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MAGNETIZATION REVERSAL BY NUCLEATION

by

W. BALTENSPERGER[1], J.S. BROZ[1] and J.S. HELMAN

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

[1]Theoretische Physik, Eidgenössische Technische Hochschule (ETH), Hönggerberg, 8093 Zürich, Switzerland

ABSTRACT

The exact static solutions of the Landau-Lifshitz equations for the magnetization, which have been identified as describing nucleation centers for the magnetization reversal in ferromagnets, are compared with those of phenomenological models of spherical and cylindrical symmetry. It is found that the latter have higher critical fields.

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TEXT

The dynamics of ferromagnets can be described phenomenologically by the Landau-Lifshitz equation for the magnetization[1]. This equation depends on a functional expression for the total energy of the system. A realistic description of a ferromagnet involves at least the Heisenberg exchange term, an axial anisotropy term, the Zeeman energy of the magnetization with the outside field along the easy axis, and the demagnetization term. The latter represents the energy of the magnetic dipole-dipole interaction. Since we shall be concerned with a static solution of the Landau-Lifshitz equation, damping is not relevant. The static equation, also called Brown's equation in micromagnetism, is a non linear partial differential equation[2] for the magnetization which has a fixed modulus M.

In the situation in which the magnetization is almost everywhere opposed to the externally applied field, the following exact solution exists[3]

$$\tan\left(\frac{\theta}{2}\right) = \beta \cosh\left(\frac{x}{\Delta\sqrt{Q}\,\delta}\right), \qquad \phi = \pm\frac{\pi}{2} \tag{1}$$

with

$$\delta^2 = \frac{4\pi M}{4\pi MQ - H}, \qquad \beta^2 = \frac{H}{4\pi MQ - H}. \tag{2}$$

Here θ and ϕ are the spherical coordinates for the direction of the magnetization. $\Delta = \sqrt{A/K}$, where A and K are the exchange and anisotropy constants, respectively. $Q = K/(2\pi M^2)$ is the qua;ity factor. $\phi = \pm\pi/2$ corresponds to the Bloch configuration where the spins turn within the wall plane. The solution only exists for $H < 4\pi MQ$. When H is close to the critical value $4\pi MQ$, the large parameters β and δ correspond to a small and localized deformation. In the other extreme, when $H \ll 4\pi MQ$, the structure looks like two separated, mirror symmetric Bloch walls; the Zeeman energy of the positive magnetization between the walls balances their tendency to annihilate. The solution is unstable and corresponds to a saddle point of the energy in configuration space. A reversal of the magnetization has to overcome this energy barrier. Actually, the energy of this structure of planar symmetry is proportional to the surface, which in this model is unlimited, making the barrier infinite for $H < 4\pi MQ$. For $H > 4\pi MQ$ the energy of these configurations, which are then not solutions, is negative, so that for overcritical fields there is no energy obstacle for the magnetization reversal.

The best known nucleation problem is the condensation of water in supersaturated vapor. In this case a water droplet with a critical radius corresponds to an unstable equilibrium[4]: a smaller droplet shrinks, while a larger one grows. It is interesting to look for a magnetic nucleation center which, in analogy to the water droplet, is finite in three dimensions, so that its energy is finite for any magnetic field. For simplicity we shall consider sufficiently large nucleation centers, to which a phenomenological theory applies. Also we shall assume it to be of spherical shape although the true optimum shape may be a rotational ellipsoid. The energy of a spherical region with reversed magnetization is

$$E = V(4\pi/3)M^2 - VHM + S\sigma + W. \tag{3}$$

The first term is the magnetization energy of the sphere, the second the Zeeman energy, the third the energy of the Bloch wall surrounding the sphere, where S is the surface and σ the energy per unit surface. Note that the anisotropy energy is relevant only inside the wall and is contained in σ. As a minor point note that at the equator ($\theta = \pi/2$) a Bloch wall is possible, since the magnetization is already parallel to the wall, whereas at the poles ($\theta = \pi$ or 0) a Néel wall is required which has a higher energy. Thus σ is actually a function of θ. In the middle of the Bloch wall the magnetization points into a direction tangential to the surface. Hence, for topological reasons, the Bloch wall magnetization must contain at least two vortices, which add an energy term W. For a sphere $V = (4\pi/3)r^3$ and $S = 4\pi r^2$. Then an equilibrium radius is obtained from $dE(r)/dr = 0$ as $r_N = 2\sigma/[M(H - 4\pi M/3)]$, which shows that a radius r_N exists for $H > 4\pi M/3$. From $d^2E(r)/dr^2 = -8\pi M(H - 4\pi M/3) < 0$ follows that the equilibrium is unstable. That the energy has in fact a saddle point at this configuration is shown by noting for instance that deviations of the nucleus from the rotational symmetry increase the energy. At the nucleation radius r_N the energy becomes

$$E_N = \frac{64\pi}{3}\frac{\sigma^3}{\left[(H - 4\pi M/3)M\right]^2} + W. \tag{4}$$

The surface energy of a Bloch wall is[5] $\sigma = 2\pi S\sqrt{JK/a}$, where S is the spin, J the exchange integral, a the lattice constant and K the anisotropy energy density. For a Néel wall K should be replaced by $K + 2\pi M^2$. We subestimate the energy of the nucleus using the Bloch value throughout.

To assess the energy of the vortices, let us, for a small system, consider the energy of the spins at the core of the vortex. At the very least 4 neighboring spin pairs will be orthogonal instead of parallel in each layer. Assuming that the vortices are at the poles, this situation prevails over about half the thickness Δ of the wall, while outside the spins are normal to the wall and parallel to each other. Then for two vortices $W > 4JS^2\Delta/a$. Typically Δ/a will be in the range 10 to 100. For a spin system with anisotropy, S will at least have the value 1.

Using these values the energy and the radius of the nucleation center become

$$E_N = (4\pi)^4\,\frac{8}{3}S^3\,\frac{(JK/a)^{3/2}}{\left[(H - 4\pi M/3)M\right]^2} + W \tag{5}$$

$$r_N = 4\pi S\,\frac{(JK/a)^{1/2}}{(H - 4\pi M/3)M}. \tag{6}$$

Let us now consider a field $H = 4\pi M(Q + 1/3) = 2K/M + 4\pi M/3$, which is larger than the critical field $4\pi MQ$ for a planar nucleation center by the amount $4\pi M/3$. Then $r_N = 2\pi a(J/Ka^3)^{1/2}$. If $J \gg Ka$, which is typical, then $r_N/a >> 4\pi$, so that these phenomenological formulas are justified. The energy becomes $E_N = (4\pi)^4(2/3)J(J/Ka^3)^{1/2}+W > 3000k_BT_C$ for a lattice with 6 neighbors, where T_C is the Curie point[6]. The probability of occurrence of such a nucleus as a thermal fluctuation contains a factor $\exp(-E_N/k_BT)$ which is a negligible number, since $T < T_C$. This proves that at the critical field for which the energy of the planar nucleation center vanishes, this three dimensional nucleation mode is precluded.

Let us now consider a nucleation center with cylindrical symmetry. It consists of a cylindrical region of radius ρ and length $L \to \infty$ in which the magnetization is inverted, surrounded by a Bloch wall. The energy $E(\rho) = L(-\pi\rho^2 HM + 2\pi\rho\sigma)$ has a maximum value $L\pi\sigma^2/HM$ at $\rho_N = \sigma/HM = r_N/2$. Again the phenomenological theory is justified and at the critical field of the planar center $E(\rho_N) = 2\pi^3 JL/a$, which for $L/a \gg 1$ will not occur as a fluctuation. Other cylindrical modes have been examined[7–14]. For the curling mode the critical field is[14] $H_c = K/2\pi M + 3.392A/M\rho^2$. In an infinite medium this coincides with the critical field for planar nucleation. Of course, the mode of unison rotation has also this same critical field.

We conclude that the phenomenological spherical and cylindrical models with a fully inverted magnetization are not relevant at the critical field of the planar nucleation. However, the unison mode and the curling mode of the infinite cylinder compete. The critical field of a small localized nucleation center[15] is suspected to be larger, but not known at present.

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