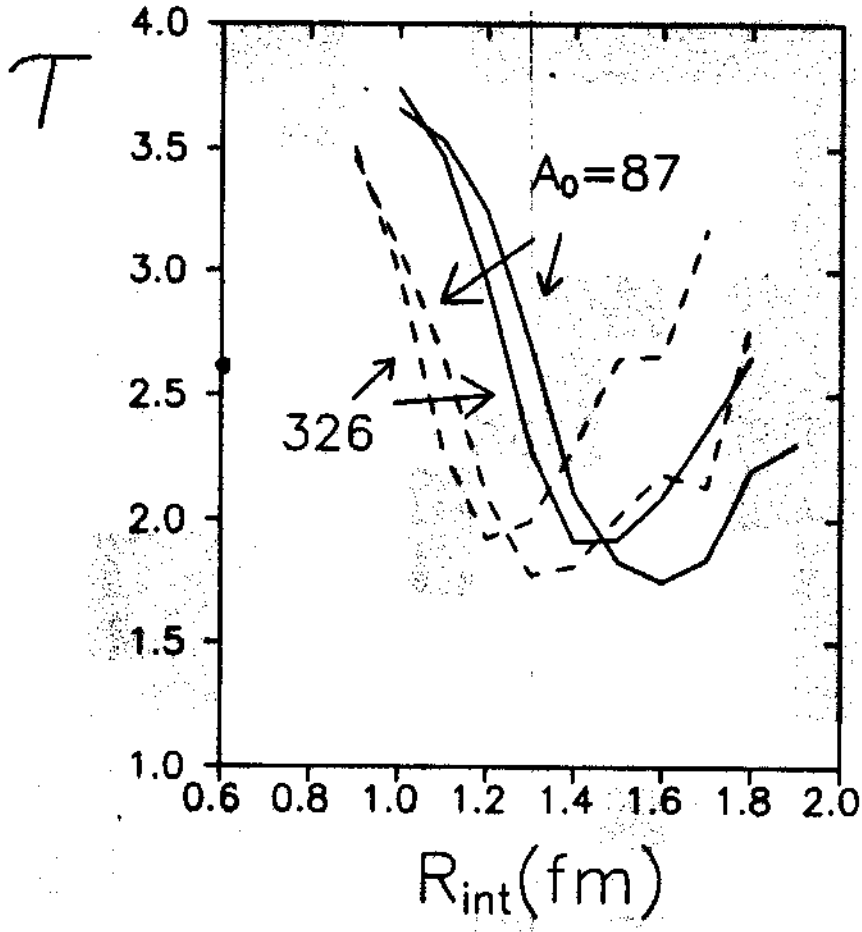

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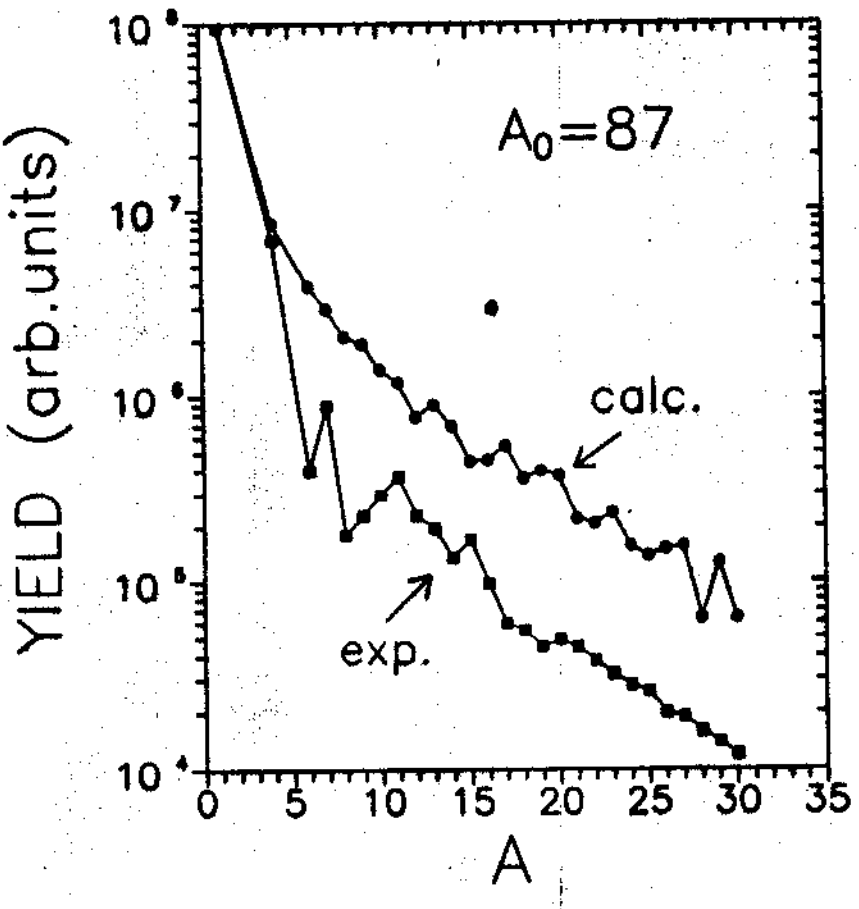
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exp.
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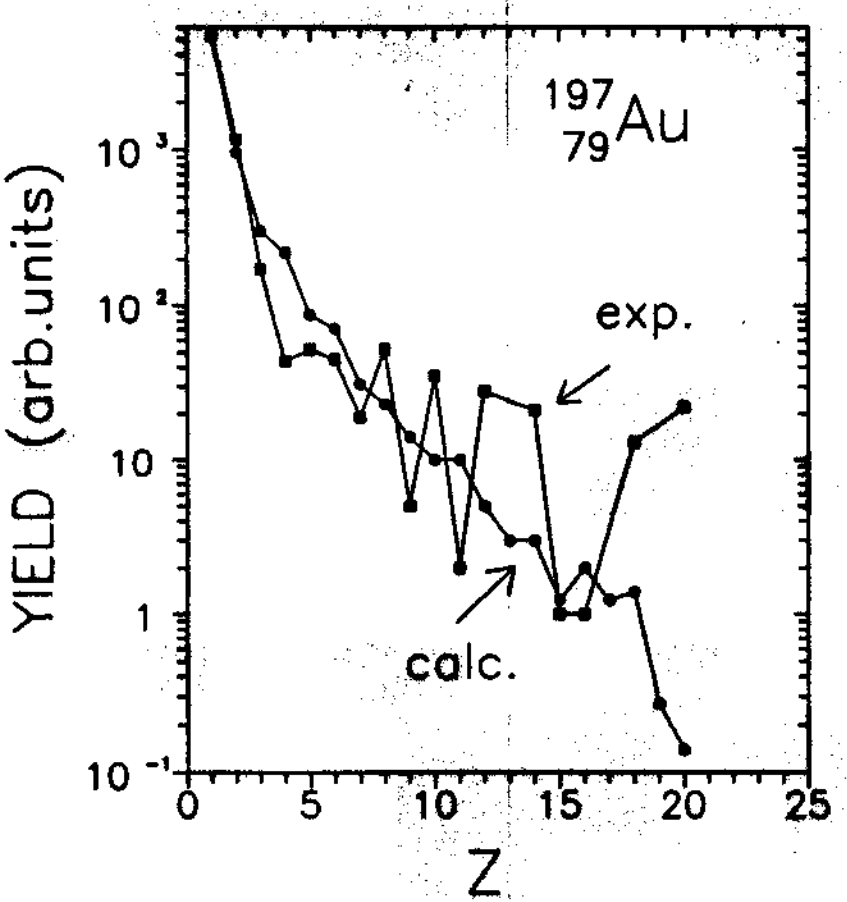
10 ⁷
10 ⁸
10 ⁹
10 ¹⁰
0
5
10
15
20
25
30
35

FIGURE 5:


$^{197}_{79}$Au
exp.
calc.
YIELD (arb.units)
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10^2
10
1
10^{-1}
0
5
10
15
20
25
Z

FIGURE 6:

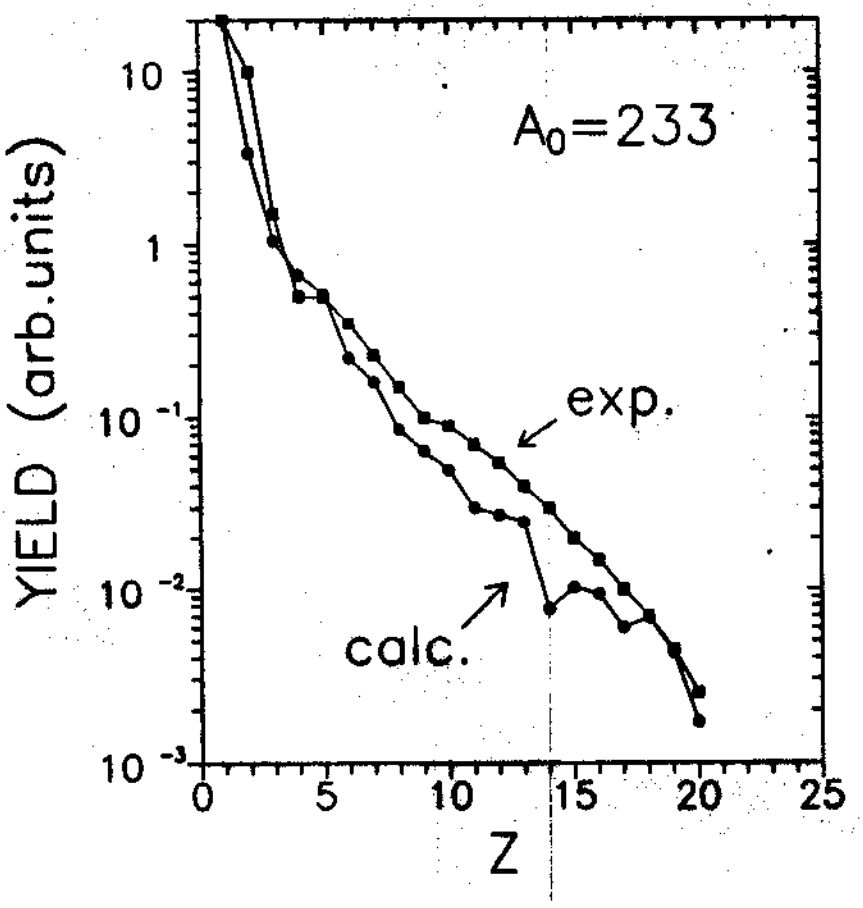
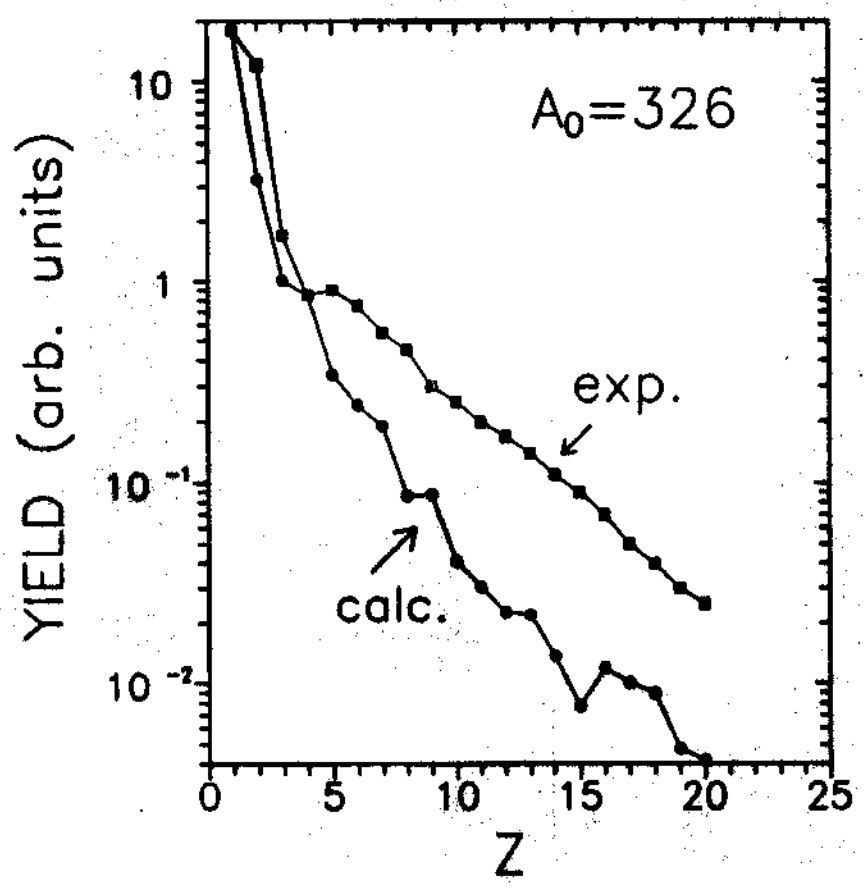


FIGURE 7:



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