ISSN 0029 - 3865

CBPF-NF-036/87

RESTRICTED HIGH-DENSITY PERCOLATION AND NUCLEAR FRAGMENTATION

by

N.-C. Chao* and K.C. Chung

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

*Departamento de Física Teórica e Experimental
Universidade Federal do Rio Grande do Norte
59000 - Natal, RN - Brasil

ABSTRACT

We propose a restricted high-density percolation that may belong to a new universality class. A Monte Carlo simulation for our model is performed with systems of finite number of FCC lattice sites within a sphere. The cluster distribution is shown to obey a power law with an exponent $\tau \cong 2.50$ at critical probability $p_c \cong 0.50$. Comparisons with the experimental data from p + Kr and p + Xe collisions are made.

PACS numbers: 64.60.Ak, 25.70.Np

Key-words: High-density percolation; FCC lattice; Nuclear fragmentation.

The most interesting phenomenon in the collision of atomic nuclei with high-energy protons or heavy ions is perhaps the multifragmentation reaction, in which target nuclei are broken into various light and medium size fragments with no residue nuclei remain. Experimentally, it has been established that fragmentation is a high energy process occurring in a central collision and that the reaction products are typically found to be less than about one third of original target mass[1]. Recently, Hirsch et al.[1,2] have found that the fragment mass yields produced in high energy (80 - 350 GeV) proton-krypton and proton-xenon collisions obey a power law:

$$\text{Yield} \propto A_f^{-\tau} \qquad (1)$$

where A_f is the fragment mass number and the exponent τ has the value 2.65 and 2.64, respectively, for Kr and Xe target. The fact that τ is practically independent of target mass for heavy targets indicates that such a specific value of τ should characterize the statistical clustering mechanism in the fragment formation.

Finn et al.[2] have mentioned that the general problem of clustering can be treated by percolation theory, in which clusters are formed by sites that are randomly occupied (site percolation) or by sites that are linked through randomly distributed bonds (bond percolation). The cluster distribution is found to obey the scaling law[3]:

$$n_s \sim s^{-\tau} f(\varepsilon s^{\sigma}) \quad , \qquad (2)$$

where n_s is the mean number of clusters of size s, τ and σ are exponents and the scaling function f has the property f(0) = 1. Also, $\varepsilon = p - p_c$ with p the probability concentration and p_c the critical probability. Near p_c, the cluster distribution has similar power-law behavior as Eq.(1), with $\tau \cong 2.1$ for percolation on three-dimensional lattices. On the other hand, Minich et al.[4] and Hirsch et al.[1] have treated the formation of fragments as a liquid-gas phase transition at the critical point, in analogy with the condensation theory of Fisher[5] which also has a power law dependence for cluster distribution.

The relevance of percolation ideas in nuclear fragmentation has further been demonstrated by Campi[6] through cross relations among various moments of the fragment size distribution. It is found that the moments of the experimental fragment size distribution in individual events[7] are strongly correlated, which can be interpreted as signature of critical phenomena. Furthermore, the critical exponents deduced from such correlations are found to be close to those of percolation for finite systems.

In this Letter, we consider a new percolation model on a finite sphere containing a number of sites A_0 on the FCC lattice. Each site has a certain probability p for being occupied and 1 - p for being vacant. Clusters are formed by particles having tetrahedron linkages with their nearest neighbors. Our model is a high-density percolation[8] having minimum number of neighbors m = 3, but with restrictions on the orientation of the neighboring particles. Such restrictions significantly modify the percolation process, as our results indicate that our restricted high-density percolation seems to belong to a new universality class.

We use the Monte Carlo technique to select randomly a certain fraction of sites pA_0 to be occupied. We then search for groups of four particles forming tetrahedron clusters. Clusters sharing at least one common particle are combined into a single one. Our percolation model excludes clusters of size two and three and the choice of the FCC lattice further excludes clusters of size five. In all systems with 87, 135 and 201 sites considered, we find nearly the same "critical probability" $p_c \cong 0.50$. Near such "critical probability", the cluster size distribution is expected to obey a power law similar to the infinite system. This is shown in Figs. 1 and 2, that illustrate the log-log plots for the number of clusters (yield) as a function of the cluster size (fragment mass number) at p = 0.50. The data points for these cases seem to lie most closely on a straight line.

In Fig.1, the maximum fragment size for the system $A_0 = 87$ is shown to be about 30, which is approximately one third of the size of the system. Furthermore, the exponent has the value $\tau = 2.55$ comparing with 2.65 for the proton--krypton (A = 86) collision. In Fig.2, the $A_0 = 135$ system is shown to have an exponent τ = 2.50, which is to be compared with 2.64 for the proton-xenon (A = 132) collision. For the A_0 = 201 system, we find that τ is nearly the same as that for the A_0 = 135 system. If the exponent τ converges to a value near 2.50, our model should belong to a new universality class different from the ordinary percolation as well as the high-density percolation previously considered.

The values of τ in our simulation has a discrepancy of about 2% between the A_0 = 87 and A_0 = 135 systems, whereas the experimental values for p + Kr and p + Xe collisions are nearly the same. However, examining the data points for the p + Kr collision of Hirsch et al.[1], we note that a slighty larger value of τ can fit more closely the data points corresponding to $A_f > 25$.

Our model can be considered as a two-stage fragmentation process. In the first stage, a certain fraction of high-energy nucleons is ejected out of the target nucleus and the residue nucleus is then consisting of clusters similar to the ordinary percolation clusters. In the second stage, these clusters undergo further fragmentation and more stable fragments as well as low-energy evaporated nucleons are produced. Such a two-stage process seems to be supported by experimental central collision such as that of a 42 GeV Ne + Au reaction, in which about one hundred fast nucleons are ejected in the first stage, then five or six fragments are produced together with about thirty slow nucleons[9].

The clusters in ordinary percolation theory can have very ramified configurations. However, it is dificult to imagine that such clusters can form observable nuclear fragments. Most previous percolation approaches to the fragmentation reaction[10-12] have included such clusters. Although Campi et al.[13] have incorporated some "compactedness" conditions, it is unclear that they were able to produce an exponent in agreement with the experimental value.

Moreover, we have performed our simulation on the FCC lattice, which is shown by Cook and Dallacasa[14] as the only lattice model that is isomorphic with the independent-particle description of nuclei.

In conclusion, we have proposed a new percolation model that seems to belong to a different universality class from the ordinary percolation as well as the high-density percolation. Our model simulates a two-stage fragmentation. The results show good agreement with the experimental fragment mass distribution for the proton-induced reactions.

We are indebted to Prof. T. Kodama for many discussions as well as valuable suggestions on performing simulations. One of us (NCC) would also like to thank the financial support from the Departamento de Física Nuclear e Altas Energias at CBPF and the Pontifícia Universidade Católica do Rio de Janeiro during the course of this research. This work was partially supported by CNPq and FINEP (Brasil).

FIGURE CAPTIONS:

Fig.1: Cluster number (yield) vs cluster size (fragment mass number) for a restricted high-density site percolation for A_0 = 87 sites on the FCC lattice within a sphere at p = 0.50. Data are the results of Monte Carlo simulation of 3000 runs.

Fig.2: Cluster number (yield) vs cluster size (fragment mass number) for a restricted high-density site percolation for A_0 = 135 (dots) and A_0 = 201 (crosses) on the FCC lattice within a sphere at p = 0.50. Data are the results of Monte Carlo simulation of 2000 runs.

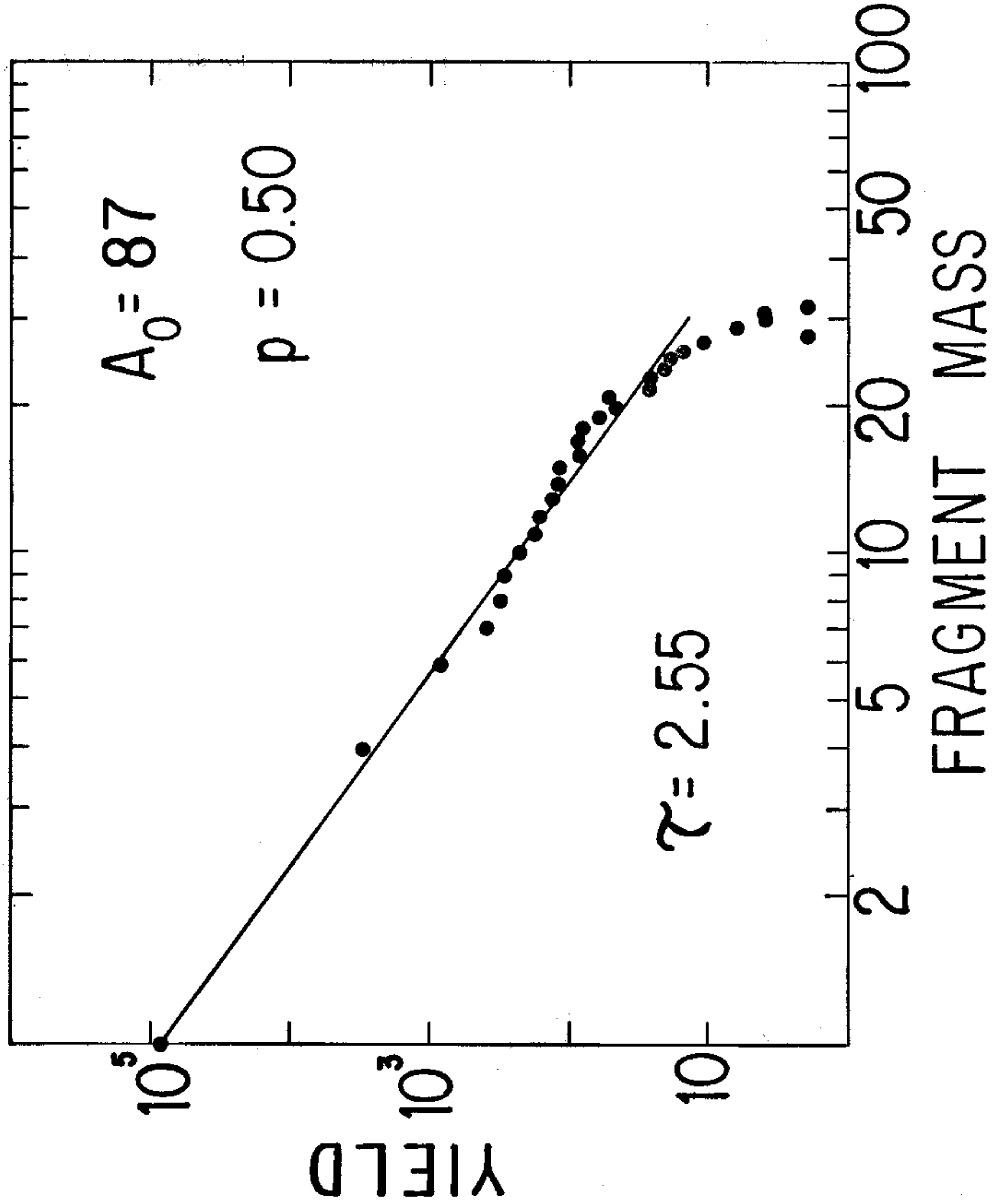

Fig. 1

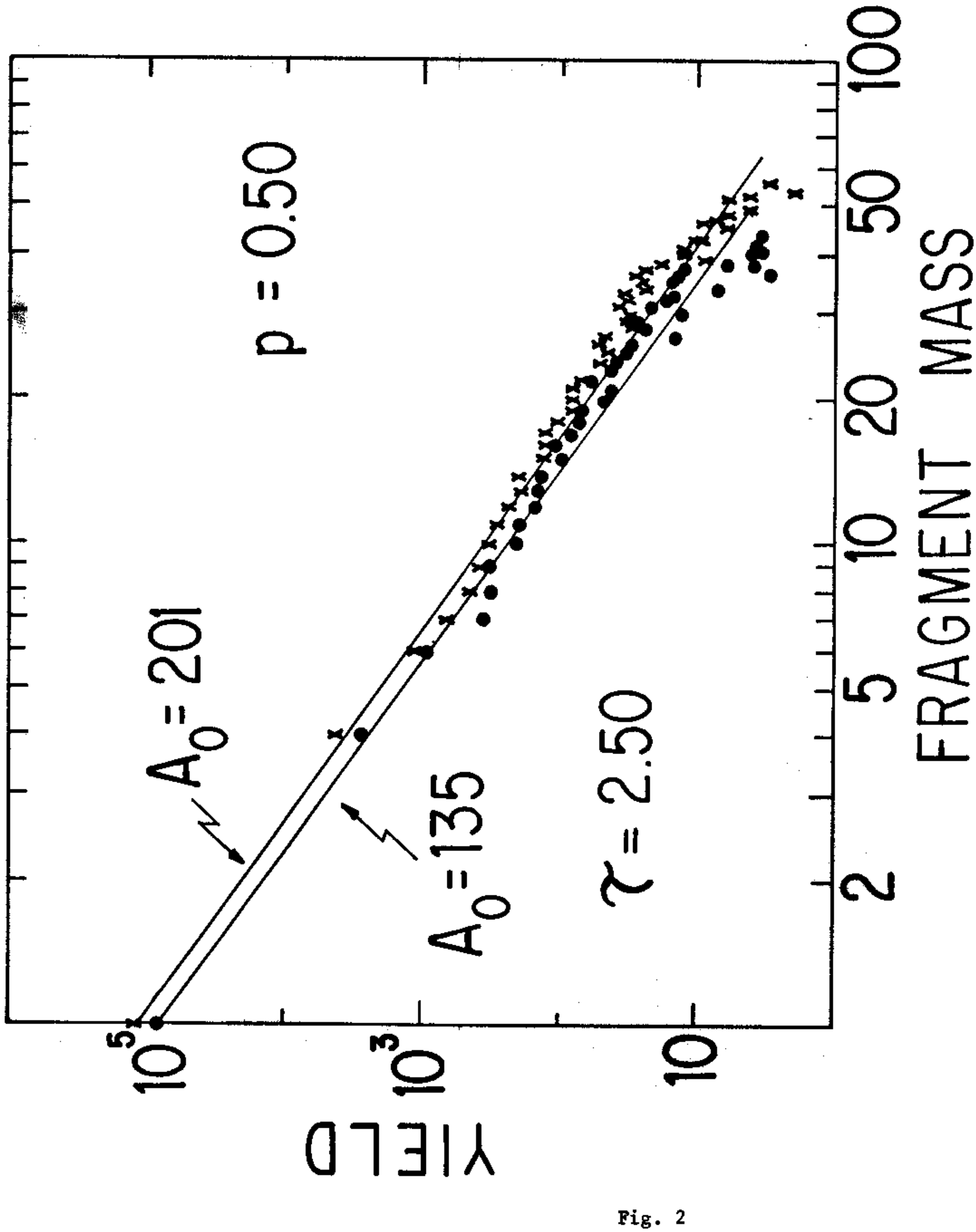
YIELD
10^5
10^3
10
FRAGMENT MASS
2
5
10
20
50
100
$A_0 = 201$
$A_0 = 135$
$\tau = 2.50$
p = 0.50

Fig. 2

REFERENCES

1. A.S.Hirsch, A.Bujak, J.E.Finn, L.J.Gutay, R.W.Minich, N.T. Porile, R.P.Scharenberg, B.C.Stringfellow and F.Turkot, Phys.Rev. C **29**, 508 (1984).
2. J.E.Finn, S.Agarwal, A.Bujak, J.Chuang, L.J.Gutay, A.S. Hirsch, R.W.Minich, N.T.Porile, R.P.Scharenberg, B.C.Stringfellow and F.Turkot, Phys.Rev.Lett. 49, 1321 (1982).
3. See, for example, J.W.Essam, Rep. Prog. Phys. 43, 833 (1980) and D.Stauffer, Introduction To Percolation Theory (London: Taylor and Francis).
4. R.W.Minich, S.Agarwal, A.Bujak, J.Chuang, J.E.Finn, L.J. Gutay, A.S.Hirsch, N.T.Porile, R.P.Scharenberg, B.C.Stringfellow and F.Turkot, Phys.Lett. 118B, 458 (1982).
5. M.E.Fisher, Physics (N.Y.) 3, 255 (1967); Rep. Prog. Phys. 30, 615 (1967).
6. X.Campi, J.Phys. A19, L917 (1986).
7. C.J.Waddington and P.S.Freier, Phys.Rev.C 31, 888 (1985).
8. P.M.Kogut and P.L.Leath, J.Phys. C15, 4225 (1982).
9. A.I.Warwick et al., Phys.Rev. C 27, 1083 (1983).
10. W.Bauer, U.Post, D.R.Dean and U.Mosel, Nucl.Phys. A452, 699 (1986).
11. H.W.Barz, J.P.Bondorf, R.Donangelo and H.Schulz, Phys.Lett. B169, 318 (1986).
12. O.Knospe, R.Schmidt and H.Schulz, Phys.Lett. B182, 293 (1986).
13. X.Campi and J.Desbois, Proc. 23rd Int. Winter Meeting on Nuclear Physics, Bormio, 1985.
14. N.D.Cook and V.Dallacasa, Phys.Rev. C 35, 1883 (1987).