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ABSTRACT

It is shown that the explicit breaking of (1,0) supersymmetry by means of a torsion-like term yields dynamical mass generation for the gauge superfields which couple to a (1,0) - supersymmetric non-linear σ-model.

Key-words: Supersymmetric gauge-invariant; Non-linear σ-models.

Two-dimensional supersymmetries of type (p,q) have been fairly well studied in a series of works [1,5]. For the Green-Schwarz-type superstrings [6], the (1,1) case is relevant [7], whereas the formulation of the heterotic string [8] requires a (1,0)-type supersymmetry on the world sheet [7]. Indeed, in the Polyakov's approach to the heterotic superstring, a (1,0) non-linear σ-model is coupled to (1,0) supergravity [9], and the critical dimension turns out to be 10.

In dimensionally reducing the theory from 10-dimensional world, effective 4-dimensional theories of interest for phenomenology should exhibit an unbroken supersymmetry [7]. So, a compactification constraint imposes that the heterotic σ-model be actually (2,0) supersymmetric [7]. Though a superfield formulation of the latter is known in (2,0) superspace itself [10], an exhaustive description of arbitrary (1,0) supersymmetric σ-models can be found in [11], where geometric constraints emerge if one wishes to actually have a supersymmetry of type (2,0).

Rich enough is the structure of anomalies of these arbitrary (1,0) supersymmetric σ-models. In ref. [2], Hull and Witten present in details mechanisms for the cancellation of the σ-model anomaly [12].

Motivated by the remarkable quantum properties exhibited by the $N=\frac{1}{2}$ supersymmetric σ-models [11], we shall in this work discuss the possibility of a dynamical mass generation for the gauge connection of the local Yang-Mills symmetry discussed in [2,11]. We shall pursue our investigation by contemplating the general (1,0) action of ref. [2] modified by the presence of a gauge invariant interaction term which explicitly (and softly) breaks (1,0) supersymmetry. This term, as we shall discuss later, admits a geometrical interpretation and is proposed so as to probe the fermionic quartic coupling characteristic of supersymmetric σ-models. On the other hand, the breaking does not affect the renormalisability of the originally exact model and induces two-loop mass-like terms for the gauge fields of the model.

Before starting our discussion, let us briefly summarise our notation and conventions concerning the (1,0) supersymmetry algebra. We parametrise our superspace by means of the coordinates $(x^{+},x^{-};\theta)$, where $x^{\pm}$ are the usual light-cone coordinates $(x^{\pm}\equiv \frac{x^0 \pm x^1}{\sqrt{2}})$ and θ is a real (Majorana) left-handed Weyl spinor. The supersymmetric covariant derivative operator is given by

$$D = i\partial_{\theta} + \theta\partial_{+} \quad , \qquad (1)$$

with $\partial_{\theta}\equiv\frac{\partial}{\partial\theta}$ and $\partial_{+}\equiv\frac{\partial}{\partial x^{+}}$, and satisfies

$$D^2 = i\partial_{+} \; . \qquad (2)$$

The types of superfields relevant for our purposes here are a real scalar superfield, $\Phi(x;\theta)$, and a real left-handed spinor superfield, $\Psi(x;\theta)$, defined by the following θ-expansions:

$$\Phi(x;\theta) \equiv \phi(x) - i\theta\lambda(x) \qquad (3a)$$

and

$$\Psi(x;\theta) \equiv \psi(x) + \theta F(x) \quad , \qquad (3b)$$

where ϕ is a real scalar field, λ and ψ are respectively right-handed and left-handed Majorana-Weyl spinors and F is a real scalar non-propagating (auxialiary field).

The action for an arbitrary (1,0)-type supersymmetric σ-model written in (1,0) superspace reads

$$S_o = \int d^2x d\theta \left[g_{ij}(\Phi) + b_{ij}(\Phi)\right](D\phi^i)(\partial_-\theta^j) + $$
$$+ \int d^2x d\theta \; G_{AB}(\Phi)\Psi^A(\nabla\Psi^B), \tag{4}$$

where

$$\nabla\Psi^B \equiv \left[\partial^B_G D + A_i{}^B{}_C(\Phi)\, D\Phi^i\right]\Psi^C . \tag{5}$$

The superfields $\Phi^i (i = 1,2,\ldots,n)$ give the coordinates of some n-dimensional manifold K (the σ-model target space) with metric tensor $g_{ij}(\Phi)$. The tensor $b_{ij}(\Phi) = -b_{ji}(\Phi)$ is defined on K and determines the (1,0) supersymmetric version of the Wess-Zumino term. The spinor superfields $\Psi^A (A = 1,2,\ldots,m)$ correspond to local cross-sections of some vector bundle V over K, with fibre metrics given by $G^{AB}(\Phi)$, connection $A_i^{AB}(\Phi) = -A_i^{BA}(\Phi)$ and structure group $G \subseteq O(m)$. The action S_o is invariant under the local rotations

$$\Psi'^A = \Lambda^A{}_B(\Phi)\Psi^B \tag{6}$$

of G. Without loss of generality, we shall from now assume $G^{AB}(\Phi) = \delta^{AB}$.

We then add to S_o an explicit (gauge invariant) supersymmetry breaking term given by

$$S_{br} = g\int d^2x d\theta\, \theta\, T_{iAB}(\Phi)(D\Phi^i)(\nabla\Psi^A)\Psi^B \quad , \tag{7}$$

where g is a dimensionless breaking parameter. The local G-invariance of S_{br} is guaranteed provided that the tensor $T_{iAB}(\Phi)$ defined over V transforms according to

$$T'^{AB}_i(\Phi) = \Lambda^A_C(\Phi)\Lambda^B_D(\Phi)T^{CD}_i(\Phi) \quad . \tag{8}$$

The explicit breaking term proposed in eq. (7) has been uniquely fixed on the basis of dimensional counting (we wish a dimensionless coupling parameter), Lorentz invariance and the request that it gives a quartic fermion interaction once the auxiliary field $F^A(x)$ is eliminated through its equation of motion. We shall now seek a possible geometric interpretation for the tensor $T^{AB}_i(\Phi)$ that determines our breaking interaction term.

We start by giving the Ψ^A-superfield equation of motion,

$$\begin{aligned}
&2D\Psi^A + (igT_i{}^A{}_B + 2A_i{}^A{}_B)(D\phi^i)\Psi^B + \\
&-g\theta T_{i,j}{}^A{}_B(D\Phi^j)(D\Phi^i)\Psi^B - ig\theta T_i{}^A{}_B(\partial_+\Phi^i)\Psi^B + \\
&+g\theta T_i{}^A{}_B(D\Phi^i)(D\Psi^B) - g\theta T_i{}^B{}_C A_{jB}{}^A(D\Phi^i)(D\Phi^j)\Psi^C + \\
&+g\theta T_{iB}{}^A A_j{}^B{}_C(D\Phi^i)(D\Phi^j)\Psi^C = 0 \quad ,
\end{aligned} \tag{9}$$

and then project on the $\theta = 0$ component, which yields the following solution for the non-propagating field F^A:

$$F^A = -(A_i{}^A{}_B + \frac{i}{2}\, g\, T_i{}^A{}_B)\lambda^i\psi^B \tag{10}$$

We indeed see that the only effect of our breaking term is to affect the spinor couplings.

Replacing this solution into the component-field action stemming from (S_0+S_{br}) yields a quartic spinor coupling of the form $\lambda\lambda\psi\psi$ whose explicit expression is:

$$\mathcal{L}_{quartic} = [\partial_i A_{jAB} + 3(A_{iAC} + \frac{i}{2} g\, T_{iAC}) \cdot$$

$$\cdot (A_{jB}{}^C + \frac{i}{2} g\, T_{jB}{}^C)]\lambda^i \lambda^j \psi^A \psi^B \tag{11}$$

The interaction term given above is all we need to geometrically motivate the meaning of the tensor $T_i{}^{AB}(\Phi)$: it is a curl-free torsion on the vector bundle V over K which modifies the connection $A_i{}^{AB}$ according to

$$\hat{A}_i{}^{AB}(\Phi) = A_i{}^{AB}(\Phi) + \frac{i}{2} g\, T_i{}^{AB}(\Phi) \ . \tag{12}$$

Thus, the breaking term proposed in eq. (7) is the (1,0) analogue of the N=1 supersymmetry breaking term that induces the contorted non-linear σ-model of ref. [13].

We move now towards the study of some quantum features of the model defined by the action S_0+S_{br}. We shall see that the breaking (7) induces a finite 2-loop mass-like contribution to the gauge connection $A_i{}^{AB}$. We pursue our investigation in terms of supergraph computations, despite the explicit breaking of supersymmetry [14], and we shall adopt the background superfield method in combination with the normal coordinate expansion to maintain manifest the invariance under the K-manifold reparametrisation.

As for our purposes in this work, the relevant normal coordinate expansions [15] are listed below:

$$T_i^{AB}(\Phi+\pi) = T_i^{AB}(\phi) + \nabla_j T_i^{AB}(\Phi)\,\xi^j +$$

$$+ \frac{1}{2}\,[\nabla_{i_1}\nabla_{i_2}T_i^{AB}(\Phi) - \frac{1}{3}\,R^j_{i_1 i i_2}T_j^{AB}(\Phi)]\xi^{i_1}\xi^{i_2} +$$

$$+ O(\xi^3)\ , \tag{13}$$

$$D_+(\Phi^i+\pi^i) = D_+\Phi^i + \nabla_+\xi^i + \frac{1}{3}\,R^i{}_{i_1 i_2 j}(D_+\Phi^j)\xi^{i_1}\xi^{i_2} +$$

$$+ O(\xi^3)\ , \tag{14}$$

where

$$\nabla_+\xi^i \equiv D_+\xi^i + \Gamma^i{}_{mn}(D_+\Phi^m)\xi^n\ , \tag{15}$$

$\Gamma^i{}_{mn}$ being the Christoffel symbol of the manifold K. The expansion for A_i^{AB} follows the same pattern as the one for T_i^{AB}. It is also worthy to mention that from now on, Φ^i denotes the background part of the superfield giving the coordinates of the K-manifold.

Though we shall be considering two-loop supergraphs, we truncate our normal coordinate expansion at second order in the quantum fluctuation ξ^i. For a complete two-loop analysis of the effective action, it is clear that the expansions (13) and (14) should be given up to quartic ξ-terms. However, by analysing all two-loop diagrams generated by vertices with 3 and 4 superfields ξ^i, we have checked that they do not contribute gauge field mass-like terms to the effective action. This justifies why our normal coordinate expansions were taken as in (13) and (14). Once they are inserted into the action (S_o+S_{br}), one can readily get the superpropagators:

$$<\xi^a(1)\xi^b(2)> = \delta^{ab}\frac{1}{k^2}D(k)\delta(\theta_1-\theta_2) \qquad (16a)$$

and

$$<\Psi^A(1)\Psi^B(2)> = \delta^{AB}\frac{ik_-}{k^2}D(k)\delta(\theta_1-\theta_2)\ , \qquad (16b)$$

and read off the background-quantum vertices from which we shall set our Feynman diagrams. In (16), a and b stand for the local frame indices of the target space $(a,b=1,2,...,n)$.

Before presenting the results for the computation of the 2-loop graphs contributing the gauge superfield a mass term, we should state a few facts concerning the 1-loop analysis. The breaking parameter λ requires an 1-loop renormalisation as already expected from power-counting argument. Its renormalisation does not however interfere on the 2-loop gauge-field mass contribution. Moreover, the breaking of (1,0)-supersymmetry by means of (7) does not generate any other 1-loop infinity we have to take care of, neither any finite mass-like term for the gauge superfield. We can therefore pass to the discussion of those 2-loop supergraphs relevant for our purposes in this work.

We draw in Fig. 1 the structure of the 2-loop superdiagrams which contribute supersymmetric mass-like terms to the superfield A_i^{AB}. They are linear in the breaking parameter λ and, despite the presence of an explicit θ-factor in one of the vertices, the supersymmetry algebra performed in the course of the supergraph evaluation eliminates the explicit θ-dependence in such a way that our final answer is supersymmetric.

Structure of the 2-loop graphs which contribute a gauge-field mass term

A_i^{AB}-field

Ψ_a - field

ξ^a - field

Fig. 1

Drawing all the graphs with the structure of Fig. 1, and using the Feynman rules derived from the actions (4) and (7), we obtain the following answer:

$$
\begin{aligned}
& ig^2\lambda\int\frac{d^2k}{(2\pi)^2}\,\frac{d^2\ell}{(2\pi)^2}\,\frac{k_-}{k^2}\,\frac{\ell_+}{\ell^2}\,\frac{(\ell+k+p)_-}{(\ell+k+p)^2} \quad . \\
& \cdot \int d\theta\,\left\{\left[T_k{}^{AB}(\Phi)D_+\Phi^k\right]\delta_i{}^j\, + \right. \\
& \left. + \left[T_i{}^{AB}(\Phi)D_+\Phi^j\right]\right\} A^{iCB}\,A_j{}^{AC}\, + \\
& + \text{(su.sy.breaking terms)} \quad , \qquad (17)
\end{aligned}
$$

where the "su.sy.breaking terms" exhibit an explicit θ-dependence and, even at the level of component fields, they do not yield mass-like contributions.

Our calculations then show that the major consequence of our torsion-like breaking term is that, at the quantum level, it triggers the dynamical mass generation for the gauge fields which couple to the non-linear σ-model as given in the action (4). For N=1-supersymmetry, ref. [16] indicates dynamical mass generation from the coupling of the gauge superfields to the non-linear σ-model but, contrary to what we have concluded in this letter, the (1,1) - case did not require any breaking term with explicit θ-dependence.

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