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A LESS-CONSTRAINED (2,0) SUPER-YANG-MILLS MODEL: THE COUPLING TO NON-LINEAR σ-MODELS

by

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ABSTRACT

Considering a class of (2,0) super-Yang-Mills multiplets characterised by the appearance of a pair of independent gauge potentials, we present here their coupling to non-linear σ-models in (2,0)-superspace. Contrary to the case of the coupling to (2,0) matter superfields, the extra gauge potential present in the Yang-Mills sector does *not* decouple from the theory in the case one gauges isometry groups of σ-models.

Key-words: Extended gauge theories.

The raise of interest on the investigation of geometrical aspects and quantum behaviour of two-dimensional systems, such as Yang-Mills theories and non-linear σ-models, especially if endowed with supersymmetry, has been broadly renewed in connection with the analysis of superstring background configurations [1,2] and the study of conformal field theories and integrable models.

As for supersymmetries defined in two space-time dimensions, they may be generated by p left-handed and q right-handed independent Majorana charges: these are the so-called (p,q) supersymmetries [1,3] and are of fundamental importance in the formulation of the heterotic superstrings [4].

Motivated by the understanding of a number of features related to the dynamics of world-sheet gauge fields [5] and the possibility of finding new examples of conformal field theories, we have recently considered the superspace formulation of a (2,0) Yang-Mills model [6,7] enlarged by the introduction of a second gauge potential transforming under the same *simple* gauge group as the ordinary Yang-Mills field of the theory.

In the works of refs. [6,7], we have attempted at an understanding of a further gauge potential on the basis of discussing constraints on field-strenght superfields in the algebra of gauge-covariant derivatives in (2,0) superspace. The minimal coupling of this sort of less-superspace. The minimal coupling of this sort of less-constrained Yang-Mills models to matter superfields has been contemplated and it has been ascertained that the additional gauge potential corresponds to non-interacting degrees of freedom in the Abelian case. For non-Abelian symmetries, the further Yang-Mills field still decouples from matter though it presents self-interactions with the gauge sector.

It is therefore our purpose in this letter to find out a possible dynamical rôle for the additional gauge potential discussed in refs. [6,7], through its coupling to matter superfields that describe the coordinates of the Kähler manifold adopted as the target space of a (2,0) non-linear σ-model [8]. To pursue such an investigation, we shall gauge the isometry group of the σ-model under consideration while working in (2,0) superspace; then, all we are left with is the task of coupling the (2,0) Yang-Mills extended supermultiplets of ref. [6] to the superfields that define the (2,0) σ-model in question.

The coordinates we choose to parametrise the (2,0) superspace are given by

$$z^{A} \equiv (x^{++},\, x^{--};\, \theta,\, \bar{\theta}), \tag{1}$$

where x^{++}, x^{--} denote the usual light-cone coordinates, whereas θ, $\bar{\theta}$ stand for complex right-handed Weyl spinors. The supersymmetry covariant derivatives are taken as:

$$D_{+} \equiv \frac{\partial}{\partial\theta} + i\bar{\theta}\partial_{++} \tag{2a}$$

and

$$\bar{D}_{+} \equiv \frac{\partial}{\partial\bar{\theta}} + i\theta\partial_{++}\ , \tag{2b}$$

where ∂_{++} (or ∂_{--}) represents the derivative with respect to the space-time coordinate x^{++} (or x^{--}). They fulfil the algebra:

$$D_+^{\ 2} = \bar{D}_+^{\ 2} = 0 \qquad (3a); \qquad\qquad \{D_+,\bar{D}_+\} = 2i\ \partial_{++}. \qquad (3b)$$

The coordinates that parametrise the Kähler target manifold are given by *complex scalar* superfields, Φ^i and $\bar{\Phi}^i \equiv \Phi^{i}$, *constrained* according to:

$$\bar{D}_+\Phi^i = 0 \qquad (4a); \qquad D_+\bar{\Phi}^i = 0, \qquad (4b)$$

$$i = 1,\ 2,\ \ldots,\ n.$$

This sort of "chirality" constraints yields the following component-field expansion for Φ^i:

$$\Phi^i(x;\theta,\bar{\theta}) = \phi^i(x) + \theta\ \eta^i(x) + i\theta\bar{\theta}\ \partial_{++}\phi^i(x), \qquad (5)$$

where the ϕ^i's are complex scalars and the η^i's are complex left-handed Weyl spinors. The (2,0)-supersymmetric σ-model action written in (2,0)-superspace reads [8]:

$$S = -\ \frac{1}{2}\int d^2x\ d\theta\ d\bar{\theta}\ \Big[K_i(\Phi,\bar{\Phi})\ \partial_{--}\Phi^i - \text{c.c.}\Big], \qquad (6)$$

where the target space vector $K_i(\Phi,\bar{\Phi})$ can be expressed as the gradient of a *real scalar* (Kähler) potential, $K(\Phi,\bar{\Phi})$, whenever the Wess-Zumino term is absent (i.e., torsion-free case) [1]:

$$K_i(\Phi,\bar{\Phi}) = \partial_i K(\Phi,\bar{\Phi}) \equiv \frac{\partial}{\partial\Phi^i}\ K(\Phi,\bar{\Phi}). \qquad (7)$$

We shall draw our attention to Kählerian target manifolds of the coset type, G/H. The generators of the isometry group, G, are denoted by Q_α (α = 1, ..., dim G) whereas the isotropy group, H, has its generators denoted by $Q_{\bar\alpha}$ ($\bar\alpha$ = 1, ..., dim H). The transformations of the isotropy group are linearly realised on the superfields Φ and $\bar\Phi$, and act as matrix multiplication, just as on flat manifolds. The isometry transformations instead are non-linear and their infinitesimal on the points of G/H can be written as:

$$\delta\Phi^i = \lambda^\alpha k^i_\alpha(\Phi) \tag{8a}$$

and

$$\delta\bar\Phi_i = \lambda^\alpha \bar k_{\alpha i}(\bar\Phi), \tag{8b}$$

where $k_{\alpha i}$ and $\bar k_{\alpha i}$ are (holomorphic and anti-holomorphic) Killing vectors of the target manifold. The finite versions of the isometry transformations above reads:

$$\Phi'^i = \exp(L_{\lambda.k})\,\Phi^i \tag{9a}$$

and

$$\bar\Phi'_i = \exp(L_{\lambda.\bar k})\,\bar\Phi_i, \tag{9b}$$

with

$$L_{\lambda.k}\,\Phi^i \equiv \left[\lambda^\alpha k^j_\alpha \frac{\partial}{\partial\Phi^j}\,,\,\Phi^i\right] = \delta\Phi^i. \tag{10}$$

Though the Kähler scalar potential can always be taken H-invariant, isometry transformations induce on K a variation given by:

$$\delta K = \lambda^\alpha[\,(\partial_i K)k_{\alpha i} + (\bar\partial^i K)\bar k_{\alpha i}] = \lambda^\alpha[\,\eta_\alpha(\Phi) + \bar\eta_\alpha(\bar\Phi)]\,, \tag{11}$$

where the holomorphic and anti-holomorphic functions η_α and $\bar{\eta}_\alpha$ can be determined up to a purely imaginary quantity as bellow:

$$(\partial_i K)k^i_\alpha \equiv \eta_\alpha + iM_\alpha(\Phi,\bar{\Phi}) \qquad (12a)$$

and

$$(\bar{\partial}^i K)\bar{k}_{\alpha i} \equiv \bar{\eta}_\alpha - iM_\alpha(\Phi,\bar{\Phi}). \qquad (12b)$$

The real functions M_α and the holomorphic and anti-holomorphic functions η_α and $\bar{\eta}_\alpha$ play a crucial rôle in discussing the gauging of the isometry group of the target manifold [9,10]. Therefore, by virtue of the transformation equation (11) and the constraints imposed on Φ and $\bar{\Phi}$, it can be readily checked that the superspace action (6) is *invariant* under *global isometry transformations*.

Proceeding further with the study of the isometries, a relevant issue in the framework of (2,0)-supersymmetric σ-models is the gauging of the isometry group G of the Kählerian target manifold. This in turn means that one should contemplate the minimal coupling of the (2,0)-σ-model to the Yang-Mills supermultiplets of (2,0)-supersymmetry [11]. An eventual motivation for pursuing such an analysis is related to 2-dimensional conformal field theories. It is known that 2-dimensional σ-models define conformal field theories provided that suitable constraints are imposed upon the target space geometry [1,2]. Now, the coupling of these models to the Yang-Mills sector might hopefully yield new conformal field theories of interest.

The study of (2,0)-sypersymmetric Yang-Mills theories has been carried out in ref. [11] and the gauging of σ-model isometries in (2,0)-superspace has been recently considered in ref. [12] in the absence of a Wess-Zumino term. However, an alternative and so to say *less constrained* version of (2,0) gauge multiplets has been proposed and discussed in refs. [6,7]. It has been shown that the relaxation of some constraints on the gauge superconnections and on field-strenght superfields leads to the appearance of an extra gauge potential that shares a common gauge transformation with the usual Yang-Mills field. Nevertheless, this extra potential is shown to decouple from the (2,0) matter superfields whenever they are minimally coupled to the Yang-Mills sector.

It is our main purpose henceforth to carry out the coupling of a (2,0)-σ-model to the *more relaxed* gauge superfields of ref. [6] and show that the extra gauge degrees of freedom *do not* now decouple from the matter fields (that is, the target, space coordinates). The extra gauge potential obtained upon relaxing constraints can therefore acquire a dynamical significance by means of the coupling between the σ-model and the Yang-Mills fields of ref. [6]. Moreover, this system might provide another example of a gauge-invariant conformal field theory.

The Yang-Mills supermultiplets are introduced by means of the gauge covariant derivatives which, according to the discussion of ref. [6], are defined as bellow:

$$\nabla_+ \equiv D_+ - ig\Gamma_+, \quad (13a) \qquad \bar{\nabla}_+ \equiv \bar{D}_+, \quad (13b)$$

$$\nabla_{++} \equiv \partial_{++} - ig\Gamma_{++} \quad (13c) \quad \text{and} \quad \nabla_{--} \equiv \partial_{--} - ig\Gamma_{--}, \quad (13d)$$

where the gauge superconnections Γ_+, Γ_{++} and Γ_{--} are all Lie-algebra-valued and g stands for the gauge coupling parameter, Γ_+ and Γ_{++} can be both expressed in terms of a *real scalar* superfield, $V(x;\theta,\bar{\theta})$ according to:

$$\Gamma_+ = e^{-gV}(D_+ e^{gV}) \tag{14a}$$

and

$$\Gamma_{++} = -\frac{i}{2}\bar{D}_+[e^{-gV}(D_+e^{gV})]. \tag{14b}$$

Therefore, the gauging of the σ-model isometry group shall be achieved by minimally coupling the action (6) to the gauge superfields V and Γ_{--}, as we shall see in the sequel.

To establish contact with a component-field formulation and to actually identify the presence of an additional gauge potential, we write down the θ-expansion for V and Γ_{--}:

$$V(x;\theta,\bar{\theta}) = C + \theta\zeta - \bar{\theta}\,\bar{\zeta} + \theta\bar{\theta}\,v_{++} \tag{15}$$

and

$$\Gamma_{--}(x;\theta,\bar{\theta}) = (A_{--} + iB_{--}) + i\theta\,(\rho + i\eta) + i\bar{\theta}\,(\chi + i\omega) + \theta\bar{\theta}\,(M + iN). \tag{16}$$

A_{--}, B_{--} and v_{++} are the light-cone components of the gauge potential fields; ρ, η, χ and ω are left-handed Majorana spinors; M, N and C are real scalars and ζ is a complex right-handed spinor.

The gauge transformations of the component fields above can be found in ref. [6] and they suggest that the component v_{++} should be identified as the light-cone partner of A_{--},

$$v_{++} \equiv A_{++}, \tag{17}$$

so that we end up with two component-field gauge potentials: $A^{\mu} \equiv (A^{0};A^{1})$ and $B_{--}(x)$.

To write down the local version of the isometry transformations (8), we have to replace the global parameter λ^{α} by a pair of *chiral* and *antichiral* superfields, $\Lambda^{\alpha}(x;\theta,\bar{\theta})$ and $\bar{\Lambda}^{\alpha}(x;\theta,\bar{\theta})$, by virtue of the constraints satisfied by Φ and $\bar{\Phi}$. This can be realised according to:

$$\Phi'^{i} = \exp\left(L_{\Lambda.k}\right)\Phi^{i} \tag{18a}$$

and

$$\bar{\Phi}'_{i} = \exp\left(L_{\bar{\Lambda}.\bar{k}}\right)\bar{\Phi}_{i}. \tag{18b}$$

In order to get closer to the case of global transformations, and to express all gauge variations exclusively in terms of the superfield parameters Λ^{α}, we propose a field redefinition that consists in replacing $\bar{\Phi}$ by a new superfield, $\tilde{\Phi}$, as it follows:

$$\tilde{\Phi}_{i} \equiv \exp\left(iL_{V.\bar{k}}\right)\bar{\Phi}_{i}. \tag{19}$$

From the expression for the gauge transformation of the prepotential V, it can be shown that:

$$\exp(iL_{V'.\bar{k}}) = \exp(L_{\Lambda.\bar{k}})\,\exp(iL_{V.\bar{k}})\,\exp(-L_{\bar{\Lambda}.\bar{k}}), \tag{20}$$

and $\tilde{\Phi}_{i}$ consequently transforms with the gauge parameter Λ^{α}:

$$\tilde{\Phi}'_{i} = \exp(L_{\Lambda.\bar{k}})\,\tilde{\Phi}_{i}\,, \tag{21}$$

which infinitesimally reads:

$$\delta\tilde{\Phi}_{i} = \Lambda^{\alpha}(x;\theta,\bar{\theta})\;\bar{k}_{\alpha i}(\tilde{\Phi}). \tag{22}$$

Now, an infinitesimal isometry transformation induces on the modified Kähler potential, $K(\Phi,\tilde{\Phi})$, a variation given by:

$$\delta K(\Phi,\tilde{\Phi}) = \Lambda^{\alpha}(\eta_{\alpha} + \tilde{\eta}_{\alpha}), \tag{23}$$

where

$$\tilde{\eta}_{\alpha} = (\tilde{\partial}^{i}K)\,\bar{k}_{\alpha i}(\tilde{\Phi}) + iM_{\alpha}(\Phi,\tilde{\Phi}), \tag{24}$$

with $\tilde{\partial}$ denoting a partial derivative with respect to $\tilde{\Phi}$. The isometry variation δK computed above reads just like a Kähler transformation and this is a direct consequence of the existence of the real scalar $M_{\alpha}(\Phi,\tilde{\Phi})$, as discussed in refs. [9,10].

The form of the isometry variation of $K(\Phi,\tilde{\Phi})$ suggests the introduction of a pair of *chiral* and *antichiral* superfields, $\xi(\Phi)$ and $\bar{\xi}(\bar{\Phi})$, whose respective gauge transformations be such that they compensate the change of K under isometries. This can be achieved by means of the Lagrangian defined as:

$$\begin{aligned}\mathscr{L}_{\xi} = {} & \partial_{i}[\,K(\Phi,\tilde{\Phi}) - \xi(\Phi) - \tilde{\xi}(\tilde{\Phi})]\ \nabla_{--}\Phi^{i} + \\ & - \tilde{\partial}_{i}[\,K(\Phi,\tilde{\Phi}) - \xi(\Phi) - \tilde{\xi}(\tilde{\Phi})]\ \nabla_{--}\tilde{\Phi}^{i}\ ,\end{aligned} \tag{25}$$

where the covariant derivatives $\nabla_{--}\Phi^{i}$ and $\nabla_{--}\tilde{\Phi}^{i}$ are defined in perfect analogy to what is done in the case of the bosonic σ-model:

$$\nabla_{--}\Phi^{i} \equiv \partial_{--}\Phi^{i} - g\Gamma^{\alpha}_{--}k^{i}_{\alpha} \tag{26a}$$

and

$$\nabla_{--}\tilde{\Phi}_{i} \equiv \partial_{--}\tilde{\Phi}_{i} - g\Gamma^{\alpha}_{--}\bar{k}_{\alpha i}(\tilde{\Phi}). \tag{26b}$$

Finally, all we have to do in order that the Lagrangian $\mathscr{L}_{\xi}$ given above be invariant under local isometries is to fix the gauge variations of the auxiliary scalar superfields ξ and $\tilde{\xi}$.

If the latter are so chosen that:

$$(\partial_i \xi) k^i_\alpha(\Phi) = \eta_\alpha(\Phi) \tag{27a}$$

and

$$(\tilde{\partial}^i \tilde{\xi}) \bar{k}_{\alpha i}(\tilde{\Phi}) = \tilde{\eta}_\alpha(\tilde{\Phi}), \tag{27b}$$

then, it can be readily verified that Kähler-transformed potential $[K(\Phi,\bar{\Phi}) - \xi(\Phi) - \tilde{\xi}(\tilde{\Phi})]$ is an isometry-invariant and the Lagrangian $\mathcal{L}_\xi$ of eq. (25) is indeed symmetric under the gauged isometry group.

The interesting point we would like to stress is that the extra gauge degrees of freedom accomodated in the component-field $B_{--}(x)$ of the superconnection Γ_{--} *do not decouple* from the scalars Φ and $\tilde{\Phi}$, unlike the case of the minimal coupling of the (2,0)-Yang-Mills supermultiplets to the (2,0)-matter superfields [6]. Hence, thanks to the coupling to a σ-model, through the gauging of the isometry group of the target manifold, we are able to present a situation where the extra gauge potential of (2,0)-supersymmetry can acquire a dynamical significance, as previously promissed.

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