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# SYSTEMATICS OF PHOTON - AND ELECTRON-INDUCED SPALLATION REACTIONS*

by

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## ABSTRACT

A semi-empirical four-parameter formula is proposed in order to systematize intermediate- and high-energy photon- and electron-induced spallation cross sections and bremsstrahlung-induced spallation yields of medium weight nuclei. Analytical expressions for the parameters are obtained from parameter-values calculated by best-fitting of experimental data. We extend the same treatment to bremsstrahlung-induced spallation yields, and we verify that analytical expressions of the same form can be applied to estimate spallation yields. Such a four-parameter formula has shown to predict reliable values of unknown spallation reaction cross sections. The reproducibility of the experimental data by the present formula is comparable with or better than those by other semi-empirical formulae with a larger number of parameters.


Key-words: Spallation reactions; Medium-weight nuclei; 0.075-5.0 GeV bremsstrahlung; 3-16 GeV electrons; Calculated yield distributions.

## INTRODUCTION

The study of photospallation reactions has been made by measuring the yields of the radioactive products mostly by the activation analysis technique. A systematization of these measurements was carried out by JONSSON and LINDGREN [1] by using the CDMD (charge distributions and mass distribution) RUDSTAM's five--parameter formula [2].

The aim of the present work is to extend the four-parameter formula presented in a previous paper by us in a study of proton-induced spallation yields [3] to photon-, electron-, and bremsstrahlung-induced spallation reactions. This four-parameter formula was deduced in a statistical framework of the spallation process proposed by GUPTA *et al.* [4]. If the statistical aspect of this process is the relevant one, we believe that the formula which has been applied to proton-induced spallation yields should also work well when applied to photospallation yields. This formula gives the formation cross section of a product nuclide $(Z,A)$ through a nominal loss of $x$ protons and $y$ neutrons from a target nucleus $(Z_t,A_t)$ as

$$\sigma(x,y) = \sigma_M \exp[-B(x-1)-Kw^2] \quad , \qquad w = x-C\alpha y \quad , \qquad (1)$$

where $\sigma_M$, $B$, $C$ and $K$ are parameters to be obtained by best fitting the experimental data, and

$$\alpha = \frac{Z_t + 1}{A_t - Z_t} \quad . \qquad (2)$$

The parameter $\sigma_M$ is the factor which normalizes the distribution with the measured cross sections and represents the cross section at the peak of the isotopic curve (one proton lost); this cross section is generally not measured. The parameter $B$ gives the rate of decreasing of the cross section for the different isotopic distribution curves; it is related to the slope of the yield-surface ridge as introduced by HALPERN *et al.* [5]. The parameter $K$ gives the width of the distribution described by equation (1) and has dimension of (number of protons)$^{-2}$. The parameter $C$ is related to the coeficient of "average retention" of charged particles due to the Coulomb potential barrier of the target nucleus.

A slight modification introduced into formula (1) will suffice to describe the main characteristics of photon- and electron--induced spallation reactions. The modification consists of changing the expression for $\alpha$ in equation (2) into a new one of the form

$$\alpha = \frac{Z_t}{A_t - Z_t} \tag{3}$$

which is appropriate to the photoreactions under study. As before [3], the same limiting conditions have been imposed to product nuclides. We consider as *true* spallation reactions only those for which emission of at least one proton and one neutron occurs, and product nuclides must have mass number greater than or equal to, at least, half of the original nucleus, i.e.,

$$Z \geq \frac{Z_t}{2} \quad , \qquad A \geq \frac{A_t}{2} \quad . \tag{4}$$

The present four-parameter formula, in its preliminar form, was tested by us in an analysis of bremsstrahlung-induced spallation yields of $^{nat}$Cu [6] by taking experimental data by BACHSCHI *et al.* [7]. Very encouraging results were then obtained.

SYSTEMATICS

The available intermediate- and high-energy photon- and electron-induced spallation cross sections have been compiled and used to construct approximate cross section distributions with equation (1). These data are mainly for intermediate-mass target nuclei, although there are a few measurements for some heavy target nuclei [8,9]. However, values of the parameters obtained from curve fitting for these latter data did not give the expected spallation pattern and this fact makes difficult to obtain a reliable spallation systematics covering the region of heaviest target nuclei.

The electrospallation cross section were included in the present analysis for, according to the concept of virtual photons, an electron-induced reaction can be treated as a photonuclear reaction as proposed by WEIZACKER [10] and WILLIAMS [11] and developed by BARBER [12]. NOGA *et al.* [13] established a very practical relationship which allows to calculate the cross section ratio of these two processes. The relationship reads

$$\frac{\sigma_Q}{\sigma_e} = \left(\frac{\pi}{2\alpha}\right) \left[\ln\left(\frac{E_0}{m_e}\right) - 0.5\right]^{-1} \quad . \qquad (5)$$

Here, $\alpha = 1/137$, $\sigma_Q$ is the cross section per equivalent photon, $\sigma_e$ is the cross section per electron, $E_0$ is the incident electron energy, and $m_e$ is the electron rest mass. The experimental confirmation of this ratio has been made from measurements of induced activities by using mainly monoisotopic or quasi-monoisotopic target elements. Recently, BATIĨ *et al.* [14] suggested the utilization of foils of a single element with a different isotopic composition which are submitted to simultaneous irradiation of high-energy electron beams. By this method, they have obtained good agreement with predictions following the cascade-evaporative model and a five-parameter RUDSTAM's formula.

We have also included bremsstrahlung spallation data, although they are dependent of the specific irradiation condition. For these cases, we are able to establish relationships for the parameters between the caracteristics of the target nucleus and the bremsstrahlung end-point energy.

The four parameters of formula (1) have been determined by least-squares fitting the experimental data, and taking the quantity

$$R = \exp(\delta) \quad , \quad \delta^2 = \frac{1}{n} \sum_{i=1}^{n} (\ln \frac{\sigma_i^e}{\sigma_i^c})^2 \qquad (6)$$

as an indicator of the goodness of fitting. In equation (6), $\sigma_i^e$ and $\sigma_i^c$ denote the experimental and calculated cross section, respectively, for the formation of the residual $(Z_i, A_i)$, and $n$ is the final number of measurements fitted to the semi-empirical distribution. Details of the calculation procedure can be seen in our previous paper [3]. The parameter-values so obtained are listed in Tables 1 and 2, together with the standard $\chi^2$-values.

DEPENDENCE OF PARAMETERS WITH IRRADIATION CONDITIONS

Photospallation and electrospallation cross sections

The parameter $\sigma_M$ (expressed in μb) has shown to be related to incident mean photon energy (or incident electron energy) per total mass number of the target nucleus, $\varepsilon = E/A_t$ (expressed in MeV/$\mathcal{N}$), as shown in Fig. 1-a. We found , from a least-

-squares fit, the following expression to estimate $\sigma_M$-values:

$$\sigma_M = 15712\ \varepsilon^{-1.356} \quad (\mu b) \qquad \text{if} \quad \varepsilon \lesssim 21\ \text{MeV}/\mathcal{N}$$
$$\sigma_M = 248 \quad (\mu b) \qquad \text{if} \quad \varepsilon \gtrsim 21\ \text{MeV}/\mathcal{N} \ . \tag{7}$$

The values of parameter $B$ are plotted *versus* $\varepsilon$ in Fig. 1-b, where we can see that $B$ decreases with increasing of $\varepsilon$ up to 10 MeV/$\mathcal{N}$; for $\varepsilon > 10$ MeV/$\mathcal{N}$, $B$ has a constant value of 0.25. An expression for $B$ has been deduced as

$$B = 3.03\ \varepsilon^{-1.06} \qquad \text{if} \quad \varepsilon \lesssim 10\ \text{MeV}/\mathcal{N}$$
$$B = 0.25 \qquad \text{if} \quad \varepsilon > 10\ \text{MeV}/\mathcal{N} \ . \tag{8}$$

The parameter $K$ is related to the standard deviation, $s = (2K)^{-1/2}$, of the isotopic distributions, or of the "reduced" distribution

$$\sigma(x,y) \times e^{B(x-1)} = e^{-Kw^2} \quad . \tag{9}$$

As shown in the Fig. 1-c, the values of $s$ are distributed normally around a constant value and, therefore, the parameter $K$ has shown to be independent of the irradiation conditions. A mean value $K = 0.466 \pm 0.060$ (proton number)$^{-2}$ units has been found.

Finally, the parameter $C$ has shown to exhibit a slight increase with increasing the ratio $\alpha$ of proton to neutron of the struck nucleus (Fig. 2). This parameter has shown to be independent of the irradiation conditions, and the relationship between $C$ and $\alpha$ obtained by least-squares fitting of available data is given by

$$C = 2.30\alpha - 1.044 \quad . \tag{10}$$

Bremsstrahlung-induced spallation yields

In the case of bremsstrahlung-induced yields, we have obtained analytical expressions for parameters $B$, $C$ and $K$ from data listed in Table 2. If $\varepsilon = E_0/A_t$ (MeV/$\mathcal{N}$), we have

$$B = 3.0/\varepsilon \qquad \text{for } \varepsilon \lesssim 10 \text{ MeV}/\mathcal{N}$$
$$B = 0.30 \qquad \text{for } \varepsilon > 10 \text{ MeV}/\mathcal{N} \quad . \qquad (11)$$

The trend of parameter $B$ can be seen in Fig. 3-a.

As expected, the parameter $K$ was obtained to be constant, and a value $K = 0.479 \pm 0.098$ has been deduced (Fig. 3-b).

Finally, the dependence of parameter $C$ with the ratio $\alpha = Z_t/N_t$ (Fig. 4) was found to be

$$C = 2.77\alpha - 1.44 \quad . \qquad (12)$$

DISCUSSION

Within large uncertainties which result for the values of parameters, the analytical expressions obtained are much the same either for spallation mean cross section data or for bremsstrahlung yield data, except for the parameter $\sigma_M$ . In general, the calculated distributions fit the experimental points very closely. As an example, a "reduced" distribution is shown in Fig. 5 for mean cross sections of photospallation residuals of $^{59}$Co target nucleus as reported in Ref. [21].

The four-parameter formula of the present work can be used to predict values for the most probable mass number of isotopic distributions. It suffices to change from $Z_t + 1$ into $Z_t$ in the formula already deduced in our study of systematics of proton-induced spallation yields [3]. Also, it is possible to derive other important quantities related to the spallation process such as the mean nominal number of nucleons lost by the struck nucleus, the charge- and mass-yield distributions, the total spallation reaction cross sections, and the $N/Z$ charge-dispersion curves, as discussed previously [3].

By means of the $\chi^2_\nu$-criterion we can compare the degree of performance of the present formula with the systematics carried out by JONSSON and LINDGREN [1]. These authors used RUDSTAM's five-parameter CDMD formula, according to which the yields of a product nuclide of charge number $Z$ and mass number $A$ from a given target nucleus $A_t$ is given by

$$\sigma(Z,A) = \frac{\hat{\sigma}\, P R^{2/3}}{1.79(e^{PA_t} - 1)} \exp\left[PA - R\left|Z - SA + TA^2\right|^{3/2}\right], \quad (13)$$

where $\hat{\sigma}$, $P$, $R$, $S$ and $T$ are free parameters. The parameter $\hat{\sigma}$ is the total inelastic yield, the parameter $P$ defines the slope of the yield-mass curve, $R$ gives the width of charge distributions, and $S$ and $T$ locate the most probable charge of the distributions. From the analysis of photospallation data, the authors [1] obtained simple expressions to calculate values of the different parameters. In the case of mean cross sections per photon, in the range 250-1000 MeV, the expressions are:

$$\hat{\sigma} = 0.3\, A_t \quad \text{(mb)}$$
$$P = 5.22\, A_t^{-0.89} \qquad S = 0.486 \qquad\qquad (14)$$
$$R = 11.8 A^{-0.45} \qquad T = 0.00038 \; .$$

A comparative study between the systematics of the present work and that by JONSSON and LINDGREN [1] is presented in Table 3. For such a comparison, we have used formula (1) with the expressions (3,7,8,10), and $K = 0.466$. In the case of bremsstrahlung-induced spallation yields, the $\chi_\nu^2$-values have been calculated from parameter-values reported in Ref. [28] and formula (13), while our $\chi_\nu^2$-data have been obtained from formula (1) and parameter-values taken from Table 2. Results are presented in Table 4. From inspection of $\chi_\nu^2$-values shown in Tables 3 and 4, we can see that the performance of formula (1) is comparable with or better than that exhibited by a RUDSTAM's formula [equation (13)].

It should be noted that in the case of bremsstrahlung-induced spallation reactions the product yields are strongly dependent of peculiar experimental conditions, such as radiator thickness, beam collimation, geometrical arrangements, and others, in such a way that the parameter $\sigma_M$ may not result the same if the yields of a given reaction are obtained at different laboratories. We can, however, make use of formula (1) with parameter-values given by equations (11) and (12), and the value $K = 0.479$ to describe the yield distributions of spallation residuals. In this case, the normalizing factor $\sigma_M$ is determined from experimental data. As an example, Fig. 6 shows the "reduced" distribution of spallation residuals measured by ARAKELYAN *et al.* [29] in the interaction of 4.5-GeV bremsstrahlung with $^{60}Ni$. Here, with $\sigma_M = 6.2$ mb, a very good agreement is obtained between experimental and calculated "reduced" yields (about 80% of experimental data have been reproduced within a factor of 2).

Recently SHIBATA *et al.* [30] have analyzed the dependence of parameter $P$ of RUDSTAM's formula with incident bremsstrahlung or kinetic energy of various nuclear projectiles. They showed that the trends of the slope of the mass yield curves are very similar, with values of $P$ for photospallation reactions larger than those for particle-induced spallation. The same behaviour is noted for parameter $B$ of formula (1) if we compare the trends of $B$ obtained for photospallation in the present work and for proton-induced spallation as reported in [3].

## CONCLUSIONS

A semi-empirical four-parameter formula has been proposed to systematize cross sections and yields of electron- and photon-induced spallation residuals. The formula has shown to be very suitable in reproducing experimental data. It can be used advantageously for predicting relative yields and cross sections not available from experiment. Expressions for the different parameters have been deduced, which allow to extend the use of the formula to both target nuclei and incident energies not considered in the present systematics.

FIGURE CAPTIONS

Fig. 1.a) Plot of parameter $\sigma_M$ *versus* electron or mean photon incident energy per total number of nucleons available, $\varepsilon = E/A_t$. Symbols refer to target nuclei as indicated. The figure has been constructed from data of Table 1. The line is the trend for $\sigma_M$ as defined by equation (7). b) plot of parameter $B$ *versus* $\varepsilon$. Points represent values of $B$ as they are listed in Table 1. The line is the trend for $B$ as defined by equation (8). c) plot of standard deviation $s$ of the "reduced" cross section distributions *versus* $\varepsilon$. Points represent data listed in Table 1. The straight line is a least-squares fit through the points.

Fig. 2. Dependence of parameter $C$ on proton to neutron ratio $\alpha = Z_t/N_t$ of the struck nucleus. For each target nucleus the value of $C$ (points) is the average value of the $C$-values for different irradiation energies on that nucleus as given in Table 1. The line is a least-squares fit through the points (equation (10)).

Fig. 3.a) Plot of parameter $B$ *versus* incident bresmsstrahlung maximum energy per mass number of the target nucleus, $\varepsilon = E_0/A_t$. Symbols refer to target nuclei as indicated. Data are taken from Table 2, and the line is the trend for $B$ as defined by equation (11). b) Plot of the standard deviation $s$ of the "reduced" yield distributions *versus* $\varepsilon$ from values of parameter $K$ listed in Table 2. The straight line is a least-squares fit through the points.

Fig. 4. The same as in Fig. 2 for bremsstrahlung induced spallation reactions. Data are from Table 2 and the line is defined by equation (12).

Fig. 5. "Reduced" mean cross section distribution of spallation residuals produced by irradiation of 0.3-1.0 GeV photons on $^{59}Co$. The "reduced" experimental cross sections (points) have been obtained from data listed in Ref. [21]. The parabola results after parameters $\sigma_M$, $B$, $K$, and $C$ of equation (1) having been determined by least--squares fitting (Table 1).

Fig. 6. Comparison between calculated and experimental "reduced"

yield distribution of spallation residuals produced by incident 4.5-GeV bremsstrahlung on $^{60}$Ni. The curve has been obtained from equation (1) with parameters $B$, and $C$ estimated, respectively, by formulas (11) and (12) and $K$ = 0.479. The best $\sigma_M$-value has been calculated from yield data of Ref. [29].

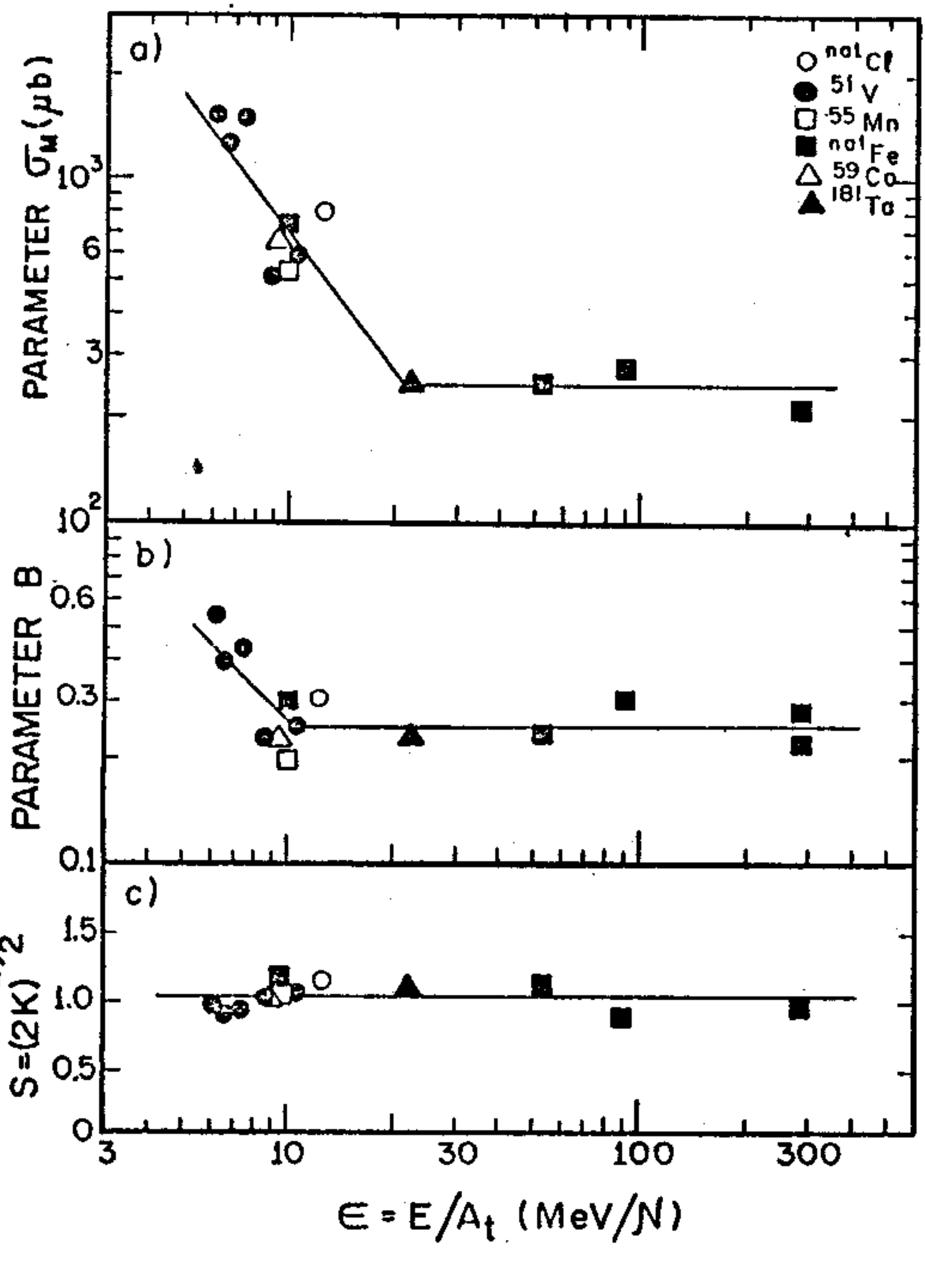

a)
b)
c)
nat Cl
51 V
55 Mn
nat Fe
59 Co
181 Ta
PARAMETER $\sigma_M$ (μb)
PARAMETER B
S=(2K)^-1/2
$10^3$
6
3
$10^2$
0.6
0.3
0.1
1.5
1.0
0.5
0
3
10
30
100
300
∈ = E/A_t (MeV/N)

Fig. 1

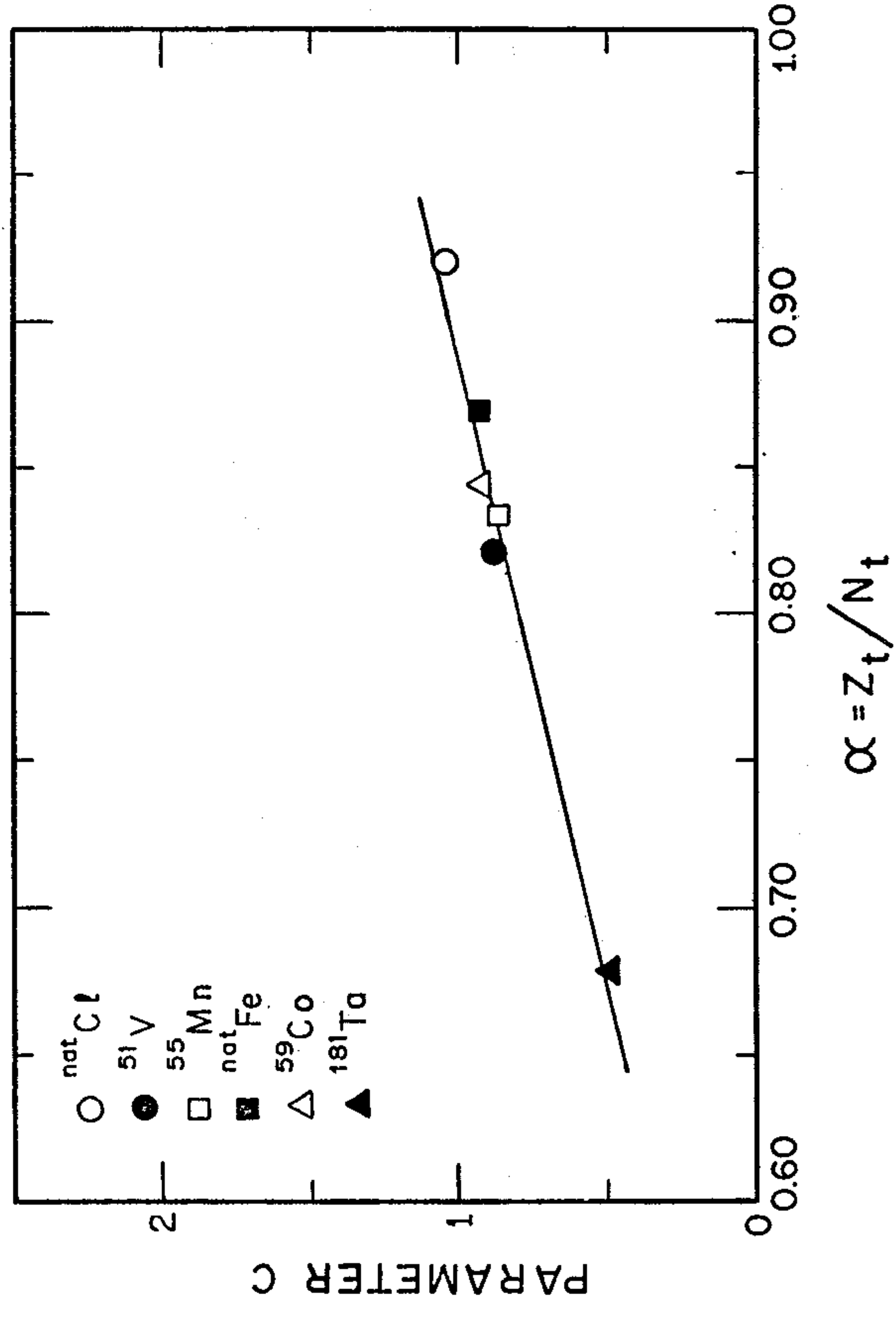

PARAMETER C
0
1
2
0.60
0.70
0.80
0.90
1.00
α = Zt/Nt
natCl
51V
55Mn
natFe
59Co
181Ta

Fig. 2

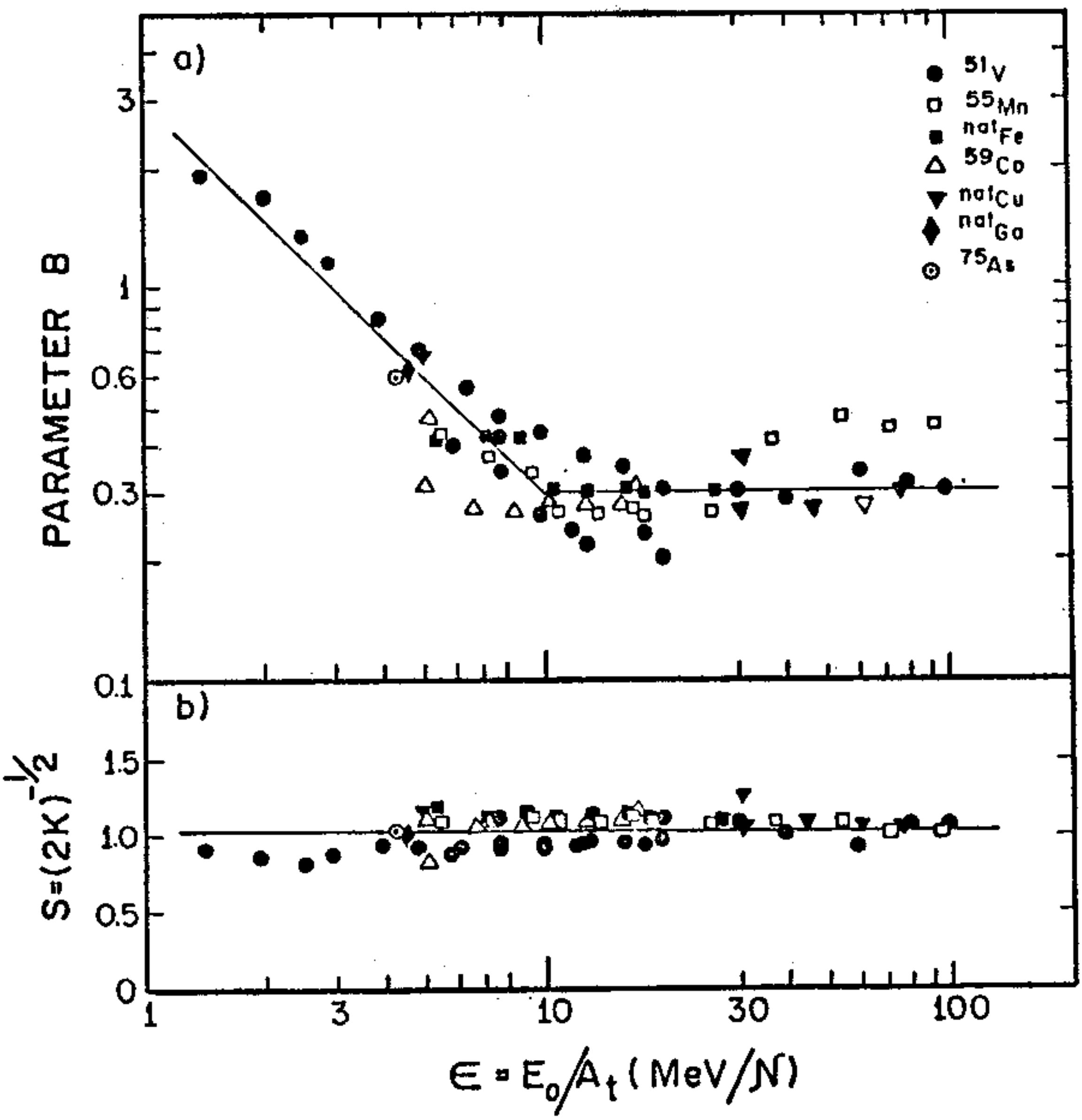

a)
b)
PARAMETER B
$S=(2K)^{-1/2}$
$\epsilon = E_0/A_t$ (MeV/N)
$^{51}V$
$^{55}Mn$
$^{nat}Fe$
$^{59}Co$
$^{nat}Cu$
$^{nat}Ga$
$^{75}As$
3
1
0.6
0.3
0.1
1.5
1.0
0.5
0
1
3
10
30
100

Fig. 3

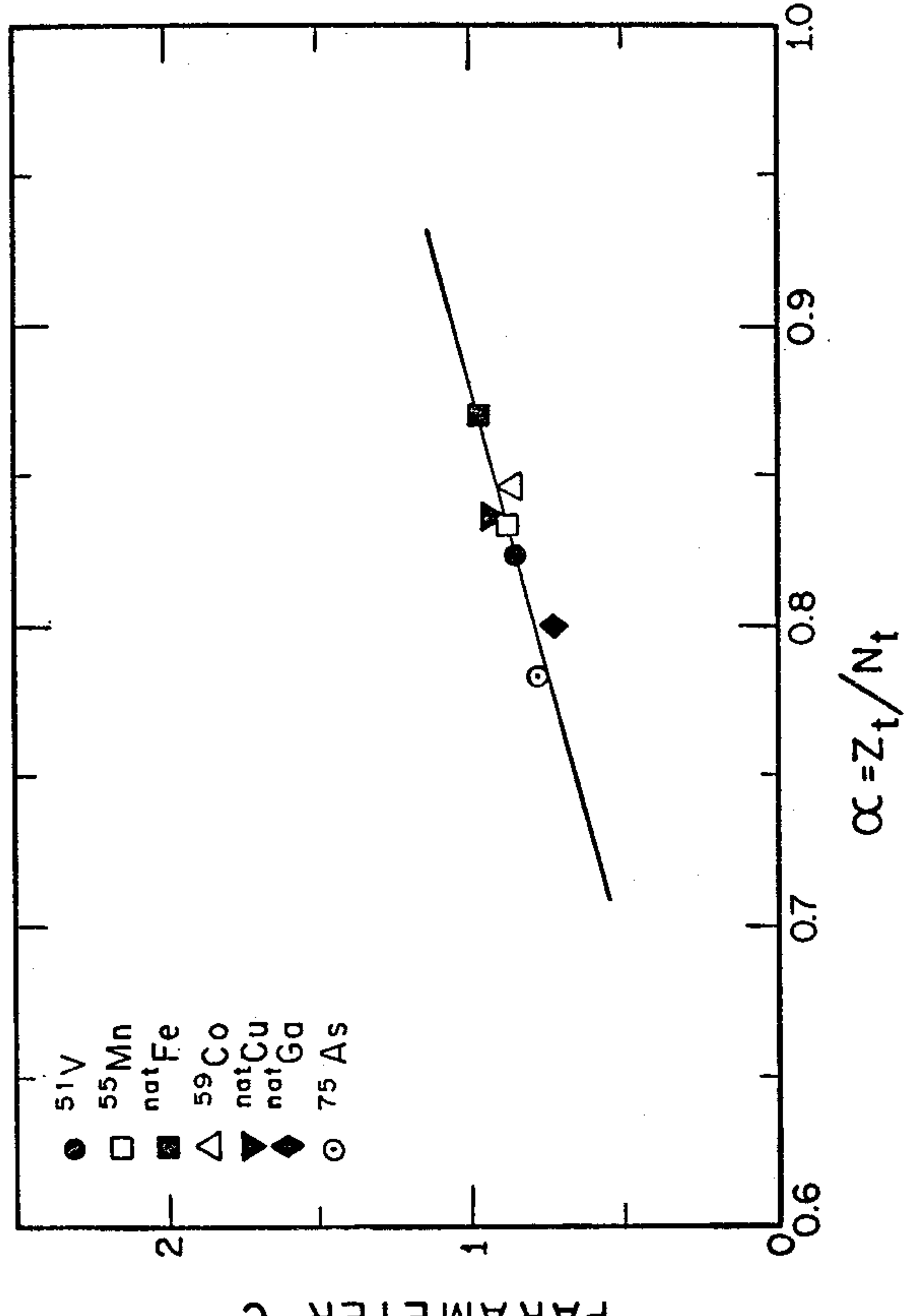
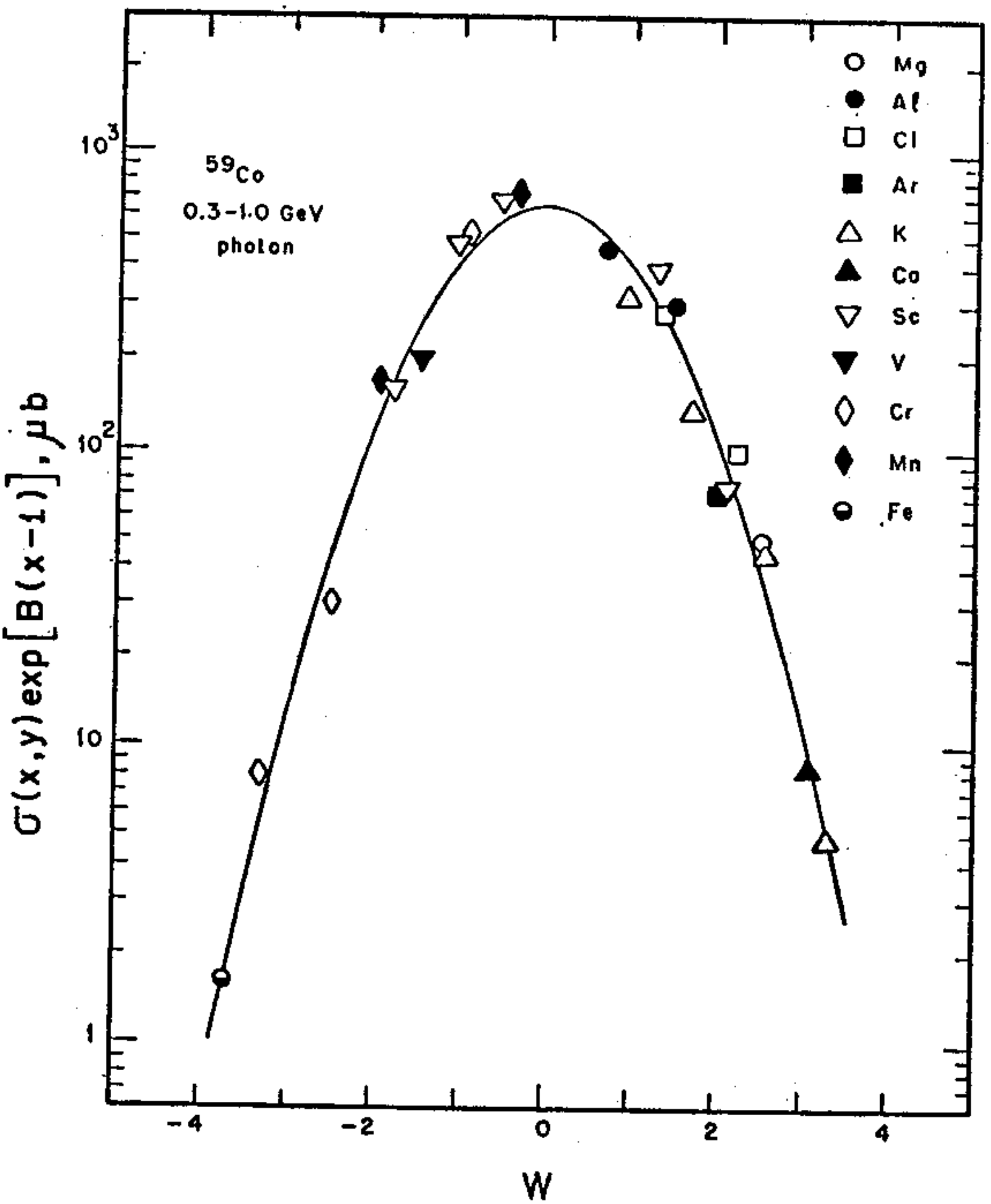
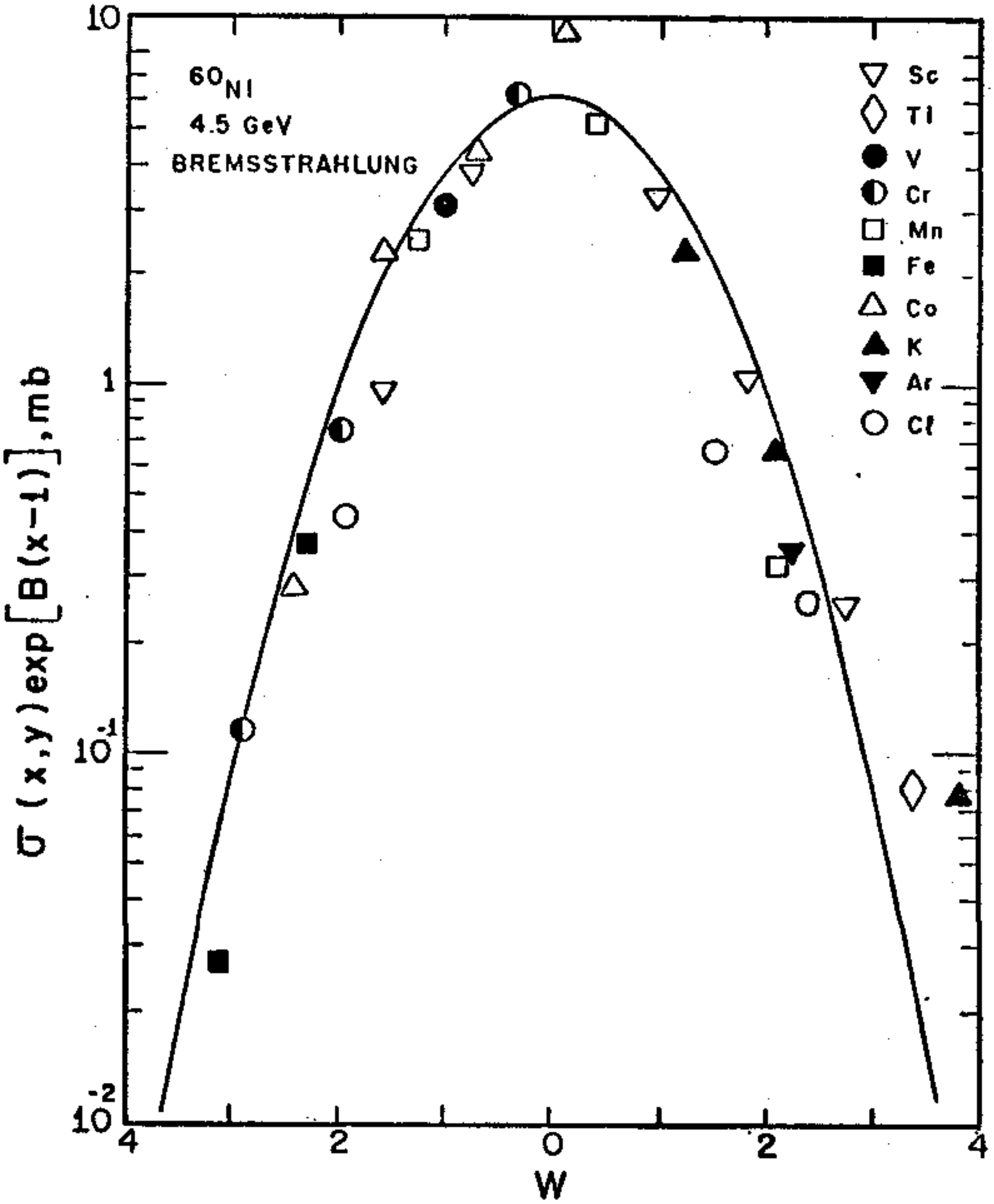

PARAMETER C
α = Zt/Nt
0
1
2
0.6
0.7
0.8
0.9
1.0
51V
55Mn
natFe
59Co
natCu
natGa
75As

Fig. 4



$^{59}Co$
0.3-1.0 GeV
photon
Mg
Al
Cl
Ar
K
Co
Sc
V
Cr
Mn
Fe
$10^3$
$10^2$
10
1
-4
-2
0
2
4
W
$\sigma(x,y)\exp[B(x-1)]$, $\mu b$


Fig. 5



60Ni
4.5 GeV
BREMSSTRAHLUNG
Sc
Ti
V
Cr
Mn
Fe
Co
K
Ar
Cl
σ (x,y)exp[B(x-1)],mb
10
1
10^-1
10^-2
4
2
0
2
4
W


Fig. 6

Table 1. *Values of the parameters obtained for different irradiation conditions of photon- and electron-induced spallation reactions.*

| Target nucleus | $a = \frac{Z_t}{N_t}$ | Irradiation condition | $\varepsilon = \frac{E}{A_t}$ (MeV/$\mathcal{N}$) | Number of measurements $n$ | Ref. | $\sigma_N$(μb) | Parameters $B$ | $K$ | $C$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $^{nat}Cl$ | 0.920 | ∿ 0.2–1.0 GeV photon | 12.60[a] | 7 | 15–18 | 800 | 0.300 | 0.394 | 1.048 | 37.7 |
| $^{51}V$ | 0.821 | 0.25–0.40 GeV photon | 6.20 | 15 | 19 | 1500 | 0.539 | 0.498 | 0.904 | 688.8 |
| | | 0.25–0.58 GeV photon | 7.47 | 6 | 19 | 1450 | 0.429 | 0.532 | 0.881 | 45.8 |
| | | 0.25–0.80 GeV photon | 8.77 | 17 | 19 | 510 | 0.229 | 0.488 | 0.881 | 1051.8 |
| | | 0.20–0.58 GeV photon | 6.68 | 6 | 20 | 1240 | 0.393 | 0.541 | 0.874 | 10.7 |
| | | 0.3 –1.0 GeV photon | 10.74 | 18 | 21 | 580 | 0.245 | 0.463 | 0.875 | 149.8 |
| $^{55}Mn$ | 0.833 | 0.3 –1.0 GeV photon | 9.96 | 20 | 21 | 540 | 0.197 | 0.434 | 0.858 | 55.2 |
| $^{nat}Fe$ | 0.869 | 0.3 –1.0 GeV photon | 9.80 | 22 | 21 | 730 | 0.288 | 0.372 | 1.007 | 73.6 |
| | | 3.0 GeV electron | 53.66 | 9 | 22 | 250 | 0.240 | 0.412 | 0.895 | 218.8 |
| | | 5.0 GeV electron | 89.43 | 8 | 22 | 280 | 0.302 | 0.595 | 0.916 | 332.7 |
| | | 16.0 GeV electron | 286.2 | 10 | 22 | 210 | 0.226 | 0.467 | 0.933 | 17.8 |
| | | 16.0 GeV electron | 286.2 | 13 | 22 | — | 0.269 | 0.476 | 0.920 | 14.8 |
| $^{59}Co$ | 0.844 | 0.3–1.0 GeV photon | 9.28 | 23 | 21 | 660 | 0.225 | 0.443 | 0.926 | 15.5 |
| $^{181}Ta$ | 0.676 | 4.0 GeV electron | 22.10 | 11 | 8 | 250 | 0.240 | 0.412 | 0.492 | 107.2 |

[a] For incident photons we take $E = (E_1 \times E_2)^{1/2}$, where $E_1$ and $E_2$ are the end-point bremsstrahlung energies of the energy-range indicated.

**Table 2.** *Values of the parameters obtained for different irradiation conditions of bremsstrahlung-induced spallation reactions.*

| Target nucleus | $\alpha = \frac{Z_t}{N_t}$ | Bremsstrahlung end-point energy $E_0$ [GeV] | $\varepsilon = \frac{E_0}{A_t}$ (MeV/N) | Number of measurements $n$ | Ref. | $\sigma_M$ (μb) | Parameters $B$ | $K$ | $C$ | $\chi^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $^{51}V$ | 0.821 | 0.075 | 1.5 | 6 | 19 | 1536 | 1.900 | 0.635 | 0.805 | 15.7 |
| | | 0.100 | 2.0 | 7 | 19 | 1742 | 1.700 | 0.747 | 0.887 | 27.8 |
| | | 0.125 | 2.5 | 7 | 19 | 1489 | 1.330 | 0.786 | 0.857 | 93.2 |
| | | 0.150 | 2.9 | 8 | 19 | 1255 | 1.118 | 0.687 | 0.838 | 66.9 |
| | | 0.200 | 3.9 | 11 | 19 | 1048 | 0.820 | 0.587 | 0.836 | 145.5 |
| | | 0.250 | 4.9 | 14 | 19 | 1125 | 0.700 | 0.570 | 0.860 | 102.9 |
| | | 0.300 | 5.9 | 16 | 21 | 689 | 0.398 | 0.664 | 0.896 | 115.4 |
| | | 0.325 | 6.4 | 16 | 19 | 1201 | 0.561 | 0.564 | 0.892 | 572.8 |
| | | 0.400 | 7.8 | 18 | 19 | 1224 | 0.468 | 0.558 | 0.888 | 126.3 |
| | | 0.400 | 7.8 | 16 | 21 | 810 | 0.330 | 0.588 | 0.883 | 104.2 |
| | | 0.400 | 7.8 | 6 | 20 | 2200 | 0.420 | 0.412 | 0.860 | 81.9 |
| | | 0.500 | 9.8 | 18 | 19 | 1278 | 0.431 | 0.531 | 0.885 | 1055.8 |
| | | 0.500 | 9.8 | 17 | 21 | 905 | 0.258 | 0.535 | 0.849 | 185.2 |
| | | 0.600 | 11.8 | 17 | 21 | 1005 | 0.234 | 0.526 | 0.844 | 277.4 |
| | | 0.640 | 12.5 | 18 | 19 | 1370 | 0.369 | 0.531 | 0.883 | 1658.2 |
| | | 0.750 | 12.7 | 17 | 21 | 1057 | 0.215 | 0.519 | 0.844 | 354.8 |
| | | 0.800 | 15.7 | 18 | 19 | 1422 | 0.347 | 0.518 | 0.884 | 2885.8 |
| | | 0.900 | 17.6 | 16 | 21 | 1080 | 0.231 | 0.515 | 0.860 | 227.2 |
| | | 1.000 | 19.6 | 17 | 21 | 1222 | 0.201 | 0.507 | 0.838 | 372.0 |
| | | 1.000 | 19.6 | 12 | 23 | 25000 | 0.300 | 0.400 | 0.300 | 229.2 |
| | | 1.500 | 29.4 | 17 | 19 | 1500 | 0.300 | 0.464 | 0.901 | 114.1 |
| | | 2.000 | 39.2 | 8 | 24 | 2130 | 0.288 | 0.489 | 0.800 | 59.1 |
| | | 3.000 | 58.8 | 9 | 24 | 2358 | 0.328 | 0.533 | 0.806 | 59.1 |
| | | 4.000 | 78.4 | 10 | 24 | 2073 | 0.307 | 0.445 | 0.804 | 5.1 |
| | | 5.000 | 98.0 | 10 | 24 | 1480 | 0.300 | 0.420 | 0.840 | 257.8 |

(continued)

(continued)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $^{55}Mn$ | 0.833 | 0.300 | 5.5 | 20 | 21 | 1208 | 0.423 | 0.408 | 0.898 | 574.3 |
| | | 0.400 | 7.3 | 21 | 21 | 1147 | 0.368 | 0.420 | 0.881 | 527.2 |
| | | 0.500 | 9.1 | 21 | 21 | 1300 | 0.334 | 0.420 | 0.870 | 420.2 |
| | | 0.600 | 10.9 | 21 | 21 | 1071 | 0.268 | 0.412 | 0.859 | 469.2 |
| | | 0.750 | 13.6 | 21 | 21 | 1220 | 0.261 | 0.415 | 0.860 | 540.5 |
| | | 0.900 | 16.4 | 21 | 21 | 1363 | 0.267 | 0.413 | 0.864 | 541.5 |
| | | 1.000 | 18.2 | 21 | 21 | 1391 | 0.260 | 0.414 | 0.863 | 613.5 |
| | | 1.500 | 27.3 | 14 | 23 | 4345 | 0.251 | 0.427 | 0.886 | 6.9 |
| | | 2.000 | 36.4 | 11 | 24 | 4604 | 0.433 | 0.426 | 0.871 | 8.7 |
| | | 3.000 | 54.5 | 12 | 24 | 4762 | 0.467 | 0.428 | 0.876 | 110.7 |
| | | 4.000 | 72.7 | 12 | 24 | 4880 | 0.434 | 0.475 | 0.858 | 91.8 |
| | | 5.000 | 90.9 | 13 | 24 | 5285 | 0.441 | 0.461 | 0.865 | 85.6 |
| $^{nat}Fe$ | 0.869 | 0.300 | 5.4 | 17 | 21 | 857 | 0.414 | 0.351 | 0.979 | 185.4 |
| | | 0.400 | 7.2 | 21 | 21 | 1300 | 0.420 | 0.412 | 0.950 | 396.9 |
| | | 0.500 | 8.9 | 22 | 21 | 1300 | 0.420 | 0.412 | 0.950 | 370.1 |
| | | 0.600 | 10.7 | 22 | 21 | 1300 | 0.306 | 0.412 | 0.961 | 322.4 |
| | | 0.750 | 13.4 | 22 | 21 | 1500 | 0.300 | 0.412 | 0.980 | 423.3 |
| | | 0.900 | 16.1 | 22 | 21 | 1750 | 0.300 | 0.412 | 0.980 | 500.7 |
| | | 1.000 | 17.9 | 22 | 21 | 1700 | 0.300 | 0.412 | 0.970 | 525.7 |
| | | 1.500 | 26.8 | 20 | 25 | 3000 | 0.300 | 0.412 | 0.950 | 133.0 |
| $^{59}Co$ | 0.844 | 0.300 | 5.1 | 15 | 21 | 856 | 0.319 | 0.396 | 0.886 | 486.7 |
| | | 0.309 | 5.2 | 8 | 26 | 436 | 0.475 | 0.769 | 0.629 | 88.6 |
| | | 0.400 | 6.8 | 22 | 21 | 1025 | 0.270 | 0.448 | 0.879 | 656.0 |
| | | 0.500 | 8.5 | 23 | 21 | 1142 | 0.262 | 0.435 | 0.876 | 480.2 |
| | | 0.600 | 10.2 | 23 | 21 | 1413 | 0.279 | 0.421 | 0.892 | 467.7 |
| | | 0.750 | 12.7 | 23 | 21 | 1522 | 0.270 | 0.420 | 0.902 | 375.5 |
| | | 0.900 | 15.3 | 23 | 21 | 1878 | 0.271 | 0.430 | 0.896 | 438.6 |
| | | 1.000 | 16.9 | 23 | 21 | 1871 | 0.300 | 0.412 | 0.895 | 455.1 |
| $^{nat}Cu$ | 0.838 | 0.320 | 5.0 | 5 | 27 | 19.0[a] | 0.690 | 0.371 | 0.936 | 5.8 |
| | | 2.000 | 31.4 | 22 | 7 | 2903 | 0.370 | 0.454 | 0.948 | 0.25 |
| | | 2.000 | 31.4 | 19 | 24 | 3046 | 0.267 | 0.326 | 0.924 | 171.3 |
| | | 3.000 | 47.2 | 17 | 24 | 3666 | 0.267 | 0.432 | 0.924 | 105.1 |
| | | 4.000 | 69.2 | 16 | 24 | 4026 | 0.282 | 0.438 | 0.920 | 65.9 |
| | | 5.000 | 78.6 | 18 | 24 | 4357 | 0.300 | 0.436 | 0.921 | 88.9 |
| $^{nat}Ga$ | 0.799 | 0.320 | 4.6 | 8 | 27 | 15.7[a] | 0.620 | 0.497 | 0.723 | 40.6 |
| $^{75}As$ | 0.786 | 0.320 | 4.3 | 10 | 27 | 11.0[a] | 0.599 | 0.468 | 0.763 | 66.5 |

[a]arbitrary units.

**Table 3.** *The quantity Chi-squared per-degree-of-freedon ($\chi_\nu^2$) for different systematics of electron- and photon-induced spallation reactions.*

| Target nucleus | Irradiation condition | $\chi_\nu^2$-values<br>JONSSON and LINDGREN [1], 5-parameter, equation (13) | This work, 4-parameter, equation (1)† | This work, 4-parameter, equation (1)‡ |
|---|---|---|---|---|
| $^{nat}Cl$ | ∿0.2–1.0 GeV photon | 757.4 | 87.9 | 12.6 |
| $^{51}V$ | 0.25–0.40 GeV photon | 287.6 | 48.9 | 62.6 |
| | 0.25–0.58 GeV photon | 1698.6 | 59.3 | 22.9 |
| | 0.25–0.80 GeV photon | 794.6 | 99.0 | 80.9 |
| | 0.20–0.58 GeV photon | 509.1 | 13.2 | 5.3 |
| | 0.3 –1.0 GeV photon | 125.2 | 13.7 | 10.7 |
| $^{55}Mn$ | 0.3 –1.0 GeV photon | 146.4 | 4.3 | 3.5 |
| $^{nat}Fe$ | 0.3 –1.0 GeV photon | 158.3 | 20.3 | 4.1 |
| | 3.0 GeV electron | 811.0 | 62.0 | 43.8 |
| | 5.0 GeV electron | 1481.1 | 99.0 | 83.2 |
| | 16.0 GeV electron | 661.6 | 4.0 | 3.0 |
| | 16.0 GeV electron | – | – | 1.6 |
| $^{59}Co$ | 0.3 –1.0 GeV photon | 126.4 | 10.4 | 0.82 |
| $^{181}Ta$ | 4.0 GeV electron | 896.8 | 19.2 | 12.6 |

†Values of parameters $\sigma_M$, $B$, and $C$ are those calculated, respectively, by equations (7), (8), and (10), and $K = 0.466$.

‡Values of parameters $\sigma_M$, $B$, $K$, and $C$ are those listed in Table 1.

Table 4. *The quantity Chi-squared per-degree-of freedom ($\chi_\nu^2$) for different systematics of bremsstrahlung-induced spallation yields.*

| Target nucleus | Incident maximum bremsstrahlung energy $E_0$ (GeV) | $\chi_\nu^2$-values JONSSON and LINDGREN [28], 5-parameter, equation (13) | This work, 4-parameter, equation (1) |
|---|---|---|---|
| $^{51}V$ | 0.100 | 884.6 | 9.3 |
| | 0.125 | 2092.1 | 31.1 |
| | 0.150 | 131.4 | 17.2 |
| | 0.200 | 74.6 | 20.8 |
| | 0.250 | 36.5 | 10.3 |
| | 0.325 | 105.6 | 47.7 |
| | 0.400 | 7.4 | 9.0 |
| | 0.500 | 505.8 | 75.4 |
| | 0.640 | 172.0 | 118.4 |
| | 0.800 | 343.2 | 206.1 |
| | 1.000 | 252.1 | 28.6 |
| | 2.000 | 7.6 | 14.8 |
| | 3.000 | 10.1 | 11.8 |
| | 4.000 | 0.45 | 0.86 |
| | 5.000 | 45.7 | 43.0 |
| $^{55}Mn$ | 1.000 | 30.8 | 36.1 |
| | 2.000 | 0.83 | 1.2 |
| | 3.000 | 14.2 | 13.8 |
| | 4.000 | 11.4 | 11.5 |
| | 5.000 | 29.5 | 9.5 |
| $^{nat}Fe$ | 1.000 | 43.5 | 29.2 |
| $^{59}Co$ | 1.000 | 21.7 | 24.0 |
| $^{nat}Cu$ | 2.000 | 15.6 | 11.4 |
| | 3.000 | 7.1 | 8.1 |
| | 4.000 | 6.0 | 5.5 |
| | 5.000 | 6.7 | 6.4 |

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