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FLUCTUATION EFFECTS ON BUBBLE GROWTH IN HOT NUCLEAR MATTER

by

A.J. SANTIAGO* and K.C. CHUNG

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

*Permanent address:
Instituto de Física
Universidade do Estado do Rio de Janeiro
20550 - Rio de Janeiro, RJ - Brasil

ABSTRACT

The evolution of bubbles with arbitrary density in an infinite nuclear system is studied in a simplified treatment. Kinetic pressure fluctuations on the bubble surface are considered. The critical radius, evolution time and probability for bubble expansion are shown to depend significantly on the initial bubble density.

Key–words: Nuclear multifragmentation, Statistical fluctuations, Relativistic heavy ion collisions.

In a recent paper, Bondorf *et al.*[1] have addressed the problem of how random forces affect the dynamical evolution of the bubbles appearing in an expanded hot nuclear matter. This important question has to do with the nuclear multifragmentation and basically involves the study of the mechanism by which an initially very excited nuclear system grows and then breaks–up into many pieces.

The reaction mechanism of the nuclear multifragmentation is not well understood yet. However, two main mechanisms are currently considered, namely, the spinodal decomposition and the bubble nucleation[2]. In this last case, the scenario is the following: bubbles (rarefaction regions) are formed in the superheated liquid and they grow and nucleate[3], then provoking the disruption of the matter. It is just in this context that the study of the effects of fluctuations on the dynamics of bubble growth in hot nuclear matter is relevant.

Concerning to the bubble growth, it was shown earlier by Blin *et al.*[4] that, for bubbles with radius R, immersed in a hot nuclear matter, the bubble critical radius is

$$R_c = -\frac{2\sigma}{\Delta P} \tag{1}$$

provided the thermodynamical potential is given by

$$\Omega = 4\pi R^2\,\sigma + \frac{4\pi R^3}{3}\,\Delta P \tag{2}$$

where σ is the surface tension and ΔP, the pressure difference between the liquid and gas phases.

In this simple case, the physical meaning of the critical radius is clear: bubbles with radius $R < R_c$ will collapse and bubbles with $R > R_c$ will expand, while bubbles with $R = R_c$ will stay in stationary regime.

The essential result from Bondorf *et al.*'s calculation is to show that this simple picture may be somewhat modified, if single–particle fluctuations are introduced. In fact, by taking into account the random forces arising from the fluctuations in the number of nucleons striking on the bubble surface per unit time, they have obtained a quite different result: for subcritical, critical or overcritical radii, both expanding and collapsing bubbles are found. This result is in striking contrast with the predictions extracted from the bubble dynamics when kinetic pressure fluctuations are not taken into account.

However, in their work, they have assumed zero initial density bubbles. Of course, this is a quite particular configuration and represents even an unrealistic approximation if one has in mind that a liquid–gas phase transition may there be underlying in the nuclear fragmentation process. Therefore, it is worthwhile to extend the study of the single–particle fluctuation effects to the case of arbitrary bubble densities.

In this letter, we report the results obtained by generalizing straightforward the equations of motion for bubbles with nonzero density. For simplicity, we assume isothermal processes, so that the bubble evolves in thermal equilibrium with the liquid background. We are confident that our results will not be changed qualitatively by this simplified treatment. Let's, first, consider a spherical bubble of number density n_b and radius R, immersed in an infinite uncharged liquid of density n_l at temperature T, with the obvious restriction $n_b < n_l$.

Assuming no mass exchange between the bubble and the medium during the evolution, the velocity field $v(r)$ is obtained with the help of the equation of continuity,

$$
\begin{aligned}
v &= \frac{\dot{R}}{R} r \qquad && r < R \\
&= \dot{R} R^2 \frac{1}{r^2} \qquad && r > R
\end{aligned}
\tag{3}
$$

where the dot denotes time derivative. Therefore, the Hamiltonian of the system is

$$
H = 2\pi m (n_l + \frac{n_b}{5}) R^3 \dot{R}^2 + 4\pi R^2 \sigma + \frac{4\pi R^3}{3} \Delta P \tag{4}
$$

where m is the nucleon mass, and σ and ΔP stand for the surface tension and the pressure difference, respectively, of the non–zero density bubble. Of course, Eq.(4) recovers the Hamiltonian of zero–density bubble case when $n_b = 0$.

In what follows, the single–particle fluctuations will be treated in the same way than in Ref.1, *i.e.*, the pressure difference is split into

$$
\Delta P = \overline{\Delta P} + \delta \Delta P \tag{5}
$$

where $\overline{\Delta P} = (\overline{\Delta P})_b + (\overline{\Delta P})_l$ is the average part and $\delta\Delta P = (\delta\Delta P)_b + (\delta\Delta P)_l$ is the fluctuating part of the pressure difference, both written as a sum of contributions from the bubble (subscript b) and the liquid (subscript l).

The equation of motion for the bubble radius R is obtained from Eq.(4). Then, we can split it into two first order differential equations. After integrating them in a time interval Δt, which is chosen to be sufficiently small compared with the hydrodynamics timescale and sufficiently large compared with the timescale of two consecutive nucleon striking on the bubble surface, we get

$$R(t+\Delta t) = R(t) + V(t)\,\Delta t \tag{6}$$

and

$$V(t+\Delta t) = V(t) - \frac{\Delta t}{(5n_l+n_b)R(t)}\left[\frac{3}{2}V^2(t)\,(5n_l-n_b) + \frac{5\overline{\Delta P}}{m} + \frac{10\sigma}{mR(t)}\right] - \frac{5}{mR(t)\,(5n_l+n_b)}\,I(t) \tag{7}$$

where

$$I(t) = \int_t^{t+\Delta t} \delta\Delta P dt$$

is the random impulse.

A simple estimate of $I(t)$ has been made by Bondorf *et al.* in the approximation of a classical independent particle gas. They have shown that the values of $I(t)$ obey a Gaussian distribution approximately, with average value equal zero and standard deviation given by

$$\Gamma = \alpha\frac{m\sqrt{\pi n v_n^3 \Delta t}}{3\pi R}, \tag{8}$$

where v_n is the average nucleon speed and $\alpha \approx T/\epsilon_F$, which is introduced in order to take into account the fact that only nucleons in the surface of the Fermi sea contribute to the fluctuations. For more details, see Ref.1.

In this work, we have adopted the equation of state used by Sneppen and Vinet[5], which is derived from a Skyrme interaction. With relation to σ, we have assumed that it is temperature independent and given by

$$\sigma = \sigma_l - \sigma_b. \tag{9}$$

where $\sigma_{l(b)}$ is the surface tension in the case a zero–density bubble is embebbed in a medium of density $n_{l(b)}$. Furthermore, in an expanding nucleus, the surface energy is assumed for simplicity to scale like its bulk energy.

Eqs.(6) and (7) are solved iteratively, with $\Delta t = 1fm/c$. However, the results are shown to be essentially the same provided $\Delta t < 10fm/c$. Each run yields either a collapsed bubble or an expanding bubble. In this last case, the criterion used is that the bubble volume is greater than four times the critical volume. This is reasonable, because beyond this volume, the expansion process is shown can no more be stopped by random forces. In the case of collapsed bubble, the radius collapses to the final value given by $R_f = (n_b^i/n_l)^{1/3}R_i$, where the label $i(f)$ denotes initial (final) value. In this limit, the bubble and the liquid background form a whole homogeneous matter.

In what follows, we present the results of our calculation, by assuming $T = 5\,MeV$ and $n_l = 0.075fm^{-3}$. Firstly, we display the critical radius R_c as function of the initial bubble density n_b^i (Fig.1). It is seen that R_c is strongly dependent upon n_b^i, with a small decreasing in the beginning and after that a rapid and monotonic increasing. At $n_b^i = 0$, we get $R_c \approx 1.86fm$ and at $n_b^i \approx 0.007fm^{-3}$, the critical radius reaches a minimum with $R_c \approx 1.68fm$, then almost doubling this value, when n_b^i tends to n_l. It should be noted that, if the Coulomb energy is taken into account, the value of R_c has been shown in Ref.4 to decrease significantly only for densities greater or of order of $0.1fm^{-3}$.

In Fig.2, the yields of expanding and collapsed bubbles are plotted as function of the elapsed time t_e, for a particular value of the initial bubble density, namely, $n_b^i = 0.05fm^{-3}$ (solid lines) and for two different values of the initial bubble radius, $R_i = 1.5fm$ and $2.0fm$. In this case, $R_c \approx 2.3fm$ and the statistics is 50,000 runs. For the sake of comparison, the results from Ref.1, corresponding to the case of $n_b^i = 0$, are also presented (dashed lines). For $R_i = 1.5fm$, the histograms show only small dependence on n_b^i , but for $R_i = 2.0fm$, the differences are significant in two respects, namely, the position and the high of the peak of the yield curves.

In Fig.3, the expansion probability P_{exp} is plotted as function of the initial bubble density, also for $R_i = 1.5fm$ and $2.0fm$. Of course, the collapsing probability may be calculated simply as the complement of the expansion probability, when single particle fluctuations are included. It should be noted that, if $R_i = 1.5fm$, the initial bubble radius is always less than R_c, for every n_b^i, but if $R_i = 2.0fm$, one finds a region in which $R_i > R_c$ (segment AB) and another region in which $R_i < R_c$ (segment BC). The point B is seen to be located at $n_b^i \approx 0.027fm^{-3}$ for which $R_i = R_c = 2.0fm$ (see Fig.1). In other words, for $R_i = 1.5fm$, we have only subcritical bubbles, while for $R_i = 2.0fm$, we have overcritical bubbles in the left–hand region and subcritical bubbles in the right–hand region. In the curve corresponding to $R_i = 1.5fm$, it is seen that a smooth decreasing in the beginning is followed by also a smooth increasing in the end, so that the value of the expansion probability does not deviate significantly from the value at $n_b^i = 0$. The maximum of P_{exp} occurs at $n_b^i \approx 0.065fm^{-3}$, when the probability is

0.20, which has to be compared with the value 0.13 at $n_b^i = 0$. The behaviour of P_{exp} for $R_i = 2.0 fm$ is much more dramatic than in the previous case. As a matter of fact, P_{exp} drops rapidly from around 0.65 at $n_b^i = 0$ to 0.26 at $n_b^i \approx 0.07 fm^{-3}$. It is clear from this result that the approximation of $n_b^i = 0$ overestimates the value of the expansion probability.

In Fig.4, the elapsed time corresponding to the peak of the yield curves is displayied as function of the initial bubble density, both for expanding bubbles (full lines) and collapsing bubbles (dashed lines). It is seen that, in the case of expanding bubbles, there is a minimum at $n_b^i \approx 0.01 fm^{-3}$, irrespectively of the value of R_i, and for $n_b^i > 0.03 fm^{-3}$, the elapsed time is almost independent of R_i. The elapsed time of collapsed bubbles is much more sensitive to R_i in the whole range of densities considered.

In summary, we simply have made a generalization of the study of the fluctuations effects on the bubble dynamics in the hot nuclear matter, for arbitrary initial bubble density. The results of our calculation show that the approximation of zero initial bubble density may be quite poor if expansion probability is concerned, especially for values of the initial radius near R_c, in which case it may overestimate P_{exp} by a factor greater than 2. Furthermore, the $n_b^i = 0$ approximation also overestimates the expanding elapsed time in the range of low (with relation to n_l) densities, and underestimates it in the range of high densities, while the collapsed time is overestimated almost in the whole range of initial bubble densities, except in the region around $n_b^i = 0.01 fm^{-3}$. One should note that, as R_c is expected to depend strongly on the equation of state of nuclear matter, the above results may be significantly changed, if a different equation of state is used. In particular, a more realistic expression for surface tension should be used. Work on this point is presently in progress.

Acknowledgment:

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FIGURE CAPTIONS

FIG.1:

The critical radius R_c as function of the initial bubble density n_b^i.

FIG.2:

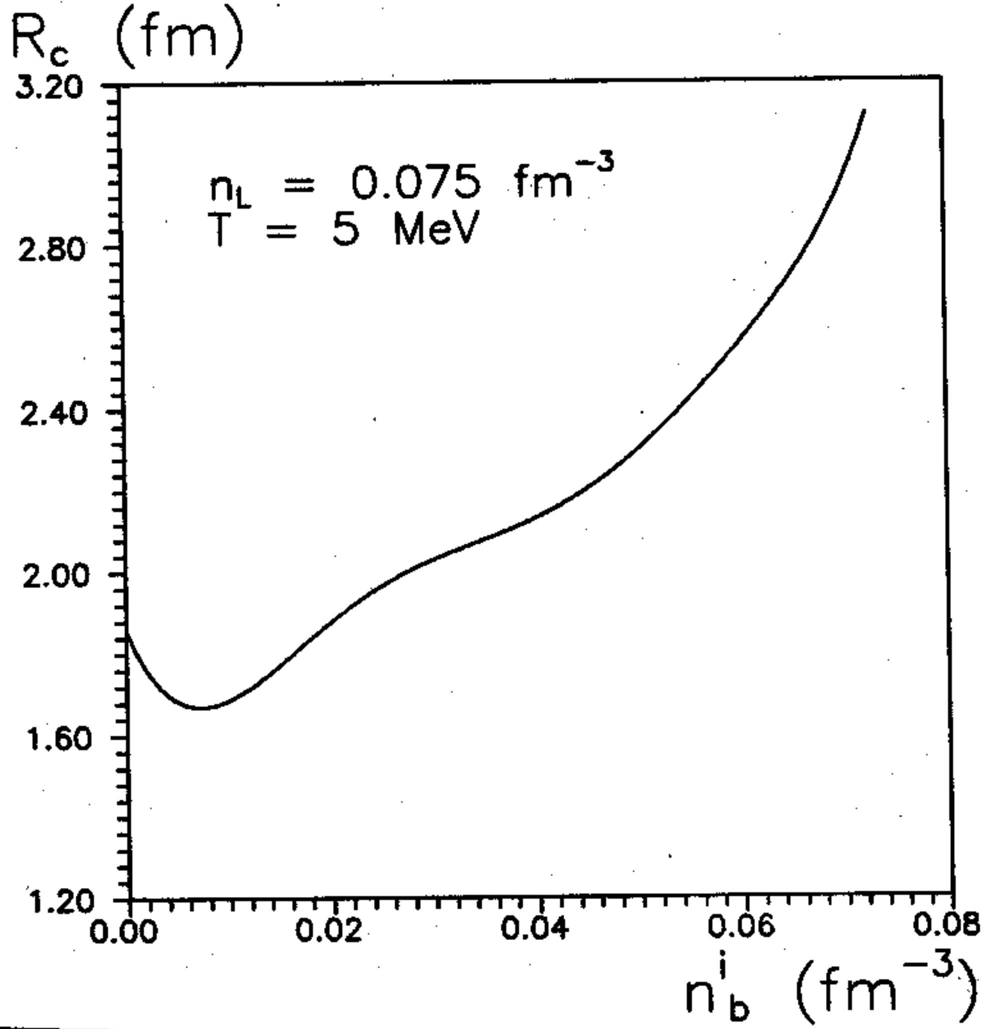
The yield of expanding and collapsed bubbles as function of the elapsed time, for $n_b^i = 0.05 fm^{-3}$ (full line) and $n_b^i = 0$ (dashed line). The upper (lower) part corresponds to $R_i = 1.5\,(2.0) fm$, while the left (right) part corresponds to expanding (collapsed) bubbles.

FIG.3:

The expansion probability P_{exp} as function of the initial bubble density n_b^i, for $R_i = 1.5 fm$ and $2.0 fm$. The point B corresponds to critical bubbles. For more details, see the text.

FIG.4:

The elapsed time t_e as function of the initial bubble density n_b^i,for expanding bubbles (full lines) and collapsed bubbles (dashed lines), both for $R_i = 1.5 fm$ and $2.0 fm$.



R_c (fm)
3.20
2.80
2.40
2.00
1.60
1.20
n_L = 0.075 fm^-3
T = 5 MeV
0.00
0.02
0.04
0.06
0.08
n_b^i (fm^-3)

FIG. 1

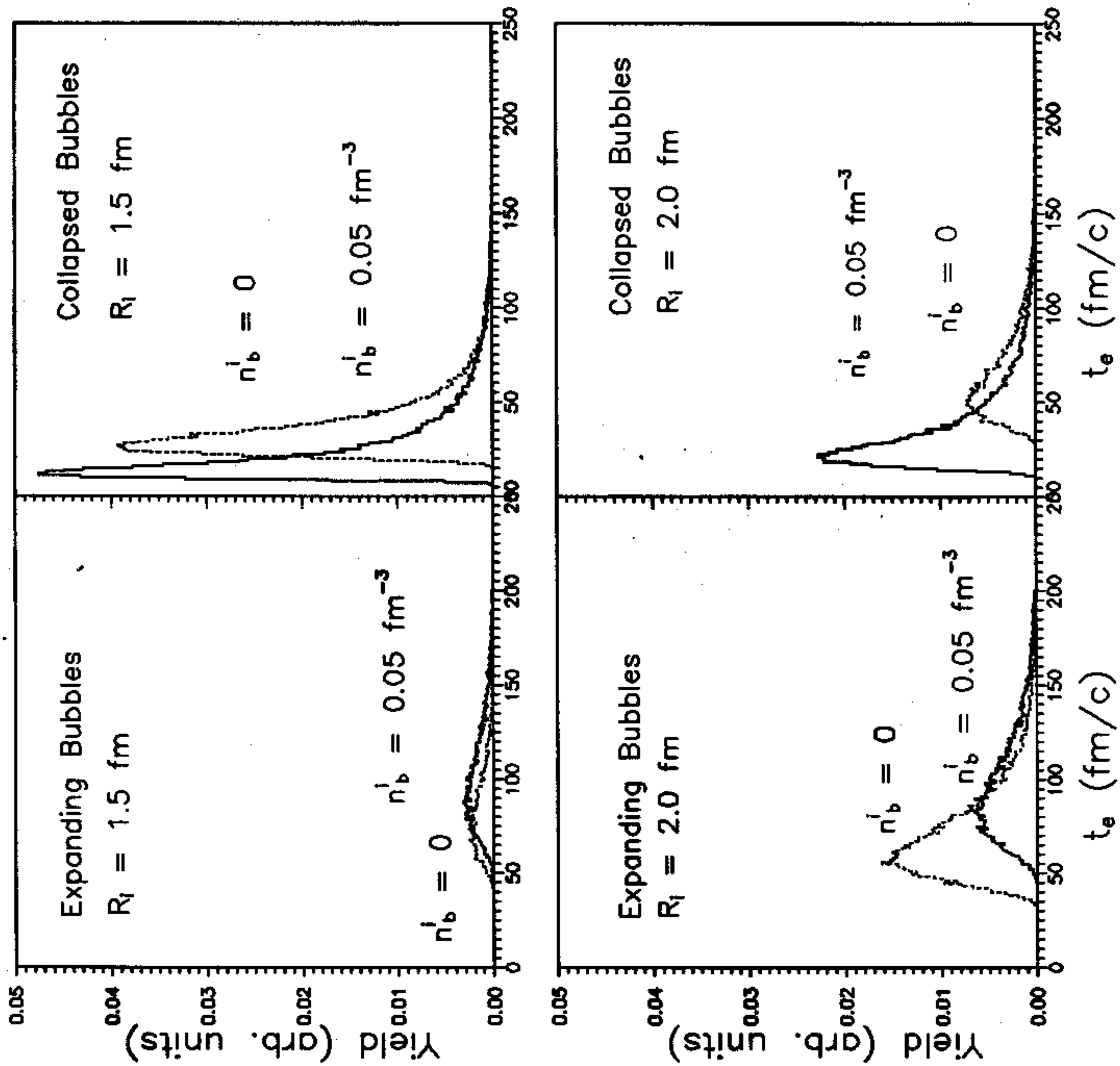

Collapsed Bubbles
R_i = 1.5 fm
n_b^i = 0
n_b^i = 0.05 fm^{-3}
Expanding Bubbles
R_i = 1.5 fm
n_b^i = 0.05 fm^{-3}
n_b^i = 0
Collapsed Bubbles
R_i = 2.0 fm
n_b^i = 0.05 fm^{-3}
n_b^i = 0
Expanding Bubbles
R_i = 2.0 fm
n_b^i = 0
n_b^i = 0.05 fm^{-3}
Yield (arb. units)
t_e (fm/c)

FIG. 2

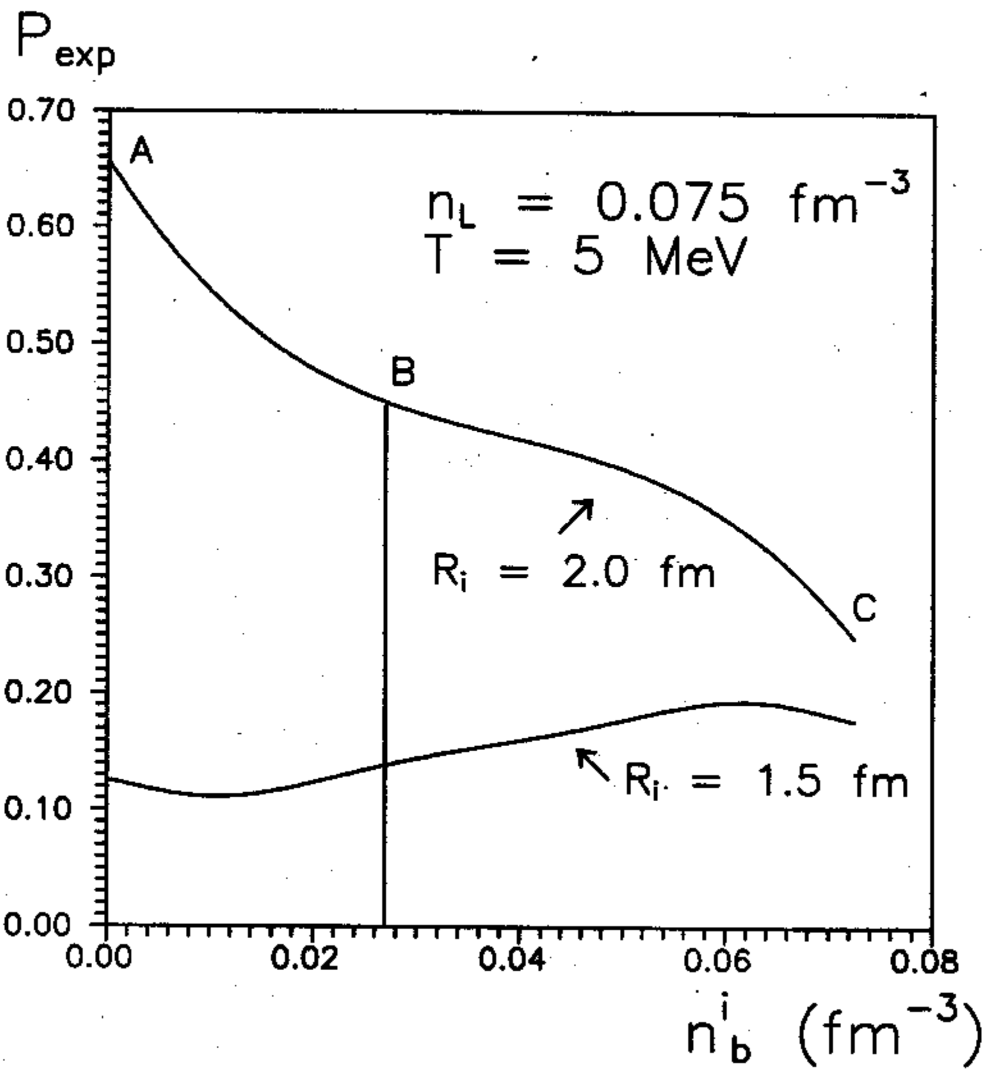

P exp
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
A
B
C
n L = 0.075 fm−3
T = 5 MeV
R i = 2.0 fm
R i = 1.5 fm
0.00
0.02
0.04
0.06
0.08
n i b (fm−3)

FIG. 3

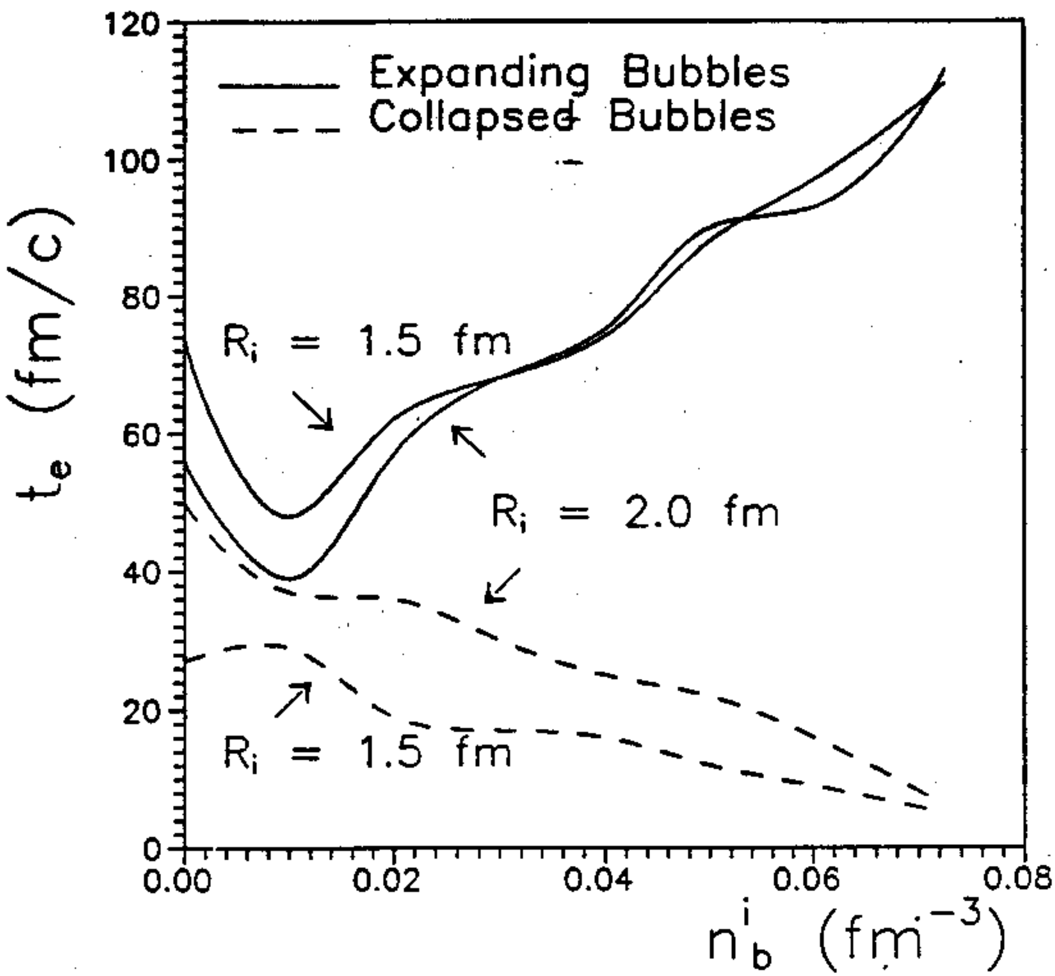

Expanding Bubbles
Collapsed Bubbles
R_i = 1.5 fm
R_i = 2.0 fm
R_i = 1.5 fm
t_e (fm/c)
n_b^i (fm^{-3})
0
20
40
60
80
100
120
0.00
0.02
0.04
0.06
0.08

FIG. 4

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