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Constraints and Hamiltonian in Light-front Quantized Field Theory

by

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Abstract

Self-consistent hamiltonian formulation of scalar theory on the null plane is constructed and quantized following the Dirac procedure. The theory contains also *constraint equations* which would give, if solved, to a nonlocal Hamiltonian. In contrast to the equal-time formulation we obtain a different description of the spontaneous symmetry breaking in the continuum and the symmetry generators are found to annihilate the light-front vacuum. Two examples are given where the procedure cannot be applied self-consistently. The corresponding theories are known to be ill-defined from the equal-time quantization.

Key-words: Light-cone quantization; Higgs Mechanism; Symmetry breaking.

1. Introduction:

The possibility of building hamiltonian formulation of relativistic dynamics on light-front surface, $(t + z) = const.$, was indicated by Dirac [1] and rediscovered [2] in the context of the infinite-momentum frame. The longitudinal momentum k^+ (say, in the massive case) being necessarily positive and conserved, the presence of $k^+ = 0$ quanta in the light-front vacuum seems excluded. This implies a simpler ground state compared to that encountered in the case of equal-time quantization. The much discussed discretized light-cone quantized (DLCQ) field theory [3] does reveal significant simplifications. Among the recent developments is the Light-front Tamm-Dancoff Field Theory which has been proposed for the non-perturbative bound state problem [4,3].

However, the description on the null-plane of the spontaneous symmetry breaking (ssb) or Higgs mechanism, for example, are not clearly understood even at the tree level [3,4]. We will address to it by quantizing the self-interacting scalar field. For this we need first to decide which Hamiltonian to use. The ingredients commonly employed, e.g., the (perturbative) vacuum, the canonical commutation relations, and the local Hamiltonian with polynomial self-interaction terms are perhaps too simple to give rise to all the non-perturbative effects, if we consider that we are faced with a constrained dynamical system. Among other motivations to clarify the problem are the string theories (e.g., heterotic strings) where the scalar fields also appear and the null plane quantization frequently adopted for a better physical understanding. The Dirac method is a systematic procedure to construct the hamiltonian formulation for constrained lagrangians. We show that a self-consistent formulation in our context does exist in the continuum (excepting some special cases) and that it contains not only the Hamiltonian but also *constraint equations*. Solving the constraints would lead to a nonlinear and nonlocal Hamiltonian. A description of the ssb parallel to that in the equal-time case, though with some important differences is shown to follow. When other fields are also present the constraint eqs. would relate the various vacuum condensates and may also be useful in suggesting new counter terms in the quantized theory. The Dirac method was attempted earlier using (the finite volume) discretized formulation [6,7] in incomplete fashion. The *standard* method requires that *all* the constraints (not associated with the gauge-fixing) be derived from inside the given

Lagrangian. In [6] the constraint $p \approx 0$ below was missed or the method not completed by finding the Dirac brackets in order to implement the constraints while in [7] constraints were added from outside. The check of the self-consistency of the procedure [5] was overlooked and the properties of the symmetry generators required to describe the ssb were not considered. It is also pointed out that the discussion may be made in the continuum or in the finite volume with a convenient boundary condition; they agree when the infinite volume limit of the latter is taken (e.g. ref. [8]). The physically significant predictions are restored only by removing the spurious finite size effects corresponding to, say, the zero modes in the finite volume sgn and delta functions. Two examples (Secs. 2,3) are given where Dirac procedure becomes inconsistent; the corresponding theories, however, are known from the equal-time quantization studies to be ill-defined. In Sec. 2 we explain the details for scalar field in two dimensions. The extensions to an iso-multiplet in $3+1$ dimensions is discussed in Sec. 3 where also the symmetry generators in the quantized theory needed to give a description of the ssb (and Higgs mechanism) are constructed.

2. Hamiltonian Formulation (Dirac procedure):

Consider first the case of a real massive scalar field ϕ in two space-time dimensions with the Lagrangian density $[\dot{\phi}\phi' - V(\phi)]$, where an overdot and a prime indicate the partial derivations with respect to the light-front coordinates $\tau = (x^0 + x^1)/\sqrt{2}$ and $x = (x^0 - x^1)/\sqrt{2}$ respectively. The eq. of motion $2\dot{\phi}' = -V'(\phi)$, where a prime on V indicates the variational derivative with respect to ϕ, shows that the solutions, $\phi = const.$, are possible. If we integrate the eq. of motion over $-L/2 \leq x \leq L/2$ we are led to the constraint equation, $\int dx\, V'(\phi) = -2\,\partial_\tau[\mathcal{C}(\tau, L)]$, where $\mathcal{C}(\tau, L) = [\,\phi(\tau, x = L/2) - \phi(\tau, x = -L/2)\,]$. The constraint thus seems to depend on the value of the surface variable $\mathcal{C}(\tau, L)$ which is absent for the periodic boundary condition but may be non-vanishing for other cases. We are thus required to formulate the (physical) problem at hand more carefully. Based on the consideration that the possible constant solutions for ϕ at the classical level are relevant for describing the ground state and the ssb we should make the separation, as is usually done in the context of gauge theories, $\phi(x, \tau) = \omega(\tau) + \varphi(x, \tau)$. The variable ω corresponds to the background field (bosonic condensate) while φ describes the (quantum) fluctuations above the former. This separation should be done independent of whether we work in the finite volume (discretized formulation) *or* directly in the continuum formulation. The Lagrangian now reads as

$$C\dot{\omega} + \int_{-L/2}^{L/2} dx\, \dot{\varphi}\, \varphi' - \int_{-L/2}^{L/2} dx\, V(\phi)\,, \tag{1}$$

where the surface variable $C(\tau, L)$ is to be treated as a dynamical one like ω and φ variables. The set of *three* lagrange eqs. then leads to the following *constraint equation*, which is independent of the boundary conditions adopted,

$$\begin{aligned}\frac{1}{L}\beta(\tau) &\equiv \frac{1}{L}\int_{-L/2}^{L/2} dx\, V'(\phi)\\ &= \omega(\lambda\omega^2 - m^2) + \frac{(3\lambda\omega^2 - m^2)}{L}\int_{-L/2}^{L/2} dx\, \varphi + \frac{\lambda}{L}\int_{-L/2}^{L/2} dx \left[3\omega\varphi^2 + \varphi^3\right] = 0\end{aligned} \tag{2}$$

Here, for concreteness we take $V(\phi) = -(1/2)m^2\phi^2 + (\lambda/4)\phi^4 + const.$, $\lambda > 0$. The canonical null plane Hamiltonian is found to be

$$H^{l.f.} \equiv \int_{-L/2}^{L/2} dx\, V(\phi) = \int_{-L/2}^{L/2} dx \left[\omega(\lambda\omega^2 - m^2)\varphi + \frac{1}{2}(3\lambda\omega^2 - m^2)\varphi^2 + \lambda\omega\varphi^3 + \frac{\lambda}{4}\varphi^4 + const.\right] \tag{3}$$

It is then clear form (2) and (3) that the elimination of ω would lead to a nonlocal Hamiltonian in contrast to the simple polynomial one obtained if we had ignored altogether the background field variable. This seems to be the price to pay for working on the null plane with the corresponding simple light-front vacuum. In the *continuum formulation* the field φ is assumed to be an ordinary absolutely integrable function of x such that its Fourier (series) transform $\tilde{\varphi}(k, \tau)$ exists together with the inverse transform. In this case C vanishes since $\varphi \to 0$ for $|x| \to \infty$ and the constraint eq. (3), with $L \to \infty$, follows directly on integrating the lagrange eq. for φ.

We remind that in the context of equal-time quantization the criterion, $V'(\omega) = 0$, obtained by minimizing the classical Hamiltonian, which determines the allowed values of the background field, plays a significant role in the tree level description of ssb. In the null plane case we do not have any physical considerations to minimize the corresponding light-front Hamiltonian. The same criterion, however, is seen to follow in the continuum limit now from the constraint eq. (2) considered at the tree level. In fact the integrals over φ, φ^2, and φ^3 are convergent (from the Fourier transform theory) and the corresponding terms drop out when $L \to \infty$. In the renormalized theory with field operators the value of the background field does get modified due to the high order quantum corrections. For the sake of the compactness we give the discussion below directly in the continuum [9] where also the variable C is absent. The relevant expressions obtained have their counterpart in the *discretized version* [8] and easily identified if we define the space integral over $[-\infty, \infty]$ by the Cauchy principal value etc.

Having discussed the Lagrangian formulation we next construct the hamiltonian formulation following the Dirac method which would latter be quantized. The Lagrangian density may be rewritten as $\mathcal{L} = \dot{\varphi}\varphi' - V(\phi)$, and it describes a constrained or singular Lagrangian. We would treat both ω and φ as canonical variables on the null-plane and let the *self-consistency* requirement [5] *determine* their properties. Indicating by p and π the momenta conjugate to ω and φ, respectively, the primary constraints are $p(\tau) \approx 0$ and $\Phi \equiv \pi - \varphi' \approx 0$ while the canonical Hamiltonian density is derived to be $\mathcal{H}_c = V(\phi)$ and $\approx$ stands for the weak equality [5]. We postulate now the standard Poisson brackets at equal τ, with the nonvanishing brackets satisfying, $\{p, \omega\} = -1$, $\{\pi(x), \varphi(x)\} = -\delta(x-y)$, and assume for the preliminary Hamiltonian the expression

$$H'(\tau) = H_c(\tau) + \mu(\tau)p(\tau) + \int dy\; u(\tau, y)\Phi(\tau, y), \tag{4}$$

where μ and u are Lagrange multipliers. Using (4) we derive

$$\dot{p} \;=\; \{p, H'\} \approx -\int dx\; V'(\phi) \;\equiv\; -\beta(\tau), \tag{5}$$

$$\dot{\Phi} \;=\; \{\Phi, H'\} \approx \;-\; V'(\phi) \;-\; 2u'. \tag{6}$$

The persistency requirement, $\dot{p} \approx 0$, then leads to a *secondary constraint* $\beta \approx 0$, while $\dot{\Phi} \approx 0$ results in a consistency condition involving the multiplier u and does not generate a new constraint. An extended Hamiltonian H'' is now defined by adding a term $\nu(\tau)\beta(\tau)$ to H' and we repeat the procedure. For the choice $\nu \approx 0$ no more constraints are generated.

The three constraints $p \approx 0$, $\beta \approx 0$, $\Phi \approx 0$ are verified to be second class [5]. The constraints may be implemented [5] by defining modified brackets to replace the standard ones. In view of $\{\beta(\tau), p(\tau)\} \equiv \alpha(\tau) = \int dx\ V''(\phi)$, $\{\beta,\beta\} = \{p,p\} = 0$ we construct the bracket $\{,\}^*$

$$\{f(x), g(y)\}^* = \{f(x), g(y)\} - \frac{1}{\alpha}[\{f(x), p\}\{\beta, g(y)\} - (\beta \leftrightarrow p)], \tag{7}$$

with the property $\{f,p\}^* = \{f,\beta\}^* = 0$ for any arbitrary functional f. We may then set $p = 0$ and $\beta = 0$ even inside these brackets and treat them as strong equalities. It is seen from (7) that among the surviving variables *only* the bracket $\{\omega,\pi\}^* = \{\omega,\Phi\}^* = -\alpha^{-1} V''(\phi)$ differs form their corresponding Poisson brackets. The remaining constraint $\Phi \approx 0$ may next be taken care of by a modification of the brackets $\{,\}^*$ themselves obtaining the (final) Dirac bracket which implements all the constraints. We check that $\Phi(x)$ is second class by itself and $\{\Phi(x), \Phi(y)\}^* = \{\Phi(x), \Phi(y)\} = -2\partial_x\delta(x-y) \equiv C(x,y) = -C(y,x)$. Its (unique) inverse with the correct symmetry property is $C^{-1}(x,y) = -C^{-1}(y,x) = -\epsilon(x-y)/4$ and hence the *final Dirac bracket* $\{,\}_D$ is constructed as

$$\{f(x), g(y)\}_D = \{f(x), g(y)\}^* + \frac{1}{4}\int\int du dv \{f(x), \Phi(u)\}^* \epsilon(u-v)\{\Phi(v), g(y)\}^*. \tag{8}$$

Inside $\{f,g\}_D$ we are allowed to set in addition $\pi = \varphi'$ so that π and p are removed from the theory and we are left with only the variables ω and φ which are related through the *constraint* $\beta = 0$ which is the same as we found at the Lagrangian level assuring us of the self-consistency.

From (8) we derive for the field φ

$$\{\varphi(x), \varphi(y)\}_D = -(1/4)\epsilon(x-y)\ , \tag{9}$$

which corresponds to the well known light-cone commutator (see below). We emphasize that in its derivation $\{\omega,\pi\}^*$ is not required. We also find

$$\{\omega, \pi(x)\}_D = \{\omega, \varphi'(x)\}_D = \frac{1}{2}\{\omega, \pi(x)\}^*, \tag{10}$$

and $\{\omega, \omega\}_D = 0$ follows from symmetry considerations. The eqs. of motion are given by $\dot{f} = \{f, H_c\}_D + \partial f/\partial\tau$ where $H_c(\tau) \equiv H^{l.f.}$ as given in (3). For the potential considered the explicit expression for α reads as

$$\alpha(\tau) = \int dx\, V''(\phi) = L\,(3\lambda\omega^2 - m^2) + 6\lambda\omega \int dx\, \varphi\, + 3\lambda \int dx\, \varphi^2, \tag{11}$$

where $L \to \infty$ as discussed above. At the *tree level* as discussed above the constraint implies $V'(\omega) = (\lambda\omega^2 - m^2)\omega = 0$ which determines the allowed values of ω. Corresponding to these values (11) shows that $\alpha \to \infty$ and consequently $\{\omega, \pi\}^* = -\alpha^{-1} V''(\phi) \to 0$ which from (10) leads in the continuum limit to $\{\omega, \varphi(x)\}_D = 0$. We then find $\dot{\omega} = 0$ which is consistent with the constant values found for ω (from $V'(\omega) = 0$) and the Lagrange eq. of motion for φ is also recovered. We are thus able to build a self-consistent hamiltonian formulation in the continuum with the separation proposed above based on physical considerations.

The quantized theory is now obtained by the correspondence [5] $i\{f, g\}_D \to [f, g]$ where the quantities inside the commutator are the corresponding quantized operators. The operator ω commutes with itself and with the nonzero modes and no operator ordering problem arises in contrast to the case of discretized finite volume formulation. For the correct sign for the mass term ω is vanishing and the self-consistency may also be checked. In the case of the massless theory ($\lambda \neq 0$), we find from (2) that ω is vanishing at the tree level and from (11) we conclude that $lim_{L\to\infty}\alpha^{-1} \neq 0$. Consequently from (10) $\{\omega, \varphi'\}_D = -\alpha^{-1} V''(\phi)/2$ is nonvanishing indicating an inconsistency and we are also unable to recover the Lagrange eqs. of motion. This is in agreement with the discussion in the corresponding equal-time case where a massless scalar theory in two dimensions is known to be ill-defined; this is remedied (Sec. 3) by the extra space dimension available in higher space-time dimensions.

The commutation relations of φ may be realized in momentum space through the expansion ($\tau = 0$)

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk\, \frac{\theta(k)}{\sqrt{2k}} \left[a(k)e^{-ikx} + a^\dagger(k)e^{ikx}\right] \tag{12}$$

where $a(k)$ and $a^\dagger(k)$ satisfy the canonical commutation relations, viz, $[a(k), a(k')^\dagger] = \delta(k-k')$, $[a(k), a(k')] = 0$, and $[a(k)^\dagger, a(k')^\dagger] = 0$ while ω commutes with them becoming essentially a c-number. The vacuum state is defined by $a(k)|vac\rangle = 0$, $k > 0$. The values of $\omega = \langle|\phi|\rangle_{vac}$ *characterize* the (non-perturbative) vacua and the Fock space built as usual. A self-consistent Hamiltonian formulation can thus be built in the continuum which also can describe ssb. The high order corrections to the (renormalized) constraint eq. (2) will alter the tree level values of ω since, we do *not* have any physical considerations to normal order the constraint equation like we have for light-front energy or the momentum operators. The phase transition in two dimensions can be described [10] in the renormalized theory based on (2) and (3) in the continuum. In the discretized formulation we do have to face the difficult problem of operator ordering of ω with the non zero modes apart from making a self-consistency check as well.

3. Spontaneous continuous symmetry breaking mechanism:

Extending the discussion to continuous symmetry in $3+1$ dimensions, the Lgrangian density with a global *isospin symmetry* may be written as

$$\mathcal{L} = [\, \dot{\varphi}_a \varphi'_a - \frac{1}{2}(\partial_i \phi_a)(\partial_i \phi_a) - V(\phi)\,]. \tag{13}$$

Here the real scalar fields ϕ_a, $a = 1, 2...$ are the components of an isospin-multiplet, $i = 1, 2$ and $\bar{x} \equiv (x^1, x^2)$ refer to the transverse directions, $V'_a \equiv \delta V(\phi)/\delta\phi_a$, and we have set now $\phi_a(\tau, x, \bar{x}) = \varphi_a(\tau, x, \bar{x}) + \omega_a(\tau, \bar{x})$ which may be justified as before by observing that the $\bar{x}$ coordinates, in contrast x behave as parameters on the null-plane. We note in this connection that for the case of free field if $\varphi(\tau, x, \bar{x})$ solves the eq. of motion so does $\varphi(\tau, x, \bar{x}) + \omega(\tau, \bar{x})$ where $(\partial_i\partial_i - m^2)\omega(\tau, \bar{x}) = 0$. The canonical (light-front) Hamiltonian is

$$H_c(\tau) = \int dx d^2x \Big[V(\phi) + \frac{1}{2}(\partial_i \phi_a)(\partial_i \phi_a)\Big], \tag{14}$$

and the bracket $\{,\}^*$ which implements the constraints $\beta_a(\tau,\bar{x}) \approx 0$, $p_a(\tau,\bar{x}) \approx 0$ is now

$$\{f,g\}^* = \{f,g\} - \int\int d^2\bar{u}d^2\bar{v}\left[\{f,p_a(\bar{u})\}C^{-1}_{ab}(\bar{u},\bar{v})\{\beta_b(\bar{v}),g\} - (\beta \leftrightarrow p)\right], \qquad (15)$$

where C^{-1} is the inverse of the matrix (suppressing τ)

$$C_{ab}(\bar{x},\bar{y}) \equiv \{\beta_a(\bar{x}),p_b(\bar{y})\} = \left[L[-\delta_{ab}\,\partial_i\partial_i + V''_{ab}(\omega)] + V'''_{abc}(\omega)\int dx\,\varphi_c + \ldots\right]\delta^2(\bar{x}-\bar{y}). \quad (16)$$

Again $\{\,\omega_a(\bar{x}),\pi_b(y,\bar{y})\,\}^* = [V''_{ac}(\phi(y,\bar{y})) - \delta_{ac}\partial^y{}_i\partial^y{}_i]C^{-1}_{cb}(\bar{x},\bar{y})$ are the only ones which differ from the corresponding standard Poisson brackets among the surviving variables. The *final* Dirac bracket which implements also the, $\Phi_a(\tau,x,\bar{x}) \equiv \pi_a - \varphi'_a \approx 0$, is

$$\{f,g\}_D = \{f,g\}^* + \frac{1}{4}\int\int d^3u d^3v\,\{f,\Phi_a(u,\bar{u})\}^*\,\epsilon(u-v)\delta^2(\bar{u}-\bar{v})\,\{\Phi_a(v,\bar{v}),g\}^*, \quad (17)$$

and we find (without requiring to use $\{\omega_a,\pi_b\}^*$)

$$\{\,\varphi_a(x,\bar{x}),\varphi_b(y,\bar{y})\,\}_D = -\frac{1}{4}\delta_{ab}\,\epsilon(x-y)\delta^2(\bar{x}-\bar{y}) \qquad (18),$$

A Taylor expansion in the constraint $\beta_a = 0$ gives

$$L\,[\,V'_a(\omega) - \partial_i\partial_i\omega_a\,] + V''_{ab}(\omega)\int dx\varphi_b + \frac{1}{2!}V'''_{abc}(\omega)\int dx\varphi_b\varphi_c + \ldots = 0, \qquad (19)$$

and at the tree level it leads to $[\,V'_a(\omega) - \partial_i\partial_i\omega_a\,] = 0$. As before this equation and (19) are in agreement with the Lagrangian formulation. The (classical level) constant solutions for ω_a are determined by solving $V'_a(\omega) = 0$ and are relevant for describing the ground state and ssb. The $\bar{x}$-dependent solutions correspond to the solitary waves of the equal-time formulation but here only in three or more dimensions.

For the other brackets we find

$$\{\omega_a(\bar{x}),\pi_b(y,\bar{y})\}_D = \{\omega_a(\bar{x}),\varphi'_b(y,\bar{y})\}_D = \frac{1}{2}\{\omega_a(\bar{x}),\pi_b(y,\bar{y})\}^* \qquad (20)$$

$$\{\omega_a(\bar{x}),\omega_b(\bar{y})\}_D = \frac{1}{4}\int\int d^3u d^3v\, \{\omega_a(\bar{x}),\pi_c(u,\bar{u})\}^*\, \epsilon(u-v)\delta^2(\bar{u}-\bar{v})\, \{\pi_c(v,\bar{v}),\omega_b(\bar{y})\}^*. \tag{21}$$

We discuss now the inverse matrix C^{-1} which is needed to implement the constraint (19). For the potential $V(\phi) = (\lambda/4)(\phi_a\phi_a - m^2/\lambda)^2$, $\lambda > 0$, the constant solutions are found from $V'_a(\omega) = (\lambda\omega^2 - m^2)\omega_a = 0$, where $\omega^2 \equiv \omega_a\omega_a$. In the broken symmetry phase $\omega^2 = (m^2/\lambda)$ while in the symmetric phase (or when the potential has the correct sign for the mass term) $\omega_a = 0$. In the latter case the leading term in (16) is, $-L(\partial_i\partial_i + m^2)\delta_{ab}\delta^2(\bar{x}-\bar{y})$, while in the former it is $L[-\delta_{ab}\partial_i\partial_i + 2m^2 P_{ab}]\delta^2(\bar{x}-\bar{y})$ where $P_{ab} = (\omega_a\omega_b)/\omega^2$ is a projection operator. The corresponding inverse matrices contain an explicit factor $1/L$ multiplying a Green's function which is well defined. Hence in the $L \to \infty$ limit the $\{\omega_a, \pi_b\}^*$ are vanishing and from (20) and (21) we find $\{\omega_a(\bar{x}),\omega_b(\bar{y})\}_D = 0$ and $\{\omega_a(\bar{x}),\varphi_b(y,\bar{y})\}_D = 0$. The self-consistency is verified like in Sec. 2. The transverse directions now present cure also the inconsistency encountered in the massless theory in two dimensions. We can also give a new demonstration of Coleman's theorem [11] on the absence of Goldstone bosons in two dimensions. There are no transverse directions on the null-plane in two dimensions and the matrix $C_{ab} = 2Lm^2P_{ab}$ contains a projection operator which can not be inverted even when $m \neq 0$. We are unable to implement the constraints and construct a self-consistent hamiltonian formulation. The Fock space operators now depend on the transverse momentum as well and they satisfy $[a_b(k,\bar{k}), a_c(k',\bar{k}')^\dagger] = \delta_{bc}\delta(k-k')\delta^2(\bar{k}-\bar{k}')$ etc. where $\bar{k} = (k^1,k^2)$ indicates the transverse components. The quantized theory is also checked to be self-consistent (and even for the case of $\bar{x}$-dependent solutions which we do not consider for our purpose).

The global invariance of (13) at the *classical* level gives rise to conserved isospin currents and the field theory generators are given by

$$G_\alpha(\tau) = \int d^2\bar{x}\, p(\bar{x})\, t_\alpha\, \omega(\bar{x}) + \int dx d^2\bar{x}\, \pi(x,\bar{x})\, t_\alpha\, \varphi(x,\bar{x}) \tag{22}$$

where α, β are the group indices, t_α are hermitian and antisymmetric, and $[t_\alpha, t_\beta] = if_{\alpha\beta\gamma}t_\gamma$. The generators in the *quantized theory* are hence given by ($p_a = 0$, $\pi_a = \varphi'_a$)

$$G_\alpha(\tau) = -i\int d^2\bar{x}\, dx\, \varphi'_a(x,\bar{x})\,(t_\alpha)_{ab}\,\varphi_b(x,\bar{x}) \;=\; \int d^2\bar{k}\, dk\, \theta(k) a_a(k,\bar{k})^\dagger\,(t_\alpha)_{ab}\, a_b(k,\bar{k}) \qquad (23).$$

which come out normal ordered and consequently on the null-plane the continuous symmetry generators always annihilate the vacuum. We find that $[G_\alpha, \varphi_a] = -(t_\alpha)_{ab}\varphi_b$, $[G_\alpha, \omega_a] = 0$, and $[G_\alpha, G_\beta] = if_{\alpha\beta\gamma}\, G_\gamma$. The tree level ssb is now described as follows. A particular solution, $(\omega_1, \omega_2, \omega_3 ...)$, of $V'_a = \omega_a(\lambda\omega^2 - m^2) = 0$ defines a preferred direction in the isospace which *characterizes* a (non-perturbative) vacuum state, $\langle|\phi_a|\rangle_{vac} = \omega_a$. The infinite degeneracy of the vacuum is described by the continuum of the allowed orientations in the isospin space of the background isovector. In the symmetric phase $\omega_a = 0$ and there is no preferred direction. In the broken phase the potential expressed in terms of the field operators φ_a and ω_a reveals that the surviving symmetry in the quantized theory is of lesser dimension than the initial one and we obtain Goldstone bosons in the theory. Not all the generators are now conserved in the quantized theory but there may survive a set of linearly independent field generators which still do so. They are evidently found by solving $(\tilde{t}_\alpha)_{ab}\omega_b = 0$ where $\tilde{t}_\alpha$ are appropriate linearly independent combinations, depending on the ω_a, of the original matrix generators. The corresponding operators $\tilde{G}_\alpha$ generate the *surviving symmetry* in the quantized theory. The number of Goldstone bosons may be counted following the arguments as in the case of equal-time quantization [12]. The implications of the lack of conservation at the quantum level of some currents conserved in the classical theory needs further study. The description of the tree level *Higgs mechanism* is straightforward and like in equal-time quantization. To give its quantized description the null plane theory of interacting gauge field (e.g., QCD) has to be understood first since the constraints would contain more terms when fermionic and other bosonic interactions are also present.

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