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BACKWARDS TIME TRAVEL INDUCED BY COMBINED MAGNETIC AND GRAVITATIONAL FIELDS

by

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ABSTRACT

In this work, we analyse the behaviour of an elementary microscopic particle submitted to combined Magnetic and Gravitational Fields on Gödel's Universe. The exam is made in a local Gaussian system of coordinates.

Key-words: Backwards time-travel; Gödel's universe; Gaussian coordinates.

There has been, recently[1,2], some speculative comments concerning the construction of time machines. The arguments, in general, make use of arbitrary modifications of topological properties of Space Time (ST) due to quantum processes and entail a bizarre, modified Einstein-Rosen bridge with two entrances — the so-called Schwarzschild wormhole[3]. This structure is such that it allows the existence of closed time like curves, a feature which is accepted as a possible way to violate the fundamental principle according to which matter travels only towards the future.

In discussing the production of an artifact that could (in principle at least) be used as a Backwards Time Travel Machine (BTTM) one faces, prior to anything, two questions:

i) Is it possible for a real particle of well-known properties to travel backwards in time ?

ii) How should a machine operate in order to accomplish for such travel ?

Here, we intend to examine some features of this problem concerning the behaviour of an elementary microscopic particle, e.g., an electron. We decided proceed in this way because it seems to us that if the above two questions could be answered for the electron, then we would quite naturally gain some insight into the corresponding question of travelling backwards in time for huge macroscopic bodies, at least as far as theoretical arguments are concerned — let aside the problem of technological investments. In dealing with such possible BTT we realize that the subject of our investigation is so delicate that in order to avoid penetrating into a metaphysical domain we must bind ourselves firmly to some fixed rules — for instance to analyse such questions only in the realm of present known laws of physics without violating any of their observed features nor making appeal to any new property of matter and/or interactions. Since the BTT question is closely related to the global properties of space-time it seems a good strategy to undertake such analysis resorting only to classical long range forces, e.g., electrodynamics and gravity. The problem can thus be stated as follows: Does there exist a particular combination of these two fields such that the phenomena proposed above could occur ? We know that a magnetic field $\vec{H}$ can provoke opposite traveling for electrons and positrons in <u>space</u>. Indeed, if a pair of particles, electron (e^-) and positron (e^+) is produced in a given point P of space-time and if a magnetic field is switched on conveniently, then particles e^- and e^+ move along a circle in the plane orthogonal to $\vec{H}$ with opposite directions of rotation, hence inducing an opposite travelling for e^+ and e^- in the space. Could a magnetic field, in a very similar way, generate for the pair e^+e^-, backwards travel <u>in time</u> ?

There are both empirical evidence and strong theoretical arguments that forbid such an occurence in flat Minkowski space-time. We can then conclude, with great confidence, that a single magnetic field is not enough to provide the necessary conditions for such BTT. By the same token, a <u>pure</u> gravitational field is not able to induce such BTT either. Thus, in order to allow this travel a possible alternative could be the combination of independent magnetic and gravitational fields.

The metric properties of space-time are governed by Einstein's General Relativity. However, Einstein's differential equations must be complemented by a set of conditions concerning its properties at large (e.g. topological features) in order to supply a physical model. The proposal by Morris et al. makes a specific choice for these features . Now, any choice, whatsoever should involve a breakdown of a global Cauchy surface. In other words, global synchronization should not be possible if we intend to allow for BTT.

Mathematical arguments are enough to convince us that it is always possible, in a given local neighborhood, to define a Gaussian system of coordinates[4]. Nevertheless this is a mere choice of characterization of a temporal order in a compact region of space-time. To allow for BTT one should somehow forbid the extension of such Gaussian system beyond a certain limit. Let us see how to construct such configuration.

Suppose, for instance, that in a given (arbitratry) neighborhood of an observer the metric written in a non--homogeneous Gaussian system of coordinates takes the form

$$ds^2 = dt^2 - g_{ij}(t,x)dx^i dx^j \qquad . \tag{1}$$

In order to fix our ideas and to allow for actual calculation let us make a definite, although arbitrary, choice for such geometry and set in the (t, ξ, η, z) Gaussian coordinate system the form:

(2) $$ds^2 = dt^2 - a^2(\mu^2-1)d\xi^2 + a^2g(t,\xi)d\eta^2 + + 2a^2h(t,\xi)d\xi d\eta - a^2dz^2$$

in which a and μ are (arbitrary) constants; g and h are functions to be specified, through Einstein's equation, after the stress--energy tensor that generates this metric is characterized. We choose matter as a perfect fluid that moves with four-velocity $V^\mu = (\mu, \frac{1}{a}, 0, 0)$, such that $\rho = -2\Lambda = \frac{4}{a^2}$ in which Λ is the cosmological constant. Note that hypersurfaces t = constant are not surfaces of homogeneity, the matter fluid being not orthogonal to the Σ hypersurface characterized by t = constant. Indeed, matter is tilted in relation to Σ. As a consequence of the above choice of the velocity field the functions h and g are indeed specified yielding:

(3a) $$g = -\frac{1}{4}\frac{\mu^2-1}{(\mu^2+1)^2}(1-\sin m)\{\mu^2 + 3 + (\mu^2-1)\sin m\}$$

(3b) $$h = \frac{\sqrt{2}}{2}\frac{\mu^2-1}{\mu^2+1}(1-\sin m)$$

with

$$m \equiv \frac{2}{a}\sqrt{\mu^2+1}\,(t-\mu a\xi) \quad .$$

This geometry, as viewed by the Gaussian observer $u^{\alpha} = \delta^{\alpha}{}_{0}$ has an expansion factor θ (Hubble red-shift law) given by

$$\theta = -2\,\frac{\sqrt{\mu^2+1}}{a}\tan m \quad . \tag{4}$$

It thus diverges at the corresponding values:

$$t - a\mu\xi = \pm\,\frac{\pi a}{4\sqrt{\mu^2+1}} \quad . \tag{5}$$

One should remark that the net effect of the presence of these singularities can be interpreted as a local dependence of the "entrance in the world" in distinct regions. This is very similar to the behaviour of lagging cores as it occurs for instance in Universes displaying white holes. However, there is an important difference between the above geometry and, say, Tolman's Universe(5). The singularities displayed at points (5) are not real ones but a consequence of our choice of the Gaussian system of coordinates.

Now comes a crucial question. How can we guarantee that it is not possible to extend the above Gaussian system beyond a certain region ? This fact is already conveyed in the existence of the infinite red-shift ($\theta = \pm\infty$) region. Besides, and this is an alternative proof, a coordinate transformation can show that the geometry (2,3) is nothing but Gödel's rotating Universe(6,7,8) which in a cylindrical coordinate system assumes the form

$$ds^2 = a^2\left[dT^2 - dr^2 - dz^2 + 2h(r)dTd\phi + g(r)d\phi^2\right] \tag{6}$$

Note that the extension of the geometry beyond the Gaussian domain causes no difficulty once this region is just a frontier and not a physical barrier. This has some extra nice features. Indeed the Gödel manifold have the peculiarity that the integral curves of the Killing vector $\frac{\partial}{\partial\phi}$ are space-like if $r < r_c$ and time--like if $r > r_c$. Thus it provides a very natural way to obtain a BTT configuration. The question then is how to force a real particle to move along this curve.

In the case of a charged particle, say an electron or a positron, the answer is simple: a magnetic field directed along the axis of rotation of matter can provide this travel. Indeed, if a magnetic field is set up in the z-direction, inside the gaussian domain, the net effect of this field in a pair e^-e^+ is precisely to induce a space separation, as in ordinary Minkowskian space-time between particle and anti-particle. However, in the region beyond the gaussian domain ($r > r_c$) the net effect of this magnetic field is to induce a separation in time. The electron (e^-) and the positron (e^+) are set to move into non-equivalent <u>time</u> directions. The simplest way to show this is the following. The weak magnetic field — whose geometrical disturbances can be neglected, satisfies Maxwell's equations in the curved geometry (6), if we set for the unique non-null component of the electromagnetic field $F^{12} = H$, the value $H(r) = \frac{H_0}{\sinh(2r)}$. Thus there exist only a magnetic field directed along the z-axis[9]. It seems worth to note that the matter that generates Gödel's geometry is neutral, that is, it is transparent for such magnetic field. However a charged particle, e.g., an electron (e^-) is accelerated by $\vec{H}$. The combined effect of gravity (that drives the metric properties and whose presence is felt through the Christoffel symbols) and the magnetic field (that acts on charges through the Lorentz force) yields for a particle moving into a circular orbit around the z-axis the acceleration $a^\mu \equiv b^\mu{}_{;\lambda} b^\lambda$, in which $b^\mu = \left(0,\ 0,\ \frac{1}{a\sinh r\sqrt{\sinh^2 r-1}},0\right)$ is the velocity of the electron in the cylindrical coordinate system. If we impose that the Lorentz force of the magnetic field on the electron induces an acceleration vector that keeps the electron in the closed time like trajectory, that is

$$a^\mu = \left(0,\ \frac{\cosh r(2\sinh^2 r-1)}{a^2\sinh r(\sinh^2 r-1)}\ ,\ 0,\ 0\right) \tag{7}$$

for a constant radius $r = r_0$ then H_0 is fixed and is given by

$$H_0 = \frac{2mc}{ea^3}\,\frac{\cosh^2 r_0(2\sinh^2 r_0-1)}{\sinh r_0(\sinh^2 r_0-1)^{3/2}} \quad . \tag{8}$$

Thus, outside the gaussian domain the trajectory of the electron accelerated by the magnetic field is given by T = constant, z = constant, r = constant = r_0 and $0 \le \phi \le 2\pi$. We thus recognize that the electron follows a closed time like trajectory, due to the influence of the magnetic field. In order to induce the positron (e^+) to follow this trajectory, we must revert the direction of the magnetic field (see figure).

Let us make a final comment.

Although the extension of the gaussian system of coordinates (2) beyond the critical radius $r = r_c$ (in cylindrical coordinates) is forbidden, one could envisage the possibility to built another gaussian system (call it Gauss-II) beyond r_c. In this Gauss-II system we define a new time t_{II}. The hypersurface Σ_{II} defined by t_{II} = constant provides thus, in this region, a causal structure and a net well defined separation between past and future. The accelerated curve of the electron cuts this surface Σ in an unique point, thus suggesting that for the gaussian observer G-II there is nothing strange with the behaviour of the electron. However, any accelerated observer that can somehow follows the electron's path will positively declare that the phenomenon of backwards time travel induced by the combined action of the magnetic and the gravitational field is indeed occuring,as one can see from equation (7). There is no contradiction at all between these two assertions once we recognize that the idea of global time suffers from a lack of observational means.

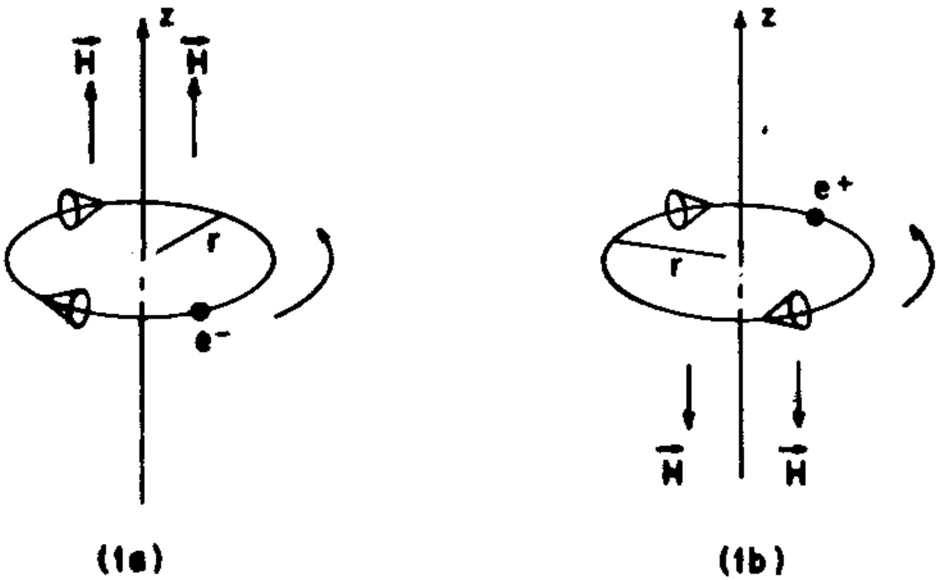

Fig. 1a

Fig. 1b

FIG. 1 - Example of BTTM - A combination of gravitational and magnetic field induce a particle [say, an electron] to travel along a closed time like curve (see 1a) . In order to induce the same path for its anti-particle (the positron e^+) the magnetic field must be reversed (see 1b).

References and Comments

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9 - Note that $F_{12}F^{12} = -\frac{a^4H_0^2}{2}\frac{\sinh^2 r-1}{\cosh^2 r}$ is bounded at the electron's world line.

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