ISSN 0029-3865

CBPF-NF-037/91

AN ALTERNATIVE PRESCRIPTION FOR GAUGING FLOREANINI-JACKIW CHIRAL BOSONS

by

S.A. DIAS[1] and A. de Souza DUTRA[1,2]

[1]Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

[2]Universidade Estadual Paulista
Campus de Guaratinguetá, DFQ
Caixa Postal 205
12500 - São Paulo, SP - Brasil

CBPF-NF-037/91

Abstract

We seek new couplings of chiral bosons to $U(1)$ gauge fields. Lorentz covariance of the resulting constrained Lagrangian is checked with the help of a procedure based in the first–order formalism of Faddeev and Jackiw. We find Harada's constraint and another local one not previously considered, besides infinite non–local couplings. We analyze the constraint structure and part of the spectrum of this second solution and show that it is equivalent to an explicitly covariant coupling of Siegel's chiral boson to gauge fields, which preserves chirality under gauge transformations.

Key-words: Two-dimensional models; Chiral bosons.

In the course of the analysis of chiral bosons properties, one natural step is to couple them to abelian and non-abelian gauge fields[1,2,3] in order to study the corresponding anomalies, or to provide an alternative approach to chiral models in two dimensions[4]. These couplings have been proposed both in Siegel's [5] explicitly covariant formalism[2,6] and in the approach of Floreanini and Jackiw (FJ)[7,8].

In the context of chiral theories in two dimensions, Harada has shown recently how to obtain a consistent coupling of FJ chiral bosons with a $U(1)$ gauge field, starting from the chiral Schwinger model (CSM) and discarding the right-handed degrees of freedom by means of a projection in phase space implemented by the *chiral constraint* $\pi_\phi = \phi'$[8]. The resulting theory had the same spectrum of the CSM with the additional characteristic that the massless mode was self-dual. There was no trace, at the end, of the right-handed fermion originally present (which, however, was necessary for the eigenvalue problem of computing the fermion determinant to be well defined [9]). It has been shown later by Bazeia[10] that Harada's approach was equivalent to the one of Bellucci, Golterman and Petcher[2] under Faddeev and Jackiw's first-order formalism[11].

We investigate, in this letter, the possibility of obtaining different couplings for the FJ chiral boson, starting from the generalized Schwinger model, where both chiralities interact with the gauge field. We follow the same strategy as Harada's in obtaining the Lagrangian of the coupled system, but we propose a check test which can straightforwardly decide whether the resulting coupling is Lorentz covariant or not. Starting with the left-handed chiral Schwinger model we find that the only possible local couplings to $U(1)$ gauge fields are those obtained by projecting either one chirality ($\pi_\phi = \phi'$) or the other ($\pi_\phi = -\phi' + e(A_0 - A_1)$). The constraints $\pi_\phi = -\phi'$ and $\pi_\phi = \phi' + e(A_0 + A_1)$ are the ones allowed for the right-handed CSM. The theory obtained using $\pi_\phi = -\phi' + e(A_0 - A_1)$ is shown to be equivalent to a specific coupling of Siegel's chiral bosons with $U(1)$ gauge fields which is symmetric under chirality-preserving gauge transformations. In addition, there is an infinity of possible non-local couplings.

Our starting point is the Lagrangian of the generalized Schwinger model (GSM),

$$\mathcal{L} \;=\; \overline{\Psi}\,\gamma^\mu \left[i\partial_\mu \,+\, e_R\, A_\mu \,\frac{(1\,+\,\gamma_5)}{2} \,+\, e_L\, A_\mu\, \frac{(1\,-\,\gamma_5)}{2} \right] \Psi \qquad (1)$$

which is equivalent to its bosonized version[12,14]

$$\mathcal{L}_B \;=\; \frac{1}{2}\,(\partial_\mu\,\phi)^2 \,+\, \frac{1}{\sqrt{\pi}}\,\left(\overline{g}_1\,\partial^\mu \,-\, \overline{g}_2\,\tilde{\partial}^\mu\right)\,\phi\,A_\mu \,+\, \frac{\bar{M}^2}{2\pi}\,A_\mu^2 \qquad (2)$$

where

$$\bar{g}_1 = \frac{\bar{e}_L + \bar{e}_R}{2}, \quad \bar{g}_2 = \frac{\bar{e}_L - \bar{e}_R}{2}, \quad \bar{M}^2 = \frac{\tilde{e}_L\,\tilde{e}_R + \bar{e}_L\,\bar{e}_R}{2}. \tag{3}$$

In (3) $\tilde{e}_L$ and $\tilde{e}_R$ are arbitrary couplings introduced by the regularization procedure[13] and $\bar{e}_L$ and $\bar{e}_R$ are defined as[12]

$$\bar{e}_L = \left[\tilde{e}_L^2 + (\tilde{e}_L - e_L)^2\right]^{1/2}, \quad \bar{e}_R = \left[\tilde{e}_R^2 + (\tilde{e}_R - e_R)^2\right]^{1/2}. \tag{4}$$

The Hamiltonian obtained from $\mathcal{L}_B$ is

$$\begin{aligned} \mathcal{H}_B &= \frac{1}{2}\left(\pi_\phi - g_1\,A_0 - g_2\,A_1\right)^2 \\ &+ \frac{1}{2}\,\phi'^{\,2} + \phi'\,(g_1\,A_1 + g_2\,A_0) - \frac{M^2}{2}\,A_\mu^2 \end{aligned} \tag{5}$$

with $\sqrt{\pi}\,g_i = \bar{g}_i$ and $\pi\,M^2 = \bar{M}^2$.

We proceed with the method of Harada, projecting one chirality with the aid of a generalized chiral constraint

$$\pi_\phi = \alpha\,\phi' \tag{6}$$

where α will be determined imposing the requirement of Lorentz covariance on the resulting theory. With this projection we obtain (after functional integration over the π_ϕ field subjected to (6) as in [8])

$$\mathcal{L}_\alpha = \alpha\,\dot\phi\phi' - \frac{(\alpha^2+1)}{2}\,\phi'^2$$
$$+\,\phi'\,((\alpha\,g_1 - g_2)\,A_0 + (\alpha\,g_2 - g_1)\,A_1)$$
$$-\,\frac{1}{2}\,(g_1\,A_0 + g_2\,A_1)^2 + \frac{M^2}{2}\,A_\mu^2. \tag{7}$$

Now, we ask which values of α are allowed in order to produce a Lorentz covariant theory. We exemplify our strategy with the non–gauged original FJ Lagrangian,

$$\mathcal{L}_{FJ} = \dot\phi\,\phi' - \phi'^2. \tag{8}$$

Performing a Lorentz rotation,

$$\begin{pmatrix}\dot\phi\\ \phi'\end{pmatrix} \longrightarrow \begin{pmatrix}\cosh\theta & \sinh\theta\\ \sinh\theta & \cosh\theta\end{pmatrix}\begin{pmatrix}\dot\phi\\ \phi'\end{pmatrix} \tag{9}$$

this Lagrangian changes to

$$\mathcal{L}^R_{FJ} = a(x)\,\dot\phi + b(x)\,\dot\phi\,\phi' + c(x)\,\phi'^2 \tag{10}$$

with

$$a(x) = \frac{(x^2-1)}{2\,x^2}\,,\quad b(x) = \frac{1}{x^2}$$
$$c(x) = -\frac{(x^2+1)}{2\,x^2}\,,\quad x = e^\theta. \tag{11}$$

Using the first–order formalism of Faddeev and Jackiw we construct a first–order Lagrangian to (10)

$$\mathcal{L}_{FJ}^{R,1} = \pi_\phi \dot{\phi} - \frac{x^2 \pi_\phi^2}{2(x^2-1)} - \frac{x^2 \phi'^2}{2(x^2-1)} + \frac{\pi_\phi \phi'}{x^2-1}. \qquad (12)$$

Now we notice that although $\mathcal{L}_{FJ}$ describes a constrained system, this is not what happens to $\mathcal{L}_{FJ}^R$. It is thus legitimate to ask whether the resulting theory is equivalent to the previous one in the new reference frame, if the constraint is taken into account. Thus imposing that $\pi_\phi = \phi'$ we obtain simply

$$\mathcal{L}_{FJ}^{R,1}\,|_{\pi_\phi=\phi'} = \mathcal{L}_{FJ}, \qquad (13)$$

thus showing that under the chiral constraint assumption, both Lagrangians ((10) and (8)) are equivalent.

Let us make the same analysis for $\mathcal{L}_\alpha$, eq. (7). The Lorentz rotation (9) produces (rotating also, obviously, A_0 and A_1),

$$\mathcal{L}_\alpha^R = a(x)\,\dot{\phi}^2 + b(x)\,\dot{\phi}\,\phi' + c(x)\,\phi'^2$$
$$+ \left\{\left(x^2-1\right)\dot{\phi} + \left(x^2+1\right)\phi'\right\}\{d_+(x)\,A_0 + d_-(x)\,A_1\}$$
$$- \frac{1}{2}\left(e_+ A_0 + e_- A_1\right)^2 + \frac{1}{2}M^2 A_\mu^2 \qquad (14)$$

with

$$a(x) = -\frac{(x^2-1)}{8x^2}\left(\alpha^2\left(x^2-1\right) - 2\,\alpha\left(x^2+1\right) + x^2 - 1\right)$$

$$b(x) = -\frac{1}{4x^2}\left(\alpha^2\left(x^4-1\right) - 2\,\alpha\left(x^4+1\right) + x^4 - 1\right)$$

$$c(x) = -\frac{(x^2+1)}{8x^2}\left(\alpha^2\left(x^2+1\right) - 2\,\alpha\left(x^2-1\right) + x^2 + 1\right) \qquad (15)$$

$$d_\pm = \frac{1}{4x^2}\left\{\alpha\left[\left(x^2\pm1\right)g_1 + \left(x^2\mp1\right)g_2\right] - \left[\left(x^2\mp1\right)g_1 + \left(x^2\pm1\right)g_2\right]\right\}$$

$$\equiv (\alpha\, e_\pm - e_\mp)$$

The first-order Lagrangian is

$$\mathcal{L}_\alpha^{R,1} = \alpha\,\dot\phi\,\phi' + \frac{4\,a\,c - \alpha^2 + b(2\,\alpha - b)}{4a}\,\phi'^{\,2}$$

$$+ \frac{[2a\,(x^2+1) + (x^2-1)(\alpha - b)]}{2a}\,\phi'\,\left[(\alpha\,e_+ - e_-)\,A_0 + (\alpha\,e_- - e_+)\,A_1\right]$$

$$-\frac{1}{2}\,(e_+A_0 + e_-A_1)^2 - \frac{(x^2-1)^2}{4a}\,\left((\alpha\,e_+ - e_-)\,A_0 + (\alpha\,e_- - e_+)\,A_1\right)^2$$

$$+ M^2 A_\mu^2 \qquad (16)$$

This expression only equals (7) if

$$\left(\alpha^2-1\right)\phi' - (g_1\alpha + g_2)\,A_0 - (g_2\alpha + g_1)\,A_1 = 0 \qquad (17)$$

Solving this equation for α we find the set of constraints which preserves relativistic covariance,

$$\pi_\phi = \pm\frac{1}{2}\left[4\phi'^2 + 4\phi'(g_2A_0+g_1A_1) + (g_1A_0+g_2A_1)^2\right]^{1/2}$$

$$+\frac{1}{2}(g_1A_0+g_2A_1)$$

$$= \pm\phi' \pm \phi'\left[\frac{1}{8\phi'^2}\left\{4\phi'(g_2A_0+g_1A_1) + (g_1A_0+g_2A_1)^2\right\} + \cdots\right]$$

$$+ \frac{1}{2}(g_1A_0+g_2A_1) \tag{18}$$

There is an infinity of allowed constraints, as long as we permit ourselves to consider non–local theories. If we seek local couplings, we have to get rid of the square root in (18). This can be obtained in two cases, namely i) $g_1 = g_2 = e$ (right handed chiral Schwinger model), with constraints

$$\pi_\phi = -\phi' \tag{19}$$

and

$$\pi_\phi = \phi' + e(A_0+A_1); \tag{20}$$

ii) $g_1 = -g_2 = e$ (left handed chiral Schwinger model), with constraints

$$\pi_\phi = \phi' \tag{21}$$

and

$$\pi_\phi = -\phi' + e(A_0-A_1). \tag{22}$$

Cases (19) and (21) are the cases studied by Harada and found elsewhere in the literature [2,4,8,10]. Cases (20) and (22) have not been previously considered. To be definite we will start from case (ii) and complete the gauging of the chiral boson within the context of the left-handed chiral Schwinger model. Imposing (22) on (2), with $\bar{g}_1 = -\bar{g}_2 = \sqrt{\pi}e$, we obtain in the same way that we did before

$$\mathcal{L}_{CH} = -\dot{\phi}\phi' - \phi'^2 + e\left(\dot{\phi} + \phi'\right)(A_0 - A_1) + \frac{M^2}{2} A_\mu A^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{23}$$

where we added a kinectical term to the resulting Lagrangian. From (23) we compute the canonical Hamiltonian

$$\mathcal{H}_C = \frac{\left(\pi^1\right)^2}{2} + \pi^1 A_0' + \phi'^2 - e\phi'(A_0 - A_1) - \frac{M^2}{2} A_\mu^2. \tag{24}$$

The constraint (22) is second class, as in the case of Harada,

$$\{\Omega_2(x), \Omega_2(x)\} = +2\delta'\left(x^1 - y^1\right) \tag{25}$$

with $\Omega_2 = \pi_\phi + \phi' - e(A_0 - A_1)$. There is another primary constraint, $\Omega_1 = \pi^0$, whose consistence under time evolution produces

$$\dot{\Omega}_1 = \left\{\Omega_1, \int dy^1 (\mathcal{H}_c + u_1\Omega_1 + u_2\Omega_2)\right\}$$

$$= \partial_1 \pi^1 + e\phi' + M^2 A_0 + e u_2 = 0. \tag{26}$$

This determines u_2, while u_1 is determined through $\dot{\Omega}_2 = 0$. The inverse of the constraint matrix is given by

$$C_{ij}(x,y) = \frac{1}{e^2} \begin{bmatrix} 2\delta'(x^1 - y^1) & e\delta(x^1 - y^1) \\ & \\ -e\delta(x^1 - y^1) & 0 \end{bmatrix} \tag{27}$$

and the non–null Dirac brackets are

$$\{\phi(x), \pi_\phi(y)\}^* = \delta\left(x^1 - y^1\right) \;\; ; \;\; \{\phi(x), A_0(y)\}^* = \frac{1}{e}\delta\left(x^1 - y^1\right) \;\; ;$$

$$\{\pi_\phi(x), A_0(y)\}^* = \frac{1}{e}\delta'\left(x^1 - y^1\right) \;\; ;$$

$$\{A_0(x), A_0(y)\}^* = -\frac{2}{e^2}\delta'\left(x^1 - y^1\right) \; ; \; \left\{A_0(x), \pi^1(y)\right\}^* = -\frac{1}{e}\delta\left(x^1 - y^1\right) \; ;$$

$$\left\{A_1(x), \pi^1(y)\right\}^* = \delta\left(x^1 - y^1\right). \tag{28}$$

One can choose ϕ to be eliminated from (24), after using the constraints strongly, and then arrive to the final Hamiltonian,

$$\mathcal{H} = \frac{(\pi^1)^2}{2} + \pi^1 A_0' + \pi_\phi^2 - e\,\pi_\phi (A_0 - A_1) - \frac{M^2}{2} A_\mu^2 \tag{29}$$

Thanks to the non–standard commutation relations obeyed by A_0, it is not easy to solve the equations of motion obtained from (29). To see something about the spectrum of this theory we can integrate functionally over the A_μ field to obtain an effective Lagrangian for the ϕ field,

$$\mathcal{L}_{eff} = \frac{1}{2}\phi\left[\frac{(e^2-M^2)\,\Box^2 - M^4\Box + M^2(\Box+M^2)\,\partial_+\partial_+}{M^2(\Box+M^2)}\right]\phi \qquad (30)$$

Using $M^2 = e^2 a$, we see that there are poles in the following regions in the (k_+, k_-)–plane:

i)$a \neq 1$,

$$k_- = \pm a\left(\frac{\left[a^2e^4 - 2(a-2)\,e^2k_+^2 + k_+^4\right]^{1/2} \pm \left(ae^2 + k_+^2\right)}{2k_+(a-1)}\right),$$
$$k_+ = 0 \qquad (31)$$

ii)$a = 1$,

$$k_- = \frac{e^2 k_+}{e^2 + k_+^2} \ ,$$
$$k_+ = 0 \qquad (32)$$

Although the expression for the k_- curve is not explicitly Lorentz covariant, we can see explicitly the presence of a self–dual pole in the spectrum of the theory, with the correct chirality.

Yet the appearance of only the A_- components of the A_μ field in the Lagrangian suggests that this kind of coupling could be obtained by a kind of "self-dual" gauging, in which only the ∂_- derivative would be covariantized. This has led us to consider Siegel's formalism for the right-handed chiral boson

$$\mathcal{L}_S = \frac{1}{2}\partial_+\phi\,\partial_-\phi + \frac{1}{2}\lambda(\partial_+\phi)^2. \tag{33}$$

Performing the substitution

$$\partial_-\phi \longrightarrow \partial_-\phi + 2\,e\,A_- \tag{34}$$

we get

$$\mathcal{L}_S^g = \frac{(\lambda+1)}{2}\dot{\phi}^2 + \lambda\dot{\phi}\phi' + \frac{(\lambda-1)}{2}\phi'^2 + e\left(\dot{\phi}+\phi'\right)(A_0 - A_1). \tag{35}$$

The first-order Lagrangian is

$$\mathcal{L}_S^{g,1} = \pi_\phi\dot{\phi} - \frac{1}{\lambda+1}\left\{\frac{1}{2}\pi_\phi^2 - \lambda\pi_\phi\phi' - e\left(\pi_\phi+\phi'\right)(A_0 - A_1\} + \frac{1}{2}e^2(A_0-A_1)\right\}. \tag{36}$$

Solving the constraint through the equation of motion for λ, we obtain

$$\pi_\phi \equiv -\phi' + e(A_0 - A_1) \tag{37}$$

and, after substitution in (36), we get $\mathcal{L}'_{CH}$ given by

$$\mathcal{L}'_{CH} = -\dot{\phi}\phi' - \phi'^2 + e\left(\dot{\phi}+\phi'\right)(A_0 - A_1) \tag{38}$$

which is the same as $\mathcal{L}_{CH}$, eq.(23), without the last two terms.

Finally, we would like to notice that the gauge symmetry of the model proposed in (35) is a kind of "chiral" gauge symmetry: the symmetry of the model is $\phi \to \phi + \varepsilon$ and $A_- \to A_- - \frac{1}{2e}\partial_-\varepsilon$, $\varepsilon = \varepsilon(x^-)$. This symmetry preserves the chirality of the chiral boson under gauge transformations. It is also responsible for more degrees of freedom than those present in the case considered by Harada[8], as we can take A_+ as a gauge invariant quantity under these restricted transformations. If this model is an alternative description for the minimal chiral Schwinger model, is a very interesting question to be adressed in the near future.

We would like to thank Marco Antonio Andrade for introducing us to the DERIVE package and Dr. Juan Alberto Mignaco for continuous encouragement and useful criticisms. One of us (A.S.D.) is partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

References

[1] S.Bellucci, R.Brooks and J.Sonnenschein, Nucl.Phys. **B304** 173 (1988).

[2] S.Bellucci, M.F.L.Golterman, and D.N.Petcher, Nucl.Phys. **B326** 307 (1989).

[3] J.Sonnenschein, Nucl.Phys. **B309** 752 (1988).

[4] K.Harada, Phys.Rev. **D42** 4170 (1990).

[5] W.Siegel, Nucl.Phys. **B238** 307 (1984).

[6] J.M.F.Labastida, A.V.Ramallo, Phys.Lett. **B222** 231 (1898).

[7] R.Floreanini, R.Jackiw, Phys.Rev.Lett. **59** 1873 (1987).

[8] K.Harada, Phys.Rev.Lett. **64** 139 (1990).

[9] L.Alvarez–Gaumé–*An Introduction to Anomalies*, HUTP–85/A092, 1985. Published in the proceedings of the Erice School of Math. Phys., 1985:93 (QCD 161:I569:1985).

[10] D.Bazeia, Mod.Phys.Lett. **A5** 2497 (1990).

[11] L.Faddeev, R.Jackiw, Phys.Rev.Lett **60** 1692 (1988).

[12] D.Boyanovsky, J.Schmidt and M.F.L.Golterman, Ann.Phys.(N.Y.) **185** 111 (1988).

[13] J.L.Alonso, J.L.Cortés, E.Rivas, Phys.Rev **D41** 2568 (1990).

[14] Our conventions are $\gamma^0 = \sigma^1$, $\gamma^1 = -i\,\sigma^2$, $\gamma_5 = \gamma^0\gamma^1 = \sigma^3$, $\eta_{\mu\nu} = \mathrm{diag}\,(+,-)$, $\epsilon^{01} = -\epsilon_{01} = +1$, $\tilde{\partial}_\mu = \epsilon_{\mu\nu}\partial^\nu$, $\gamma_5\,\Psi_{L,R} = \mp\Psi_{L,R}$, $\dot{\phi} = \partial_0\phi$, $\phi' = \partial_1\phi$, $\partial_\pm = \partial_0 \pm \partial_1$, $A_\pm = A_0 \pm A_1$.