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A HIGHER-ORDER ANALYSIS OF THE EFFECTS OF BREAKING SUPERSYMMETRY FOR NON-LINEAR σ-MODELS

by

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ABSTRACT

Arbitrary (1,1) and (1.0) two-dimensional non-linear σ-models, modified by the addition of coupling terms which explicitly break supersymmetry, are studied. The geometrical meaning of these additional terms is discussed. Supergraph methods, suitably extended to include the case of broken supersymmetry, are set and employed in explicit higher-loop computations to keep track of the effect that the explicit breaking of supersymmetry has on the ultraviolet behaviour of the originally supersymmetric models.

Key-words: Supersymmetry; Non-linear σ-models; World sheet supersymmetry.

1.Introduction

Two-dimensional supersymmetric non-linear σ-models provide a very attractive connection between extended supersymmetries and complex manifold theory[1 – 5]. While simple supersymmetry sets no constraints on the nature of the target manifold, the existence of additional supersymmetries is guaranteed by taking a Kähler manifold, while $N = 4$ is realized when the manifold is hyperkähler.

The quantum corrections modify the structure of the manifold, but the existence of further supersymmetries strongly constrains the form of the counterterms [6 – 9]. In particular, when the manifold is hyperkähler, the theory is so constrained that it is on-shell ultraviolet finite to all orders in perturbation theory[10 – 13]. This outstanding link between extended supersymmetries, complex manifold geometry and ultraviolet finiteness would be sufficient to justify the considerable amount of attention one pays to the study of the supersymmetric non-linear σ-models.

More recently, however, with the resurgence of string theories, the interest in these two-dimensional models has been much more stressed. The reason being that σ-models defined on a Riemann surface and taking values in an arbitrary d-dimensional Riemannian space keep a close relationship with string theories[14 – 25]. Actually, the dynamics of a superstring propagating in a fixed background space-time can be described by a two-dimensional supersymmetric non-linear σ-model. If the superstring is of the Green- Schwarz type, the models of relevance are those with $N = 2$. Instead, in connection with the heterotic string, the models which arise are the so-called $(2,0)$ supersymmetric non-linear σ-models. The latter belong to a more general category: the (p,q) models, whose construction is based on p right-handed and q left-handed independent Majorana generators[18].

The conformal invariance required for the consistency of the string theory has its counterpart in the finiteness of the associated non-linear σ- model. In the case of a purely gravitational background, Ricci-flatness of the target manifold is the constraint stemming from the requirement of finiteness at the first non-trivial order in perturbation theory[9]. If the background is also characterized by the presence of a rank-two anti-symmetric tensor, one ends up with a torsion for the target space and a supersymmetric generalized Wess-Zumino term accounts for this fact[26-33].

In the present work, we endeavour to investigate how strong the world- sheet supersymmetry is for the requirement of conformal invariance or, in other words, for the finiteness of the associated non-linear σ-model. To do that, we shall add several terms which respect all the symmetries but supersymmetry of the σ-model classical action; they are terms which explicitly break simple supersymmetry. Using suitably modified superspace Feynman rules which account for the explict breakings of supersymmetry[34], we shall pursue our investigation of the ultraviolet divergence structure of the σ-models modified by the inclusion of these breaking terms.

By performing explicit supergraph calculations, we shall see that finiteness can be achieved at higher-loop orders provided that the additional breaking terms satisfy certain conditions. Nevertheless, we shall notice that the cases of $(1,1)$ and $(1,0)$ supersymmetry present remarkable differences in what concerns the new supersymmetric divergent terms induced by the breaking of supersymmetry. The β-function of the former receives contributions proportional to the breaking parameters from higher loops whereas, for the latter, new contributions appear already at one loop. The basic reason for such a difference is that in the $(1,0)$ case Lorentz invariance imposes the presence of a higher number of field derivatives appearing in the supersymmetry breaking vertices.

The outline of our work is as follows. In Section 2, we present and discuss several terms which may have interesting consequences in breaking $(1,1)$ supersymmetry. The modified super-Feynman rules are derived in detail and applied to derive three- and four-loop contributions to the metric tensor β-function. In Section 3, we focus our attention on the possible ways of breaking $(1,0)$ supersymmetry. Suitably modified supergraph calculations are displayed and a one-loop finite σ-model with the torsion generated by an explicit breaking term is presented. The two-loop analysis of such a model is briefly commented. Finally, in Section 4,we present some concluding remarks. Two appendices follow. In Appendix A, we list the relevant operators for performing calculations in broken $(1,1)$ supersymmetry and give their multiplicative table. The analogous for the $(1,0)$ case can be found in Appendix B.

2. Softly broken $N = 1$ $D = 2$ supersymmetric non-linear σ-models

In this section, explicit breaking terms of $N = 1$ $D = 2$ supersymmetry are added to the $N = 1$ non-linear σ-model action. With the help of supergraph methods, we shall analyse the structure of ultraviolet divergences induced by the breaking terms. Especially, we wish to investigate how the breaking parameters modify the β-function of the exact σ-model and then understand whether the Ricci-flatness condition can be relaxed without loosing the conformal invariance.

Before going over into the σ-model calculations, let us start by discussing $N = 1$ supersymmetry in the presence of mass breaking terms which shall play some rôle when dealing with the σ-models.

The following (anti-) commutation relations for $N = 1$ supersymmetry in two dimensions will be of use in the course of our algebraic manipulations:

$$\{D_\alpha, \theta_\beta\} = C_{\alpha\beta}, \tag{2.1}$$

$$[D^2, \theta_\alpha] = D_\alpha, \tag{2.2}$$

$$[D_\alpha, \theta^2] = \theta_\alpha, \tag{2.3}$$

$$[D^2, \theta^2] = -1 + \theta^\alpha D_\alpha, \tag{2.4}$$

where

$$D_\alpha \equiv \partial_\alpha + i\theta^\alpha \partial_\alpha, \tag{2.5}$$

θ is a Majorana spinor and C is the charge conjugation matrix. We shall adopt here the notation and conventions of ref.[35].

Scalar superfields, $\Phi(x, \theta)$, are defined by the following projections:

$$A(x) = \Phi(x, \theta)|_{\theta=0}, \tag{2.6}$$

$$\psi(x) = D_\alpha \Phi(x, \theta)|_{\theta=0} \tag{2.7}$$

and

$$F(x) = D^2 \Phi(x, \theta)|_{\theta=0}. \tag{2.8}$$

$A(x)$ and $F(x)$ are respectively physical and auxiliary scalars and $\psi(x)$ is the physical fermionic component of $\Phi(x,\theta)$.

Considering now a set of scalar superfields, $\Phi^i(x,\theta)$ $(i = 1,..,M)$, we add to the quadratic supersymmetric action,

$$\mathcal{L}_0 = -\frac{1}{4}\int d^2\theta(D^\alpha\Phi^i)(D_\alpha\Phi^i) + \frac{1}{2}\int d^2\theta M_{ij}\Phi^i\Phi^j, \tag{2.9}$$

terms which explicitly break $N = 1$ supersymmetry and whose net effect is to shift the masses of the physical scalar and fermion fields accommodated in $\Phi^i(x,\theta)$. They are collected into the breaking Lagrangian, $\mathcal{L}_B$, given by:

$$\mathcal{L}_B = \frac{1}{2}\int d^2\theta\theta^2[m^2_{ij}\Phi^i\Phi^j + \mu_{ij}(D^\alpha\Phi^i)(D_\alpha\Phi^j)], \tag{2.10}$$

where m^2 and μ are real and symmetric $M \times M$ mass matrices.

Putting the Lagrangians (2.9) and (2.10) together, we shall derive the expression for the superpropagator $< \Phi^i\Phi^j >$ in such a way to take the breaking parameters, m^2_{ij} and μ_{ij}, into account to all orders. The most genereal expression for the superpropagator has in principle the following form:

$$P(x_1,\theta_1;x_2,\theta_2) = -(1 + \sum_{n=1}^{12} X_n A_n(x_1,\theta_1))(D_1^2 + M)^{-1}\delta^2(x_1 - x_2)\delta^2(\theta_1 - \theta_2), \tag{2.11}$$

where the coefficients X_n are c-number valued $M \times M$ matrices to be determined. The explicit expressions for the operators A_n and their multiplicative table can be found in Appendix A.

Taking into account that the truly independent operators are just A_1, A_2, A_4, A_8 and A_{12}, the general expression for the superpropagator of eq.(2.11) reduces to

$$P(x_1,\theta_1;x_2,\theta_2) = -(1+XA_2+YA_4+ZA_8+WA_{12})(D_1^2+M)^{-1}\delta^2(x_1-x_2)\delta^2(\theta_1-\theta_2), \quad 2.12)$$

with the matrices X, Y, Z and W being determined from

$$(1 + A)X + \Box CZ = -A, \tag{2.13a}$$

$$CX + (1+A)Z = -C, \tag{2.13b}$$

$$2(A-B)X - (1+2A-B)Y + 2\Box CZ = B \tag{2.13c}$$

and

$$2i\Box CX - 2i\Box(A-B)Z + (1+2A-B)W = D. \tag{2.13d}$$

A, B, C and D are matrices expressed in terms of the mass matrices M, m^2 and μ:

$$A = \frac{1}{\Box - M^2 - m^2}(M\mu + m^2), \tag{2.14a}$$

$$B = \frac{1}{\Box - M^2 - m^2}(2M\mu + m^2), \tag{2.14b}$$

$$C = i\frac{1}{\Box - M^2 - m^2}\mu \tag{2.14c}$$

and

$$D = \frac{1}{\Box - M^2 - m^2}(Mm^2 + 2\mu\Box). \tag{2.14d}$$

After manipulating the algebra of the D's and θ's, we find that our superpropagators take the following final form:

$$P(k;\theta_1,\theta_2) \equiv < T(\Phi(1)\Phi(2)) > = \alpha(k^2)(D_1^2 - M)\delta^2(\theta_{12}) - \beta(k^2)\theta_1^\alpha D_{1\alpha}\delta^2(\theta_{12}) +$$

$$-\gamma(k^2)\theta_1^2 D_1^2\delta^2(\theta_{12}) + i\eta(k^2)k^{\alpha\beta}\theta_{1\alpha}D_{1\beta}\delta^2(\theta_{12}) - \epsilon(k^2)\theta_1^2\delta^2(\theta_{12}), \tag{2.15}$$

where

$$\alpha(k^2) \equiv \frac{1}{k^2 + M^2 + m^2}, \tag{2.16a}$$

$$\beta(k^2) \equiv X\alpha(k^2)M + iZk^2\alpha(k^2), \tag{2.16b}$$

$$\gamma(k^2) \equiv Y\alpha(k^2)M - W\alpha(k^2), \tag{2.16c}$$

$$\eta(k^2) \equiv iX\alpha(k^2) + Z\alpha(k^2)M \tag{2.16d}$$

and

$$\epsilon(k^2) \equiv Yk^2\alpha(k^2) + W\alpha(k^2)M. \tag{2.16e}$$

It can be easily checked that, by setting the breaking parameters m_{ij}^2 and μ_{ij} to zero, we recover the superpropagators of the usual supersymmetric treatment.

We now turn to σ-model considerations. We first add to the action of the $N = 1$ supersymmetric non-linear σ-model,

$$S = -\frac{1}{4}\int d^2x d^2\theta g_{ij}(\Phi)(D^\alpha\Phi^i)(D_\alpha\Phi^j), \tag{2.17}$$

new coupling terms which explicitly break supersymmetry but, contrary to the mass terms appearing in (2.10), respect the diffeomorphism invariance of the target manifold. The terms we propose to study here are:

$$\frac{1}{2}\int d^2x d^2\theta\theta^2\mu g_{ij}(\Phi)(D^\alpha\Phi^i)(D_\alpha\Phi^j) \tag{2.18}$$

and

$$\frac{1}{4}\int d^2x d^2\theta\theta^2\lambda R_{ijkl}(\Phi)(D^\alpha\Phi^i)(D_\alpha\Phi^l)D^\beta\Phi^j)(D_\beta\Phi^k), \tag{2.19}$$

where μ and λ are respectively mass dimensional and dimensionless parameters and R_{ijkl} is the Riemann tensor of the target space.

The breaking term (2.18), besides modifying the coupling between the physical scalars and fermions of the σ-model, may also shift the masses of the fields ψ^i. Indeed, when the target manifold is a homogeneous space like the n-sphere, for example, μ is nothing but the mass of the fermionic component fields. As for the breaking term (2.19), masses are not shifted; it only affects the scalar-spinor couplings of the originally supersymmetric σ-model and, for a suitable choice of λ, the quartic spinor coupling by means of the Riemann tensor can be completely suppressed.

Following now the usual procedure of the normal coordinate expansion, one can show that, with the help of (2.15) and (2.16), the superpropagator for the quantum field ξ^a reads as below:

$$< T(\xi^a(1)\xi^b(2)) >= \delta^{ab}\frac{1}{k^2}D_1^2\delta^2(\theta_{12}) + \delta^{ab}\frac{\mu}{k^2+\mu^2}(\theta_1^\alpha D_{1\alpha}+$$

$$+2\theta_1^2 D_1^2 + \frac{\mu}{k^2}k_{\alpha\beta}\theta_1^\alpha D_1^\beta + 2\mu\theta_1^2)\delta^2(\theta_{12}), \tag{2.20}$$

where the index a labels the local frame coordinates of the target space.

With our method for deriving the superpropagators including the breaking terms (2.10), the parameter μ appearing in (2.18) can be summed up to all orders in the superpropagator, which would correspond to a sum of an infinite number of insertions into the superpropagator of the exact supersymmetry case. However, through the quantum-background vertices arising from (2.18) upon the normal coordinate expansion, the parameter μ has still to be taken into account when calculating graphs. As for the dimensionless coupling parameter λ, its effect cannot be introduced into the superpropagator $< \xi^a \xi^b >$. It plays the rôle of a usual coupling constant associated to the quantum-background vertices stemming from the normal coordinate expansion of (2.19).

We are now ready to start presenting and discussing the results of the loop corrections involving the supersymmetry breaking parameters.

Besides the well-known metric tensor renormalization of the exact supersymmetry case[6], the tadpole supergraph of fig.1 induces a renormalization of the supersymmetry breaking term (2.19) by means of the contribution:

$$\frac{1}{32\pi\epsilon}\int d^2\theta\theta^2\lambda(\frac{1}{2}D_m D^m R_{ijkl}+\frac{4}{3}R_{mi}R^m{}_{jkl})(D^\alpha\Phi^i)(D_\alpha\Phi^l)(D^\beta\Phi^j)(D_\beta\Phi^k). \tag{2.21}$$

Notice that such a renormalization is required in the cases of both locally symmetric and Ricci-flat target spaces. The mass breaking parameter μ does not require an independent renormalization: the metric tensor counterterm automatically removes such an infinity. Actually, based on power-counting and reparametrization invariance arguments, one can readily conclude that only the breaking parameter λ, and not μ,can trigger divergent supersymmetric (i.e. non- explicitly θ-dependent) higher-order corrections into the effective action. However, at the one-loop approximation, the supersymmetric contributions to the effective action induced by the supersymmetry breaking terms are all finite.

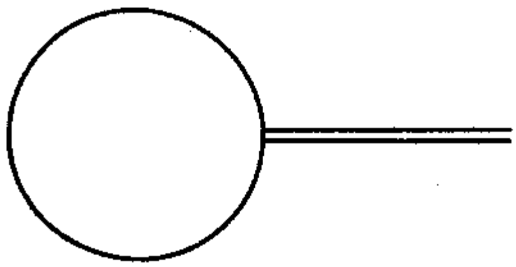

FIG. 1

Tadpole supergraph responsible for the one-loop infinities.

Considering, for example, the type of diagrams drawn in fig.2, one can show that the following finite supersymmetric contributions to the effective action are generated:

$$-\frac{3}{64}\mu \int \frac{d^2k}{(2\pi^2)} \frac{1}{k^2} \frac{1}{k^2+\mu^2} \int d^2\theta R_{imnj} R_k{}^{mn}{}_l (D^\alpha \Phi^i)(D_\alpha \Phi^j)(D^\beta \Phi^k)(D_\beta \Phi^l) \tag{2.22}$$

and

$$\frac{1}{128}\lambda \int \frac{d^2k}{(2\pi^2)} \frac{1}{(k^2+m_{reg}^2)^2} \int d^2\theta R_i{}^{pq}{}_j (D_p D_q R_{klmn} + \frac{1}{3} R^h{}_{pqk} R_{hlmn} +$$

$$\frac{1}{3} R^h{}_{pql} R_{khmn} + \frac{1}{3} R^h{}_{pqm} R_{klhn} + \frac{1}{3} R^h{}_{pqn} R_{klmh})$$

$$(D^\alpha \Phi^i)(D_\alpha \Phi^j)(D^\beta \Phi^k)(D_\beta \Phi^n)(D^\gamma \Phi^l)(D_\gamma \Phi^m), \tag{2.23}$$

where m_{reg} denotes an infra-red cut-off mass.

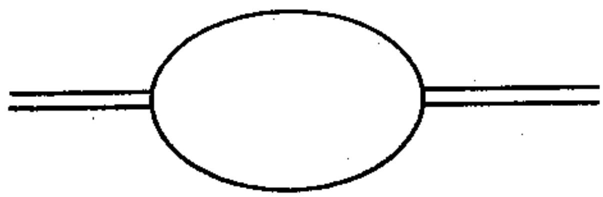

FIG. 2

Finite one-loop diagram.

At the two-loop approximation, no genuine divergence (i.e., an infinity of the type $\frac{1}{\epsilon}$) appears in connection with the metric tensor renormalization. Graphs exhibiting the topology drawn in fig.3 do contribute divergent two-loop corrections of order λ to the metric tensor renormalization. However, such divergences are of the type $\frac{1}{\epsilon^2}$, and so they do not introduce any correction to the two-loop β-function of the exact model, which is known to be vanishing.

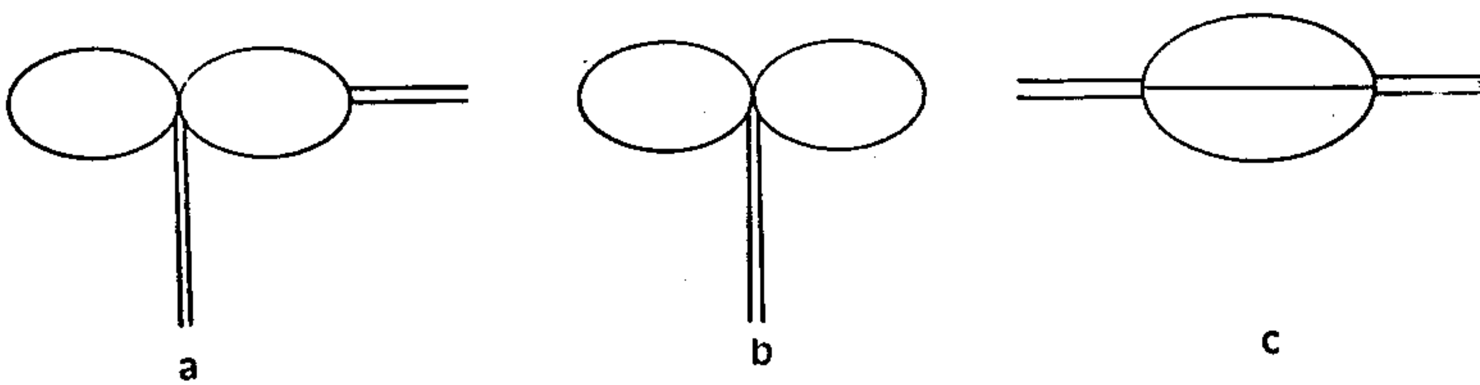

FIG. 3

Divergent two-loop graphs.

Going over to the three-loop approximation, the picture changes, as it could already be expected from power-counting considerations. Indeed, by studying the supergraphs whose topology is as shown in fig.4, and taking at the vertex 1 the quantum-background coupling following from the normal coordinate expansion of the breaking term (2.19), one can show that a genuine $\frac{1}{\epsilon}$ three-loop supersymmetric correction is induced which renormalizes g_{ij} and is non-vanishing in the Ricci-flat case. This divergent contribution is of the form

$$\lambda \int d^2\theta R_{ilmn} R_j{}^{kmh} R^n{}_{kh}{}^l (D^\alpha \Phi^i)(D_\alpha \Phi^j), \tag{2.24}$$

and, as it can be checked, it is not cancelled against any other three-loop contribution. This result clearly shows that the breaking term (2.19) yields a non-zero three-loop contribution to the metric tensor β-function of the non-linear σ-model which persists even when the target manifold is chosen to be Ricci-flat. This appears as the lowest non-trivial contribution to β_{ij} induced by the breaking interaction term of eq.(2.19). In the particular case of a maximally symmetric three-dimensional target manifold, like the three-sphere, for example, such a correction turns out to be identically vanishing.

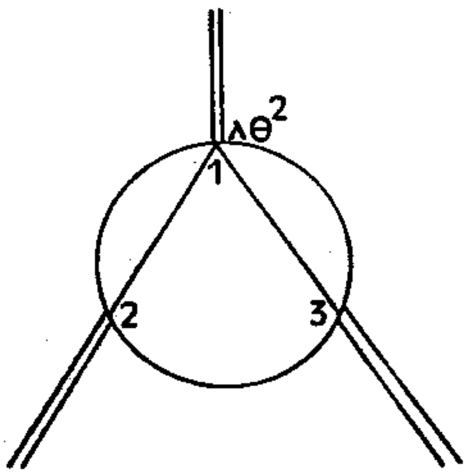

FIG.4

Three-loop supergraph inducing an order-λ correction to the β-function.

Finally, considering four-loop graphs, it can be shown that $\frac{1}{\epsilon}$-like supersymmetric corrections induced by the breaking term (2.19) are generated and contribute to the metric tensor β- function. This can be explicitly seen if we consider the four-loop supergraph depicted in fig.5 and take at the vertex 1 the quantum-background vertex associated to the coupling (2.19). From the combination of such a vertex with the supersymmetric part of the propagator (2.20), one can show that the following tensorial form is associated to this genuine four-loop divergent correction:

$$\lambda \int d^2\theta (D_i R_{klmn})(D_j R^k{}_{pq}{}^l) R^{mpqn} (D^\alpha \Phi^i)(D_\alpha \Phi^j) \qquad (2.25)$$

Such an infinity also survives the Ricci-flatness condition on the target space.

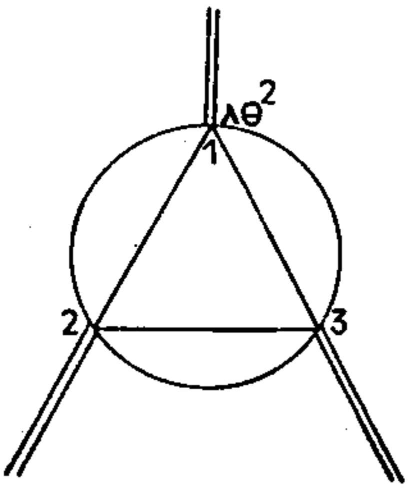

FIG. 5

Four-loop supergraph inducing an order-λ contribution to the β-function.

Therefore, as a final conclusion of this study, we can state that the curvature-breaking term we add is soft in that it does not induce any new divergent contribution to the effective action. The renormalization of the model is not spoiled in its generalized sense and, for Ricci-flat spaces, the metric tensor β-function receives non-trivial additional terms from three (and not four, as in the exact case) loops on.

3. Broken (1,0) Supergraphity and A One-Loop Finite Contorted σ-Model

The so-called two-dimensional (p,q) supersymmetries[18] have recently raised a great interest due to the rôle they play in the construction of the heterotic string theory[36-40]. Their relevance is well-justified by the fact that the four-dimensional effective theory, following from the string compactification, exhibits one unbroken supersymmetry whenever the associated non-linear σ-model has at least a $(2,0)$ world-sheet supersymmetry[41, 42].

The simplest case for which p≠q is the $(1,0)$ supersymmetry. The $(2,0)$ non-linear σ-model can be completely described in terms of the former with the second supersymmetry being non-linearly realized[18]. The $(1,0)$ superspace has been fairly well exploited and the corresponding supergraph techniques have been set in detail and have proven to be very efective for calculations involving string world-sheet supersymmetry[36-40]. We shall in this section extend the $(1,0)$ super-Feynman rules in order to treat the case of broken supersymmetry while working in superspace. They shall be next applied to a general $(1,0)$ non-linear σ-model to which one adds a torsion-like term which breaks the $(1,0)$ supersymmetry, and whose finiteness shall be investigated through two loops.

The $(1,0)$ superspace is parametrized by the coordinates $Z^A = (x^+, x^-; \theta_-)$, where $x^\pm$ are the usual light-cone coordinates and θ_- is a left-handed (real) Majorana spinor. The corresponding supersymmetry covariant derivative is defined by

$$D_+ \equiv i\frac{\partial}{\partial\theta_-} + \theta_-\partial_+ \quad , \quad D_+^2 = i\partial_+, \tag{3.1}$$

where $\partial_\pm = \frac{\partial}{\partial x^\pm}$.

The relevant "matter" superfields appearing in the formulation of the $(1,0)$ non-linear σ-models are the scalar, $\Phi(x,\theta_-)$, and the spinor, $\Lambda_-(x,\theta_-)$, real superfields, whose respective θ- expansions are given below:

$$\Phi(x,\theta_-) = A(x) - i\theta_-\psi_+(x), \tag{3.2a}$$

$$\Lambda_-(x,\theta_-) = \lambda_-(x) + \theta_- F(x). \tag{3.2b}$$

A, ψ_+ and λ_- are all physical fields, whereas F is an auxiliary degree of freedom.

The supersymmetric kinetic and mass Lagrangians built up in terms of these superfields read as below:

$$\mathcal{L} = -\int d\theta_-(D_+\Phi^i)(\partial_-\Phi^i) + i\int d\theta_-\Lambda^i_- D_+\Lambda^i_- + \int d\theta_- M_{ij}\Phi^i\Lambda^j_-, \tag{3.3}$$

where $i,j = 1,2,...,n$ are some internal indices and M is a real $n \times n$ matrix.

The explicit breaking terms whose effect is to shift the masses of the physical fields are collected in the following breaking Lagrangian:

$$\mathcal{L}_{breaking} = \int d\theta_-\theta_- m^2_{ij}\Phi^i\Phi^j + i\int d\theta_-\theta_-\mu_{ij}(D_+\Phi^i)\Lambda^j_-, \tag{3.4}$$

where m^2 and μ are real $n\times n$ matrices corresponding to the mass-breaking parameters.

Upon superspace partial integrations and use of a $2n\times 2n$ matrix notation, the quadratic superfield Lagrangians (3.3) and (3.4) can be combined and rewritten as given below:

$$\mathcal{L} + \mathcal{L}_{breaking} = \int d\theta_- \,(\, \Phi^t \quad \Lambda^t_- \,)\, \mathcal{O}\begin{pmatrix}\Phi\\ \Lambda_-\end{pmatrix}, \tag{3.5}$$

where the label "t" stands for transposition in the internal space and the operator $\mathcal{O}$ is a $2n \times 2n$ matrix with superspace operator entries defined as:

$$\mathcal{O} = \begin{pmatrix}\mathcal{A} & \mathcal{B}\\ \mathcal{C} & \mathcal{D}\end{pmatrix}, \tag{3.6}$$

with

$$\mathcal{A} \equiv 1A_2 + m^2A_5, \tag{3.7a}$$

$$\mathcal{B} \equiv \frac{1}{2}M + \frac{i}{2}\mu A_3, \tag{3.7b}$$

$$\mathcal{C} \equiv \frac{1}{2}M^t + \frac{i}{2}\mu^t A_4 \tag{3.7c}$$

and

$$\mathcal{D} \equiv iA_1. \tag{3.7d}$$

The operators A_1, A_2, A_3, A_4 and A_5, their multiplicative table and some useful inversion formulae can be found in the Appendix B.

Next, by coupling the quantum superfields to external sources, completing the squares and formally performing the Gaussian integrations over Φ and Λ_-, one can show that the superpropagators are given by:

$$< \Phi\Phi >= -\frac{1}{2}(\mathcal{A} - \mathcal{B}\mathcal{D}^{-1}\mathcal{C})^{-1}, \tag{3.8a}$$

$$< \Lambda_-\Phi >= \mathcal{D}^{-1}\mathcal{C}(\mathcal{A} - \mathcal{B}\mathcal{D}^{-1}\mathcal{C})^{-1} \tag{3.8b}$$

and

$$< \Lambda_-\Lambda_- >= -\frac{1}{2}(\mathcal{D} - \mathcal{C}\mathcal{A}^{-1}\mathcal{B})^{-1}. \tag{3.8c}$$

With the help of the multiplicative table and the inversion formulae collected in the Appendix B, one obtains the following explicit expressions for the superpropagators in configuration space:

$$< \Phi(1)\Phi(2) >= -(iD_{1+} + P\partial_{1+}\theta_{1-})\frac{1}{1-iP} \cdot$$
$$\cdot \frac{1}{\Box_1 - \frac{1}{2}MM^t - \frac{1}{2}\mu\mu^t + \frac{1}{2}M\mu^t + \frac{1}{2}\mu M^t}\delta^2(x_1 - x_2)\delta(\theta_{1-} - \theta_{2-}), \tag{3.9a}$$

$$< \Lambda_-(1)\Phi(2) >= [M^t - M^tPD_{1+}\theta_{1-} + i\mu^t(1-iP)D_{1+}\theta_{1-}]\frac{1}{1-iP} \cdot$$
$$\cdot \frac{1}{\Box_1 - \frac{1}{2}MM^t - \frac{1}{2}\mu\mu^t + \frac{1}{2}M\mu^t + \frac{1}{2}\mu M^t}\delta^2(x_1 - x_2)\delta(\theta_{1-} - \theta_{2-}) \tag{3.9b}$$

and

$$< \Lambda_-(1)\Lambda_-(2) >= -(\frac{1}{\Box_1}\partial_{1-}D_{1+} + \frac{i}{2}Q\theta_{1-})\frac{1}{1-iQ} \cdot$$
$$\cdot (1 - \frac{1}{2}M^t\frac{1}{\Box_1 - 2m^2}M)^{-1}\delta^2(x_1 - x_2)\delta(\theta_{1-} - \theta_{2-}). \tag{3.9c}$$

The matrices P and Q are defined by:

$$P \equiv -2i(\Box - \frac{1}{2}MM^t - \frac{1}{2}\mu\mu^t + \frac{1}{2}M\mu^t + \frac{1}{2}\mu M^t)^{-1}[m^2 - \frac{1}{4}(\mu\mu^t - M\mu^t - \mu M^t)] \tag{3.10a}$$

and

$$Q \equiv \frac{i}{2}(1 - \frac{1}{2}M^t\frac{1}{\Box - 2m^2}M)^{-1}(M^t\mu + \mu^tM + M^t\frac{2m^2}{\Box - 2m^2}M - \mu^t\mu)\frac{1}{\Box}. \tag{3.10b}$$

Notice that in these expressions the breaking parameters m_{ij}^2 and μ_{ij} have been taken into account to all orders with our method for inverting the operator O of eq.(3.6).

One can also introduce explicit breaking coupling terms for the superfields Φ and Λ_-. Restricting ourselves to the case of dimensionless coupling parameters, possible interactions are:

$$\mathcal{L} = i\lambda_{ijk} \int d\theta_- \theta_- P(\Phi)(D_+\Phi^i)(D_+\Phi^j)(\partial_-\Phi^k) \tag{3.11a}$$

and

$$\mathcal{L} = \lambda_{ij} \int d\theta_- \theta_- P(\Phi)(D_+\Lambda_-^i)(D_+\Lambda_-^j). \tag{3.11b}$$

The Feynman rules for such a type of vertices can be directly read off from the corresponding interaction Lagrangians, with an associated θ_--factor. Care has to be taken with the anti-commutative character of the variable θ_-.

We are now ready to apply these results to a general (1,0) non-linear σ-model to which one adds new couplings which break supersymmetry.

The basic relations to be employed in our loop calculations with broken supersymmetry are:

$$\{D_+, \theta_-\} = i, \tag{3.12a}$$

$$D_{1+}\delta(\theta_{1-} - \theta_{2-})|_{\theta_1=\theta_2} = i$$

and

$$D_{1+}\theta_{1-}\delta(\theta_{1-} - \theta_{2-})|_{\theta_1=\theta_2} = -i\theta_{1-}.$$

The superspace action for an arbitrary (1,0) non-linear σ-model with the Wess-Zumino term is known[26-28] to be given by:

$$S = -\int d^2x d\theta_- [g_{ij}(\Phi) + B_{ij}(\Phi)](D_+\Phi^i)(\partial_-\Phi^j), \tag{3.13}$$

where B_{ij} is an anti-symmetric tensor defined on M.

Our next step, and this is our main goal, is to find reparametrization invariant coupling terms which explicit break (1,0) supersymmetry. Restricting ourselves to those interaction terms governed by a dimensionless coupling parameter, λ, the only vertex we can build up which fulfill these requirements and affect the spinor-scalar couplings is given by:

$$\mathcal{L}_{breaking} = i\lambda \int d\theta_- \theta_- T_{ijk}(\Phi)(D_+\Phi^i)(D_+\Phi^j)(\partial_-\Phi^k), \tag{3.14}$$

where $T_{ijk}(\Phi)$ is a tensor globally defined over the target manifold. Since only its anti-symmetric part in the first two indices appears in (3.14), there is no loss of generality in taking T_{ijk} to be anti-symmetric in i,j.

In order to find a geometrical interpretation for the breaking term (3.14), let us consider its component-field expression which is readily shown to be:

$$\mathcal{L}_{breaking} = i\lambda T_{ijk}(A)\psi_+^i\psi_+^j\partial_- A^k. \tag{3.15}$$

Taking now the component-field fermionic Lagrangian arising from the action (3.13), and bringing it togheter with (3.15), the ψ_+-field Lagrangian can be shown to read:

$$\mathcal{L}_{fermionic} = -ig_{ij}(A)\psi_+^i(\nabla_-\psi_+)^j, \tag{3.16}$$

where the covariant derivative ∇_- is defined to be

$$(\nabla_-\psi_+)^j \equiv \partial_-\psi_+^j + \hat{\Gamma}^j{}_{kl}(\partial_- A^l)\psi_+^k. \tag{3.17}$$

$\hat{\Gamma}^j{}_{kl}$ is the connection containing the Christoffel symbol $\Gamma^j{}_{kl}$, the torsion term induced by the Wess-Zumino action and an additional contribution generated by our breaking Lagrangian:

$$\hat{\Gamma}^j{}_{kl} = \Gamma^j{}_{kl}(A) + H^j{}_{kl}(A) - \lambda T^j{}_{kl}(A). \tag{3.18}$$

Therefore, a suitable choice of the tensor T_{ijk} in our breaking Lagrangian could be made in such a way to completely cancel out the torsion contributed by the Wess-Zumino term. This would actually be the case if

$$T_{ijk} = \frac{1}{\lambda}H_{ijk}. \tag{3.19}$$

On the other hand, if the Wess-Zumino term is not present, our breaking Lagrangian by itself could describe the torsion to which the fermions ψ_+ couple. It is important to notice that, though our torsion term explicitly breaks supersymmetry, as long as the fermionic sector is concerned, it perfectly accounts for the torsion effects of the target space.

Next, we start the discussion on the quantization of the model. To do that, we proceed as it is usually done: the background field method is adopted and a normal coordinate expansion in terms of the elementary quantum fluctuaction ξ^a ("a" denotes the local frame index) is performed. The superpropagators we shall employ in the course of our work are given by:

$$< \xi^a(1)\xi^b(2) >= \delta^{ab}\frac{i}{k^2}D_{1+}(k)\delta(\theta_{1-}-\theta_{2-}). \tag{3.20}$$

For our diagrammatic calculations, we read the Feynman rules for the vertices directly from the quantum-background couplings following from the normal coordinate expansions of (3.13) and (3.14), with the remark that the latter introduces an explict θ_--factor at each vertex with coupling parameter λ.

The tadpole graph of fig.1 gives the following contributions to the effective action:

$$\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2+\mu_r^2}\int d\theta_-[R_{ij}+\frac{1}{2}(D^kD_kB_{ij}+R^m{}_iB_{mj}+R^m{}_jB_{mi})](D_+\Phi^i)(\partial_-\Phi^j) \tag{3.21}$$

and

$$-\frac{1}{2}(i\lambda)\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2+\mu_r^2}\int d\theta_-\theta_-[D^lD_lT_{ijk}+R^m{}_iT_{mjk}+R^m{}_jT_{imk}+R^m{}_kT_{ijm})$$
$$(D_+\Phi^i)(D_+\Phi^j)(\partial_-\Phi^k), \tag{3.22}$$

where μ_r stands for an infra-red cut-off mass.

The correction (3.21) contributes to the renormalization of the metric tensor, whereas the result (3.22) renormalizes the breaking interaction (3.14). Tadpoles with a $(D_+\Phi^i)$, $(\partial_-\Phi^i)$ or $(D_+\Phi^i)(D_+\Phi^j)$ sitting on the external leg of the graph are all vanishing due to the supersymmetry algebra and arguments of symmetric integration.

Contrary to what happens for an arbitrary $(1,1)$ non-linear σ- model, supergraphs of the type shown in fig.2 may contribute further one-loop infinities. We organize their contributions in three different categories: both vertices are taken from the supersymmetric quantum-background couplings, one is chosen supersymmetric and the other of the supersymmetry-breaking type (graphs linear in λ) and, finally, both vertices are taken from the supersymmetry-breaking Lagrangian (graphs of the order λ^2).

We list below the results we have found by calculating the supergraphs of the type as drawn in fig.2. At order λ^0, the divergent contributions we find are:

$$\int \frac{d^2p}{(2\pi)^2} \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2+\mu_r^2} \frac{1}{(k+p)^2+\mu_r^2} (k+p)_+(k+p)_-$$

$$\int d\theta_- (D_k B_{li})(D^k B^l{}_j)(D_+\Phi^i)(\partial_-\Phi^j) \qquad (3.23)$$

and

$$\int \frac{d^2p}{(2\pi)^2)} \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2+\mu_r^2} \frac{1}{(k+p)^2+\mu_r^2} k_+(k+p)_-$$

$$\int d\theta_- (D_k B_{li})(D^l B^k{}_j)(D_+\Phi^i)(\partial_-\Phi^j). \qquad (3.24)$$

They both contribute to the metric tensor renormalization.

The divergences encountered at the order λ are listed below:

$$-2(i\lambda) \int \frac{d^2p}{(2\pi)^2} \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} (k+p)_+(k+p)_-$$

$$\int d\theta_- \theta_- (D_m B_{ni})(D^m T_j{}^n{}_k)(D_+\Phi^i)(D_+\Phi^j)(\partial_-\Phi^k), \qquad (3.25)$$

$$-2(i\lambda) \int \frac{d^2p}{(2\pi)^2} \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} k_+(k+p)_-$$

$$\int d\theta_- \theta_- (D_m B_{ni})(D^n T_j{}^m{}_k)(D_+\Phi^i)(D_+\Phi^j)(\partial_-\Phi^k), \qquad (3.26)$$

$$2(i\lambda) \int \frac{d^2p}{(2\pi)^2} \int \frac{d^2k}{(2\pi)^2} \frac{1}{k^2} \frac{1}{(k+p)^2} (k+p)_+ k_-$$

$$\int d\theta_- (D^m B^{ni})[iT_{jnm}(D_+\Phi^j) - \theta_- D_p T_{jnm}(D_+\Phi^p)(D_+\Phi^j) +$$

$$-i\theta_- T_{jnm}(\partial_+\Phi^j)](\partial_-\Phi_i), \qquad (3.27)$$

$$-2(i\lambda)\int\frac{d^2p}{(2\pi)^2}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2}\frac{1}{(k+p)^2}k_+(k+p)_-$$

$$\int d\theta_- B^{mn}[i(D_nT_{imj})(D_+\Phi^i)(\partial_-\Phi^j)+$$

$$-\theta_-(D_pD_nT_{imj})(D_+\Phi^p)(D_+\Phi^i)(\partial_-\Phi^j)+$$

$$+\theta_-(D_nT_{imj})(D_+\Phi^i)(\partial_-D_+\Phi^j)+$$

$$-i\theta_-(D_nT_{imj})(\partial_+\Phi^i)(\partial_-\Phi^j)], \tag{3.28}$$

$$\lambda\int\frac{d^2p}{(2\pi)^2}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2}\frac{1}{(k+p)^2}k_+(k+p)_-(k+p)_-$$

$$\int d\theta_-\theta_- B_{mn}(D^mT_{ij}{}^n)(D_+\Phi^i)(D_+\Phi^j) \tag{3.29}$$

and

$$2(i\lambda)\int\frac{d^2p}{(2\pi)^2}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2}\frac{1}{(k+p)^2}k_+(k+p)_-(k+p)_-$$

$$\int d\theta_- B_{mn}[iT_i{}^{mn}(D_+\Phi^i)-\theta_-(D_pT_i{}^{mn})(D_+\Phi^p)(D_+\Phi^i)+$$

$$-\theta_-T_i{}^{mn}(i\partial_+\Phi^i)]. \tag{3.30}$$

It is interesting to notice the presence of supersymmetric corrections induced by the breaking of supersymmetry and their contribution to the metric tensor renormalization.

Finally, taking both vertices to be of the supersymmetry-breaking type, one can show that there will not appear any order-λ^2 contribution to the metric tensor renormalization. This can be understood since both θ_- coming from the vertices cannot be simultaneously eaten up by operators D_+ coming from the loop upon partial integrations. One of the θ_- will always remain untouched to produce a contribution which explicitly breaks supersymmetry. Indeed, the only order-λ^2 infinity induced by this class of graphs is given by:

$$4(i\lambda^2)\int\frac{d^2p}{(2\pi)^2}\int\frac{d^2k}{(2\pi)^2}\frac{1}{k^2}\frac{1}{(k+p)^2}(k+p)_+k_-$$

$$\int d\theta_-\theta_- T_{imn}(D^mT_j{}^m{}_k)(D_+\Phi^i)(D_+\Phi^j)(\partial_-\Phi^k). \tag{3.31}$$

Power-counting indicates that one-loop diagrams with at most three external background lines may diverge. Actually, explicit computations show that the supergraph of fig.6 gives an infinite correction to the breaking term (3.14). Its expression is given by:

$$2(i\lambda^3)\int \frac{d^2p}{(2\pi)^2}\int \frac{d^2k}{(2\pi)^2}\frac{1}{k^2}\frac{1}{(k+p)^2}\frac{1}{(k+p+q)^2}$$

$$k_-k_-(k+p)_+(k+p+q)_+\int d\theta_-\theta_-T_i{}^m{}_nT_j{}^n{}_pT_k{}^p{}_m(D_+\Phi^i)(D_+\Phi^j)(\partial_-\Phi^k). \tag{3.32}$$

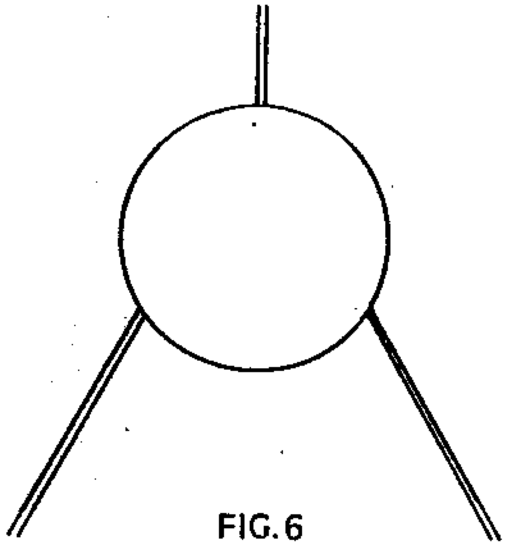

FIG.6

Divergent one-loop diagrams with the highest number of external legs.

As for the one-loop approximation, these calculations suffice to give a general picture. Besides the well-known modification of the metric tensor renormalization due to the presence of the Wess-Zumino term[29-33], we obtain that, by virtue of an interference with the latter, our torsion-like breaking piece alters the metric tensor β-function by corrections of order λ. However, if $B_{ij}=0$, our breaking coupling does not interfere with the supersymmetric action governed by g_{ij} and the result of β_{ij} for the exact model is not modified: it remains vanishing for Ricci-flat target spaces.

Still, it is worthy to mention that the interference between the Wess-Zumino term and our breaking Lagrangian leads to new divergent breaking terms of the form

$$\int d\theta_-\theta_-(D_+\Phi)(D_+\Phi), \tag{3.33}$$

$$\int d\theta_-\theta_-(D_+\Phi)(\partial_-D_+\Phi) \tag{3.34}$$

and

$$\int d\theta_- \theta_- (\partial_+ \Phi)(\partial_- \Phi) \tag{3.35}$$

But, whenever $B_{ij} = 0$, these new infinities are absent.

So, the final outcome of our one-loop analysis is the following: switching off the Wess-Zumino term, and describing the torsion of the target manifold by means of the breaking Lagrangian(3.14), we conclude that Ricci-flatness still suffices for the vanishing of the metric tensor β-function. No new breaking term is induced and the supersymmetry breaking vertex itself does not require any renormalization provided that the tensor T_{ijk} be covariantly constant with respect to the connection without torsion and symmetric in the indices j,k.

These conditions work to eliminate many of the two-loop supergraphs exhibiting the topology as drawn in fig.3a. The only divergent correction arising from such a type of diagrams contributes to renormalize the breaking vertex of eq.(3.14). Moreover, this infinity is of the type $\frac{1}{\epsilon^2}$ and, since at one-loop there is no counterterm, one might expect that $\frac{1}{\epsilon^2}$-divergences coming from the graphs $(3b)$ and $(3c)$ cancel the above infinities. However, the important point to be noticed here is that at two loops no new divergence of the type $\frac{1}{\epsilon}$ shows up, so that no contributions to the β-function appear at this order. One can then finally state that the conditions imposed at one loop are sufficient to keep the metric tensor β-function vanishing at two loops if the target space is Ricci-flat.

4. Concluding remarks

We have in this work contemplated arbitrary (1,1) and (1,0) non-linear σ-models modified by the addition of terms which explicitly break supersymmetry. They are chosen in such a way to respect the diffeomorphism invariance of the target manifold and to be associated to a dimensionless breaking coupling parameter.

In the (1,1) case, the breaking terms we have proposed to investigate do not change the β-function of the exact model up to two loops. They lead to new corrections to β_{ij} from three loops on, and the contributions they induce do not vanish for Ricci-flat manifolds.

As for the (1,0) case, the situation changes with respect to the (1,1) model. Reparametrization and Lorentz invariances, along with the request of a dimensionless coupling parameter, uniquely fix the form of the breaking term. Moreover, such a term appears with an interesting geometrical interpretation: it describes the torsion of the manifold to which the fermions of the model couple. Explicit supergraph computations, suitably extended to include the case of broken supersymmetry, show that the (1,0) model broken by the torsion-like term is one-loop finite under certain assumptions on the torsion tensor introduced by means of the supersymmetry breaking coupling. Also, at two loops the metric tensor β-function keeps vanishing under the same constraints on the torsion and for Ricci-flat target manifolds.

Just to end our discussion, we would like to stress that the breaking of the two-dimensional supersymmetry is not disastrous for the consistency of the related string model. The breaking parameters and the geometry can be suitably adjusted in such a way to give a vanishing β-function. This is the key ingredient for the unitarity of the string. On the other hand, since the breaking of the world-sheet supersymmetry affects the σ-model β-function, and the latter provides equations of motion from which one can integrate a string effective action, it would be interesting to analyze how the breaking parameters appear at the level of the α'-corrected string effective action.

Appendix A

We collect here the operators A_n $(n = 1,..,12)$ relevant for the calculations carried out in Section 2. They are

$$A_1 \equiv D^2, \qquad A_7 \equiv \partial_{\alpha\beta} D^\alpha \theta^2 D^\beta,$$

$$A_2 \equiv \theta^\alpha D_\alpha, \qquad A_8 \equiv \partial_{\alpha\beta} \theta^\alpha D^\beta,$$

$$A_3 \equiv D^\alpha \theta_\alpha, \qquad A_9 \equiv \partial_{\alpha\beta} D^\alpha \theta^\beta,$$

$$A_4 \equiv \theta^2 D^2, \qquad A_{10} \equiv D^2 D^\alpha \theta^2 D_\alpha,$$

$$A_5 \equiv D^2 \theta^2, \qquad A_{11} \equiv D^\alpha \theta^2 D_\alpha D^2,$$

$$A_6 \equiv D^\alpha \theta^2 D_\alpha, \qquad A_{12} \equiv \theta^2.$$

Due to the (anti-) commutation relations amongst the D's and θ's given in eqs.(2.1) – (2.4), the truly independent operators can be shown to be just A_1, A_2, A_4, A_8 and A_{12}: all the others can be written as linear combinations of them. Indeed:

$$A_3 = -2 + A_2,$$

$$A_5 = -1 + A_2 + A_4,$$

$$A_6 = A_2 + 2A_4,$$

$$A_7 = A_8 + 2i \,\Box\, A_{12},$$

$$A_9 = -A_8,$$

$$A_{10} = -iA_8 + 2 \,\Box\, A_{12}$$

and

$$A_{11} = A_{10} = -iA_8 + 2 \,\Box\, A_{12}.$$

The independent operators can be shown to form a closed set under multiplication, as it can be seen from the table below:

	A_1	A_2	A_4	A_8	A_{12}
A_1	$\Box$	$2A_1 + iA_8$	$-A_1 - iA_8 + \Box A_{12}$	$-i\Box A_3$	A_5
A_2	$-iA_8$	$A_2 - 2A_4$	$2A_4$	$A_8 - 2i\Box A_{12}$	$2A_{12}$
A_4	$\Box A_{12}$	$2A_4$	$-A_4$	$2i\Box A_{12}$	$-A_{12}$
A_8	$i\Box A_{12}$	A_7	0	$-\Box A_6$	0
A_{12}	A_4	0	0	0	0

Multlplicative table of the operators relevant in the case of broken (1, 1) supersymmetry.

Appendix B

We discuss here the operators $A_n (n = 1, .., 5)$ relevant for our calculations with broken (1,0) supersymmetry reported in Section 3. They are

$$A_1 \equiv D_+,$$

$$A_2 \equiv \partial_- D_+,$$

$$A_3 \equiv D_+ \theta_-,$$

$$A_4 \equiv \theta_- D_+$$

and

$$A_5 \equiv \theta_-.$$

They can be shown to form a closed set under multiplication, according to the table given below:

	A_1	A_2	A_3	A_4	A_5
A_1	$i\partial_+$	$-\frac{i}{2}\Box$	$i\partial_+ A_5$	$iA_1 - i\partial_+ A_5$	A_3
A_2	$-\frac{i}{2}\Box$	$-\frac{i}{2}\Box\partial_-$	$-\frac{i}{2}\Box A_5$	$i\partial_- A_1 + \frac{i}{2}\Box A_5$	$\partial_- A_3$
A_3	$iA_1 - i\partial_+ A_5$	$i\partial_- A_1 + \frac{i}{2}\Box A_5$	iA_3	0	0
A_4	$i\partial_+ A_5$	$-\frac{i}{2}\Box A_5$	0	iA_4	iA_5
A_5	A_4	$\partial_- A_4$	iA_5	0	0

Multiplicative table of the operators relevant in the case of broken (1,0) supersymmetry.

We give below three useful inversion formulae:

$$A_1^{-1} = 2i\frac{1}{\Box}A_2,$$

$$A_2^{-1} = 2i\frac{1}{\Box}A_1$$

and

$$(1 - RA_3)^{-1} = 1 + R\frac{1}{1 - iR}A_3,$$

where R denotes a general c-valued matrix.

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