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PHYSICAL PROPERTIES OF THE CHIRAL QUANTUM BARYON

by

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ABSTRACT

We present a comprehensive account of the development of the understanding of the quantum chiral baryon proposed recently. It is a stable chiral soliton with baryon number one obtained after first quantization through collective coordinates. Starting from the exact series solution to the non-linear sigma model with the hedgehog configuration, we calculated the values of several physical quantities (mass, axial weak coupling, gyromagnetic ratios and radii) as a function of the order of Padé approximants used as approximate representations of the solution. It turns out that consistent results may be obtained, but a better approximation should be developed.

Key-words: Chiral soliton; Non-linear sigma model; Baryonic properties; Skyrme model.

1 INTRODUCTION

The correct representation for the nucleon states is a long standing problem in elementary particle physics. By this we mean an understanding of the physical properties of the nucleon starting from adequate first principles, like the existence of quarks and the flavour symmetry as displayed through current algebra.

An important step forward seems the recall of the pioneering work by Skyrme(1) done by Pak and Tze(2), Balachandran and the group at Syracuse University(3) and Witten(4). It was soon recognized the soundness of this approach and it received a lot of attention afterwards. Applied to the description of the nucleon(5) and to pion nucleon scattering(6) it was relatively successful.

In short, the Skyrme approach in its modern version consists on the "hedgehog" configuration as input for the non-linear sigma model lagrangean to which is added a special appropriate stabilizing term including and "ad hoc" dimensionless parameter. This classical stable solution, up to now mainly treated approximately or numerically, is used in applications after introducing quantization through collective coordinates. Results for physical quantities are then expressed in terms of the two parameters of the theory, the pion decay constant and the Skyrme parameter whose values are obtained using as inputs the masses of the nucleon and $\Delta(1232)$ resonance(5).

We discovered recently(7) an important feature of the hedgehog radial equation in the classical non-linear sigma model: the exact solution is dependent on a completely undetermined parameter, with dimension of mass. This is peculiar of ordinary differential equations when the Lipshitz condition for uniqueness of the solution is not fulfilled[8]. This point has not been given enough weight in the previous literature, and even in some recent one. More important, this characteristic persists when the Skyrme term is added[9], since it is as singular at the origin as the sigma model term. It is not apparent to us by now how this feature was included in most of previous numerical work. In the works by Balachandran et al.[3], an equivalent "profile" or "shape" parameter was introduced in the ansatz proposed for the solution of the Skyrme lagrangean.

Two important consequences follow from this nonuniqueness of the exact classical hedgehog solution. First, the instability problem of the classical soliton mass can be completely phrased in terms of the parameter. Second, as we discovered later on[10] (and contemporarily, was proposed from approximate solutions by Jain, Schechter and Sorkin[11] of Syracuse University), the quantum hamiltonian obtained from the rotating soliton resulting from the introduction of collective coordinates presents a stable minimum. We find appropriate to christen this solution as the chiral quantum baryon. As shown in our previous work, encouraging results for the mass and the weak axial coupling constant g_A are obtained as we approximated the exact solution for the hedgehog by means of appropriate Padé approximants.

In the present work we elaborate on details of the proposal and numerical treatment of the chiral quantum baryon (from now on, abbreviated as χQB) and present results for its physical parameters, like mass, g_A, the isoscalar and isovector gyromagnetic factors and the corresponding radii using higher Padé approximants. These results are given as functions of the order of Padé approximants that we take as good approximate representations of the exact chiral soliton. By the way, we analyze and exhibit some problems on the consistency of the current treatment of the problem.

In section 2 we expand considerations on the treatment of the problem at the classical level, with special emphasis on the properties of the solution. We consider the quantum hamiltonian and the solution at the minimum in section 3. We show that the quantum stable solution, χQB, is different qualitatively from the usual Skyrme classically stabilized solution, and comment on the quantum minimum for the theory including a Skyrme term.

In section 4 we introduce the approximation by suitable Padé approximants of the solution, and discuss several features which turn to be important in the following.

Section 5 presents the application of the former results to the evaluation of physical properties. We are led to relations which , while being consistent in the calculation with the non linear sigma model lagrangean, seem not to be fulfilled by the correct (physical) solution describing a baryon.

Section 6 is devoted to a discussion of all the subjects considered previously, and to suggest patterns for further research. We emphasize the difficulty in representing the short and long distance regimes for the exact solution, and how this translates into our approximate solutions.

We detail in appendix A a calculation of the second functional derivative for the lagrangean of the full model with a Skyrme term. In any case, it is of indefinite sign in the pure sigma model and with the Skyrme term.

2 THE EXACT SOLUTION FOR THE HEDGEHOG IN SU(2) NON-LINEAR SIGMA MODEL AT THE CLASSICAL LEVEL

The starting point will be the non-linear sigma model classical lagrangean:

$$\mathcal{L} = -\frac{1}{2} f_\pi^2 \int d^3x \quad \mathrm{Tr}\left[\sum_{k=1}^{3} (\partial_k U^\dagger)(\partial_k U)\right] \tag{1}$$

where U is a unitary operator

$$UU^\dagger = U^\dagger U = 1$$

and

$$\partial_k = \frac{\partial}{\partial x^k}$$

In the context of applications of (1) to hadronic physics as an effective lagrangean, f_π is the pion decay constant. We shall a dopt this view to begin with, pointing also to its need for (1) to have the required dimension.

We shall be concerned with a candidate solution for (1), the SU(2) hedgehog configuration

$$U = U_0 = \exp\,(i\ \vec{\tau}.\vec{n}\ F(r)) \tag{2}$$

where:

$$\tau_k \ (k=1,2,3) \text{ are the Pauli matrices;} \tag{2a}$$

$$r^2 = \sum_{k=1}^{3} (x^k)^2 \tag{2b}$$

$$\vec{n} = \vec{r}/|r| \tag{2c}$$

Substituting (2) in (1) and minimizing to obtain the classical Euler-Lagrange equation, we get:

$$\frac{d^2F(r)}{dr^2} + \frac{2}{r}\,\frac{dF(r)}{dr} = \sin 2F(r) \tag{3}$$

To eliminate the first derivative, we put

$$F(r) = \frac{\chi(r)}{r} \tag{4}$$

and changing the independent variable

$$x = \frac{r}{2} \tag{5}$$

we arrive finally at

$$\frac{d^2\chi(x)}{dx^2} = \frac{2}{x}\sin\left(\frac{\chi(x)}{x}\right) \tag{6}$$

In order to solve (6), we try an expansion in power series:

$$\chi(x) = \chi(0) + \chi'(0)x + \frac{1}{2!}\chi''(0)x^2 + \frac{1}{3!}\chi'''(0)x^3 + \ldots + \frac{1}{n!}\chi^{(n)}(0)x^n + \ldots \tag{7}$$

Comparing both sides of (6), we need then:

$$\chi(0) = 0 \quad , \tag{8a}$$

$$\chi'(0) = 0 \ , \ 2n_0\pi \ , \ (n_0 \in \mathbb{Z}) \tag{8b}$$

and find:

$$\chi''(0): \text{ undetermined} \tag{8c}$$

$$\chi^{(2n+1)}(0) = 0 \quad , \quad n \in Z \tag{8d}$$

Notice that $\chi''(0)$ has dimension of mass (or(length)$^{-1}$).

The even derivatives are all expressed in terms of powers of the undetermined second derivative. We obtain:

$$\chi^{IV}(0) = -\frac{1}{10}\chi''(0)^3 \tag{9a}$$

$$\chi^{VI}(0) = \frac{3^2}{2^5 \times 7}\chi''(0)^5 \tag{9b}$$

$$\chi^{VIII}(0) = -\frac{17}{2^5 \times 3 \times 5}\chi''(0)^7 \tag{9c}$$

$$\chi^{X}(0) = \frac{3 \times 7 \times 73}{2^9 \times 5 \times 11}\chi''(0)^9 \tag{9d}$$

and so on. We may rearrange terms, and finally find that we can write:

$$\chi(x) = 2n_o\pi x + \frac{1}{2}\chi''(0)x^2 \; X((\chi''(0)x)^2) \tag{10}$$

with:

$$X((\chi''(0)x)^2) = X(s^2) = \sum_{n=0}^{\infty} f_n \, s^{2n} \tag{11}$$

The coefficients f_n are given by:

$$f_o = 1 \tag{12a}$$

$$f_1 = -\frac{1}{120} \tag{12b}$$

$$f_2 = \frac{1}{8960} \tag{12c}$$

$$f_3 = -\frac{17}{9\,676\,800} \tag{12d}$$

$$f_4 = \frac{73}{2\,433\,024\,000} \tag{12e}$$

$$f_5 = -\frac{3337}{6\,199\,345\,152\,000} \tag{12f}$$

In terms of the chiral angle, we have:

$$F(r=0) = n_0\pi \tag{13a}$$

$$F'(r=0) = \frac{1}{8}\chi''(r=0) \tag{13b}$$

$$F'(x=0) = \frac{1}{4}\chi''(x=0) \tag{13c}$$

and:

$$F(s) = n_0\pi + \frac{s}{4}X(s)\ , \qquad n_0 \in \mathbb{Z} \tag{14}$$

This is a most important result. The chiral angle appears naturally as a function of a dimensionless variable. The consequences of this fact constitute the main content of this article.

Some comments on these expressions. The series for $X(s)$ is of alternating signs, and with coefficients having a ratio of a hundred approximately. If some structure results from it we must expect that $s\sim 10$, which in turn means $\chi''(0)$ of the order of 10GeV. Also to make the differential equation regular, the value at the origin must be a multiple of π (see Eqs. (8a),(8b)). This we shall relate soon to the value of the baryon number.

Notice that what we have is a set of EXACT SOLUTIONS to the chiral non-linear sigma model lagrangean not known until we discovered them(7). Many of the features of these solutions survive the addition of a Skyrme term(9):

$$\mathcal{L}_{SK} = -\frac{1}{32e^2}\int d^3x \; \mathrm{Tr}[U^\dagger(\partial_k U), U^\dagger(\partial_\ell U)]^2 \tag{15}$$

In particular the undeterminacy of the second derivative at the origin persists, since this term is not more singular than the original non-linear sigma model term. This makes us inquire about the nature of the numerical solution found in the literature(5,6), since apparently they do not take care of the undeterminacy.

The next step concerns the behaviour at infinity. First, we change the independent variable in Eq. (6) such as:

$$y = 1/x = \frac{2}{r} \tag{16}$$

and the dependent variable into:

$$\chi(x) = \frac{K(y)}{y} = \frac{r}{2}\,K\left(\frac{2}{r}\right) = rF(r) \tag{17}$$

The differential equation is now:

$$\frac{d^2K(y)}{dy^2} = \frac{2}{y^2}\sin K(y) \tag{18}$$

Again, we try a solution by power series:

$$K(y) = K(0) + K'(0)y + \frac{1}{2!}K''(0)y^2 + \ldots + \frac{1}{n!}K^{(n)}(0)y^n + \ldots \tag{19}$$

and find:

$$K(y) = 2n_\infty\pi + \frac{1}{2!}\,K''(0)y^2 + \frac{1}{6!}\left(-\frac{15}{14}\right)K''(0)^3y^6 +$$

$$+\frac{1}{10!}\left(\frac{5\times 3^4}{11}\right)K''(0)^5y^{10} + \ldots,\quad n_\infty \in \mathbb{Z} \tag{20}$$

Again, the consistency of Eq. (18) imposes the value for $K(0)$ to be an integer multiple of π. Notice that a new undetermined parameter, $K''(0)$, with dimension $(\text{length})^2$, appears. This is consistent with Eq. (8c), or, in other terms, it is to be expected to express $K''(0)$ in terms of $\chi''(0)^{-2}$. That is, a unique solution for any value of $\chi''(0)$ (or $K''(0)$) should be allowed. At the classical level, however, there is no preferred value for the undetermined parameter.

The addition of the Skyrme term is accomplished in a very different spirit. It is thought as a device to provide a stable suitable minimum at the classical level introducing a dimensionless parameter. However in the work by Balachandran et al.[3] the need for the introduction of an additional "shape" or "profile" dimensional parameter was acknowledged. It opened the road to the subsequent development by Jain, Schechter and Sorkin[11]. The point, however, is that the introduction of this (dimensional) parameter in the hedgehog stems from the original non-linear sigma model lagragean, as we showed in our previous work[7,10]. Is the prize to be paid for the Euler-Lagrange equation of the model to be consistent.

We anticipate here and shall show below that this property persists in the full Skyrme lagrangean, though hindered by the introduction of the Skyrme parameter. This gives a new meaning to the numerical fit of the asymptotic behaviour of the solution to provide the value of the weak axial coupling as done by Adkins, Nappi and Witten[5] (from now on, referred as ANW).

It can be inferred from Eqs. (17) that $K(y)$ (or $F(r)$) may be written in terms of another appropriate dimensionless variable,

$$K(\sigma) = 2n_\infty\pi + \frac{1}{2}\,\sigma Y(\sigma) \quad , \quad n_\infty \in \mathbb{Z} \tag{21}$$

$$\sigma = K''(0)\,y^2 = K''(0)\,\chi''(0)^2 s^{-2} \tag{22}$$

and we have:

$$Y(\sigma) = \sum_{n=0}^{\infty} \eta_n \sigma^{2n} \tag{23}$$

The first coefficients are

$$\eta_0 = 1$$

$$\eta_1 = -\frac{1}{2^4 x 3 x 7} = -\frac{1}{336}$$

$$\eta_2 = \frac{1}{2^7 x 5 x 7 x 11} = \frac{1}{49280}$$

$$\eta_3 = -\frac{1}{2^8 x 3^2 x 5 x 7^2 x 11} = -\frac{1}{6\ 209\ 280}$$

To describe baryon states a connection is needed between the numbers n_0 in (10) and n_∞ in (20). This results from the value of the topological current whose associated charge is usually related to the baryon number. That is:

$$J_\mu = \frac{1}{24\pi^2}\,\epsilon_{\mu\nu\rho\lambda}\,\mathrm{Tr}[U^\dagger(\partial_\nu U)U^\dagger(\partial_\rho U)U^\dagger(\partial_\lambda U)] \tag{24}$$

The conserved charge is:

$$B = \int J_0 d^3x = \frac{2}{\pi}\int_0^\infty dr' r'^2 \frac{dF}{dr'}\frac{\sin^2 F}{r'^2}$$

$$= \frac{1}{\pi}\,[F(\infty) - F(0)] \tag{25}$$

We see that the baryon number one corresponds to:

$$n_\infty - n_0 = 1$$

It is usual to put $n_0 = -1$, $n_\infty = 0$. With these conditions, we need to have, as a result, from Eqs. (14) and (21)

$$X(s) \xrightarrow[s\to\infty]{} 4\pi/s \tag{26a}$$

and

$$Y(\sigma) \xrightarrow[\sigma\to\infty]{} -4\pi/\sigma \tag{26b}$$

We are now able to express the stability condition of the soliton solution to the non-linear sigma model in the hedgehog approximation at the classical level in terms of $\chi''(0)$ or $K''(0)$. The mass of the soliton is given by:

$$M_0 = 4\pi\, f_\pi^2 \int_0^\infty dr' \left[r'^2\left(\frac{dF(r')}{dr'}\right)^2 + 2\sin^2 F(r')\right] \tag{27}$$

Using now the expressions (14) or (21) for the chiral angle, F(r):

$$F(r) = F(s) = n_0\pi + \frac{s}{4}\,X(s) \quad , \quad n_0 \in \mathbb{Z}, \quad s = \frac{r}{2}\,\chi''(0) \tag{28a}$$

$$= F(\sigma) = n_\infty \pi + \frac{1}{4}\,\sigma Y(\sigma) \quad , \quad n_\infty \in \mathbb{Z} \quad , \quad \sigma = K''(0)\,\chi''(0)^2 s^{-2} \tag{28b}$$

we substitute in (27) and have:

$$M_0 = 2\pi f_\pi^2 \frac{1}{\chi''(0)} \int_0^\infty ds' \left[\frac{1}{4} s'^2 \left(\frac{d}{ds'}[s'X(s')]\right)^2 + 8\sin^2\left(\frac{s'}{4} X(s')\right) \right] \tag{29a}$$

$$= 8\pi f_\pi^2 K''(0)^{1/2} \int_0^\infty d\sigma' \left[\sigma'^{1/2} \left[\frac{d}{d\sigma'}(\sigma' Y(\sigma'))\right]^2 + \frac{1}{2\sigma'^{3/2}} \sin^2\left(\frac{1}{4}\sigma' Y(\sigma')\right) \right] \tag{29b}$$

The problem of the stability of the classical soliton solution is translated in terms of the dimensional parameters $\chi''(0)$ or $K''(0)$, which are completely undetermined because of the singularities of the differential equations (Eqs. (6) and (18))). As remarked to us by R. Méndez and J.E. Stephany Ruiz, one can interpret the instability of the classical hedgehog soliton as a consequence of the fact that the solutions for the chiral angle, as evidenced by Eqs. (14) and (21) are scale invariant[9]. It is this crucial property precisely that is important for the stability at the quantum level.

Let us now analyze the addition of the Skyrme term. If the number e is assigned the value e=1, we see that, since F is a dimensionless variable, it may depend on r only through a combination kr, where k has dimension $(\text{length})^{-1}$. If this is so, we have that the Skyrme term is of the form:

$$k \times \text{a number}$$

Let us look more carefully. The expression for the Skyrme model lagrangean, with a hedgehog for the unitary field, is:

$$L = -4\pi f_\pi^2 \int_0^\infty dr' \left\{ r'^2 \left(\frac{dF}{dr'}\right)^2 + 2\sin^2 F + \frac{8}{e^2 f_\pi^2} \sin^2 F \left(\frac{dF}{dr'}\right)^2 + \right.$$
$$\left. + \frac{4}{e^2 f_\pi^2} \frac{\sin^4 F}{r'^2} \right\} \tag{30}$$

The Euler Lagrange equation is:

$$\left(\frac{1}{4} r^2 + \frac{2}{e^2 f_\pi^2} \sin^2 F\right) \frac{d^2 F}{dr^2} + \frac{1}{2} r \frac{dF}{dr} + \frac{1}{e^2 f_\pi^2} \sin 2F \left(\frac{dF}{dr}\right)^2 - \frac{1}{4} \sin 2F$$
$$- \frac{1}{e^2 f_\pi^2} \frac{1}{r^2} \sin^2 F \sin 2F = 0 \tag{31}$$

If we try a series expansion around $r=0$:

$$F(r) = F_0 + F_1 r + \frac{1}{2!} F_2 r^2 + \frac{1}{3!} F_3 r^3 + \ldots \tag{32}$$

one finds, order by order[12]:

$$0(r^0):\ F_0 = n_0 \pi$$
$$0(r^1):\ \frac{1}{2} F_1 - \frac{1}{4} 2F_1 + \frac{1}{e^2 f_\pi^2} (F_1^2\, 2F_1 - 2F_1^3) = 0 \tag{33}$$

Notice that F_1 is, again, undetermined (see Eq.(13b)e(13c)); and, curiously, that the terms with $1/e^2$ cancel separately. Continuing in this way, we find

$$F_{2n} = 0 \qquad (n \in \mathbb{Z})$$

$$F_3 = -\frac{4}{5} F_1^3 \frac{1+2\phi^2}{1+8\phi^2}$$

$$F_5 = \frac{24}{7} F_1^5 \frac{1+\frac{32}{5}\phi^2 + \frac{88}{5}\phi^4 + \frac{448}{5}\phi^6}{1+24\phi^2 + 192\phi^4 + 512\phi^6},$$

....

$$\phi = \frac{F_1}{e f_\pi}$$

It is easy to see the relations of the original non-linear sigma model with corrections order $1/e^2$.

The moral of the story is that the Skyrme term is again scale invariant and so the undetermined parameter, namely F_1, takes the same meaning as before.

Even more curious is the fact that around infinity, is the non-linear sigma model that commands again the solution, the contribution of the Skyrme term giving the 8^{th}, 12^{th}, 16^{th}... derivatives, absent in the pure sigma model.

Thus, we can write, in terms of the dimensionless variable $\eta = F_1 . r$, for the mass of the classical soliton with a Skyrme term:

$$M_{o,sk} = \frac{1}{2}\frac{1}{F_1} \cdot 2\pi f_\pi^2\, a + \frac{F_1}{e^2 f_\pi^2}\, 2\pi f_\pi^2 c \tag{34}$$

$$a = 4\int_0^\infty d\eta' \left[\eta'^2 \left(\frac{dF}{d\eta'}\right)^2 + 2\sin^2 F(\eta')\right] \tag{35a}$$

$$c = 8\int_0^\infty d\eta' \left[2\sin^2 F(\eta') F'(\eta')^2 + \frac{\sin^4 F(\eta')}{\eta'^2}\right] \tag{35b}$$

The minimum (at the classical level) is located at

$$\frac{F_1}{ef_\pi} = \bar{\phi} = \frac{\left[\frac{1}{2}\, a(\bar{\phi}) + \bar{\phi}^2 c(\bar{\phi})\right]}{\frac{d}{d\phi}\left[\frac{1}{2} a(\bar{\phi}) + \bar{\phi}^2 c(\bar{\phi})\right]} \tag{36}$$

The role of e, the Skyrme parameter, is that of a fixed number taken to obtain a right value for the minimum in the variable F_1. In other words, it is there just to provide a stable classical solution, and it is determined to provide some needed result.

To summarize this section: we have exhibited in detail the features of the possible soliton solution to the hedgehog configuration of the SU(2) non-linear sigma model. In particular we have emphasized the need for a completely undetermined dimensional parameter for the differential equation to make sense. By the way, this also fixes the possible values of the chiral angle at the origin and infinity, which in turn determine the value of the baryon charge. In terms of this new parameter is possible to understand the instability of the classical soliton in the non-linear sigma model.

We have also sketched that the full Skyrme model also shares these features, being in some sense a natural extension of the non-linear sigma model, since it is also scale invariant. It is, however, classically stable.

An intriguing fact is that the stability of the Euler Lagrange equation for both cases is not guaranteed. As shown in Appendix A, the second functional derivative of the classical lagrangean is of not definite sign.

3 SEMICLASSICAL QUANTIZATION. THE CHIRAL QUANTUM BARYON

The problem of the correct introduction of quantization for solitons deserves considerable attention [13]. In the realm of the study of baryons as solitons, the current procedure is rather heuristic and limited in scope. It only pretends an approximate description for the lower energy configurations, and it would be urgent to devise a more formal justification for it.

The starting point is to recognize that all configurations obtained from the hedgehog by an isospin rotation are of the same fi nite energy. It can be expected that for the lower energy states, flavour rotation generate modes that approximate them. The flavour rotations are then introduced as collective, time dependent, coordinates, and are quantized applying standard procedures. In more concrete form, in the original lagrangean (1), we replace U_o by

$$U_o(r,t) = A(t)U_o(r)A^{\dagger}(t) \quad , \; A(t) \in SU(2) \tag{38}$$

$$= \cos F(r) + i\tau_i D_{ik}(t) n_k \sin F(r) \tag{39}$$

The final result of introducing $U_o(r,t)$ in Eq. (1) is

$$L = - M_o + \theta \, \mathrm{Tr}[\partial_o A \, \partial_o A^{-1}] \tag{40}$$

$$\theta = \frac{16}{3} \pi f_\pi^2 \int_0^\infty dr' r'^2 \sin^2 F(r') \tag{41}$$

and the quantization of such a system is well known. We refer the reader to the abundant literature on the subject[3,4,14,15] to save space, and recall two results. The first is that the states of the quantum hamiltonian for the SU(2) hedgehog should be labelled by the same eigenvalue of the angular momentum, $\vec{J}^2$, and of isospin, $\vec{I}^2$:

$$\vec{J}^2 = \vec{I}^2 \tag{42}$$

The second is that the hamiltonian is the one of a rigid rotator, and the energies are given by:

$$E = M_0 + \frac{\vec{J}^2}{2\theta} \tag{43}$$

Notice that both M_0 and θ are functionals of the chiral angle in the classical description. For quantization of the soliton as a fermion the eigenvalues of the angular momentum should be half-integers.

What was done[5] with Eq. (43) for the Skyrme model was to fit the masses of the nucleon and the $\Delta(1232)$, and get values for the parameters e and f_π.

What we do, instead[10], is to take profit of the exact solution for the hedgehog soliton, Eqs. (14) or (21), and write θ as follows.

$$\theta = 2\pi f_\pi^2 \frac{1}{\chi''(0)^3} \int_0^\infty ds' \, \frac{64}{3} \, s'^2 \sin^2 F\left(\frac{s'}{4}\, X(s')\right) \tag{44}$$

$$= 2\pi f_\pi^2 \frac{1}{\chi''(0)^3}\, b$$

In terms of this, and introducing the integral a from Eq. (35) we have:

$$E = 2\pi f_\pi^2 \frac{1}{\chi''(0)} a + \frac{1}{2} \vec{J}^2 \frac{1}{2\pi f_\pi^2 b} \chi''(0)^3 \tag{45}$$

Notice that a and b are uniquely given as numbers once the exact solution for the classical hedgehog soliton is known. Clearly Eq. (45) has a minimum as a function of the parameter $\chi''(0)$ (or Eq. (43) as a functional of the classical chiral angle). We get:

$$\chi_0'' = f_\pi \left[\frac{2}{3} \frac{(2\pi)^2}{\vec{J}^2}\, ab\right]^{1/4} \tag{46}$$

We have for the quantum semiclassical energy of a baryon from the classical hedgehog exact solution, the chiral quantum baryon:

$$E_0 = f_\pi \frac{4}{3} \sqrt{2\pi} \left(\frac{3}{2} \vec{J}^2 \frac{a^3}{b}\right)^{1/4} \tag{47}$$

Notice that, were $\vec{J}^2 = 0$ allowed, the mass of the state should be null. It is easy to see that it is a minimum. The value of the second derivative of Eq. (45) with respect to $\chi''(0)$ is positive:

$$\left.\frac{d^2E}{d\chi''(0)^2}\right|_{\chi''(0)=\chi''_0} = \frac{\sqrt{2\pi}}{f_\pi} \frac{3}{2} \left(\frac{2}{3} \frac{a}{b^3} \frac{1}{\vec{J}^2}\right)^{1/4} \left(\frac{\vec{J}^2}{\pi} + 1\right) \geq 0 \tag{48}$$

Of course, χ''_0 depends on $\vec{J}^2$, as E_0 does. An immediate prediction is the ratio of the masses for the states 3/2 and 1/2. That is:

$$\frac{E_0(3/2)}{E_0(1/2)} = (5)^{1/4} \cong 1,4953 \tag{49}$$

For a comparison,

$$\frac{M(\Delta(1232))}{M(N(938))} = 1,313$$

It is interesting to have an estimate for the quantities appearing in Eq.(45). In Nature, provided it is applicable to nucleons and deltas, Eq. (43) for the Skyrme model gives:

$$M(\Delta(1232)) - M(N(938)) = 294\text{MeV}/c^2 = \frac{3}{2}\frac{1}{\theta}$$

That is:

$$\theta \simeq \frac{1}{196} (\text{MeV}/c^2)^{-1}$$

With this, it turns out that

$$M_o \approx 865 \text{ MeV}/c^2$$

The contribution of the rotation for the nucleon is of 73,5 MeV/c^2 approximately. The ratio of the contribution of the rotation part to the "inertial" part is:

$$\frac{\frac{3}{4}\frac{1}{2\theta}}{M_o} = 0{,}085$$

For the $\Delta(1232)$ baryon, the ratio is 0,425.

We can perform the same estimate for the χQB, using the result of Eq. (46):

$$\frac{\frac{1}{2}\,\vec{J}^2\,\frac{1}{2\pi f_\pi^2 b}\,\chi_o''^{\,3}}{2\pi f_\pi^2\,\frac{1}{\chi_o''}\,a} = \frac{1}{8\pi^2}\,\vec{J}^2\,\frac{\chi_o''^{\,4}}{ab} = \frac{1}{3} \tag{50}$$

We see that the χQB in any case rotates rather fast, and somehow it contradicts the hope that we are dealing with a slowly rotating soliton. It should be required the addition of strangeness and of pion degrees of freedom to find whether some slowing down is possible.

Before coming to numerical analysis of the χQB, we devote next section to consider the approximation of the exact solution for the chiral angle.

4 APPROXIMATION OF THE SOLUTION FOR THE CHIRAL ANGLE FOR THE χQB

We have to devise an approximate solution for the chiral angle starting from the series developments at the origin and infinity. This (Eqs. (6) and (18)) could be done by means of the standard procedures of analytic continuation. Starting from both ends of the positive real axis, one could develop successively at nearby points until both solutions overlap somewhere.

Instead, we have proposed to use the series at the origin for the construction of Padé approximants[16]. In short, an [N,M] Padé approximant to a function f is a rational function made up of polynomials of degree N and M at the denominator and numerator respectively:

$$f[N,M](x) = \frac{n_0+n_1 x+ \ldots + n_M x^M}{1+ d_1 x+ \ldots +d_N x^N} \tag{51}$$

where the coefficients n_k, d_k are determined by the condition that the approximant reproduces the first N+M coefficients of the series expansion for f:

$$f(x) - f[N,M](x) = 0 \ (x^{N+M+1}) \tag{52}$$

Notice that our previous notation[10] differs a little from the standard one.

We have made use of the series expansion for X(s) (Eq.(11)). It should appear natural to consider the Padé approximants in the variable s^2. Since we intend to represent the function in the whole positive real axis, we are forced by the condition on X(s) at infi nity (Eq. (26a)) to use all powers of s. Moreover only approximants of the kind $[N+1,N](s)$ should be envisaged as they naturally have the correct asymptotic behaviour.

However, since the chiral angle should be a function of definite sign, we can easily convince ourselves that the only admissible approximants are of the kind $[2K,2K-1](s)$ $(K=1,2,\ldots)$. This comes out because from Eq. (21) we know that $F(s) \underset{s\to\infty}{\sim} \frac{1}{s^2}$, it means that the coefficient of the term s^{-2} in $X(s)$ should be zero; hence, at infinity F(s) has a zero. Now, by the fundamental theorem of algebra a polynomial of order K had K real roots at most. Then, it has K changes of sign at most (or an even or odd number, modulo 2). We need then an odd number of changes of sign on the negative real axis; eliminating the zero at infinity this leaves only odd powers for the numerator and even numbers for the denominator.

Let us be precise. Notice that the asymptotic condition on X(s), Eq. (26a) imposes:

$$X[2K,2K-1](s) \xrightarrow[s\to\infty]{} \frac{n_{2K-1}}{d_{2K}\, s} = \frac{4\pi}{s}$$

and we have:

$$n_{2K-1} = 4\pi\, d_{2K} \tag{53}$$

We must now ask that at infinity F(s) goes as $\frac{1}{s^2}$. Let us pass to the variable $t = 1/s$.

$$X[2K,2K-1](t) \cong 4\pi t + O(t^3) \tag{54}$$

Which in turn means:

$$\frac{n_{2K-2}}{n_{2K-1}} = \frac{d_{2K-1}}{d_{2K}} \tag{55}$$

or, since Eq. (53) is satisfied, an analogous relation should hold for the next to leading order coefficients in the numerator and denominator:

$$n_{2K-2} = 4\pi d_{2K-1} \tag{56}$$

Let us give simple examples. Take for X(s) just the first coefficient, 1, and look at the first Padé approximant.

$$X[2,1](s) = \frac{n_o+n_1 s}{1+d_1 s+d_2 s^2} \tag{57}$$

The relations (55) and (56) means:

$$n_1 = 4\pi\, d_2$$

$$n_o = 4\pi\, d_1$$

From the condition (52) we obtain

$$d_1 = \frac{1}{4\pi} \quad , \quad d_2 = \frac{d_1}{4\pi}$$

and

$$X[2,1](s) = \frac{1+\frac{s}{4\pi}}{1+\frac{s}{4\pi}+\left(\frac{s}{4\pi}\right)^2} \tag{58}$$

The coefficient of the term t^3 in Eq.(54) is related to the value of weak axial coupling constant, and in this case is:

$$c_1[2,1] = -\left[\frac{n_0}{n_1}\,\frac{d_1}{d_2} - \left(\frac{1}{d_2} - \left(\frac{d_1}{d_2}\right)^2\right)\right]$$

$$= -(4\pi)^2 \tag{59}$$

We have, for the chiral angle, near $t \approx 0$:

$$F(t) \cong -\pi(4\pi)^2 t^2 = -496{,}100\ t^2 \tag{60}$$

Let us now go into the next Padé approximant, [4,3],

$$X[4,3](s) = \frac{n_0+n_1 s+n_2 s^2+n_3 s^2}{1+d_1 s+d_2 s^2+d_3 s^2+d_4 s^4} \tag{61}$$

The corresponding truncated series expansion goes up to the power s^4. We then have from Eqs. (52), (53) and (56):

$$d_1 = \frac{17}{15}\,\frac{4\pi}{(3(4\pi)^2-136)} = \frac{17}{15}\,\frac{4\pi}{d}$$

$$d_2 = \frac{135(4\pi)^2 - 3808}{3360\ d}$$

$$d_3 = \frac{17}{1120}\,\frac{4\pi}{d}$$

$$d_4 = \frac{289}{50400}\,\frac{1}{d}$$

$$n_1 = d_1$$

$$n_2 = 4\pi d_3$$

$$n_3 = 4\pi d_4$$

For (54), we have now:

$$X[4,3](t=1/s) \cong 4\pi t - 4\pi c_1[4,3]t^3+4\pi c_2[4,3]t^4 + O(t^5) \tag{62}$$

with:

$$c_1[4,3] = \frac{d_2}{d_4} - \frac{d_1}{4\pi d_4} = 711{,}194 \tag{63a}$$

$$c_2[4,3] = \frac{1}{4\pi d_4} - \frac{d_1}{d_4} - \frac{d_1 d_3}{4\pi d_4^2} + \frac{d_2 d_3}{4\pi d_4^2} = -1965{,}3398 \tag{63b}$$

Notice that, from Eq.(20), the value of $c_2[4,3]$ should be zero, as is the corresponding derivative. Repeating the same steps for higher order Padé approximants for the series at the origin, it is easy to show that

$$c_1[2N,2N-1] = \frac{d_{2N-2}}{d_{2N}} - \frac{d_{2N-3}}{4\pi d_{2N}} \tag{64}$$

5 THE APPROXIMATE CALCULATION OF PHYSICAL PROPERTIES FOR THE χQB

Let us use now the framework of the chiral quantum baryon to get the values of several interesting physical quantities. This may help to figure how far it is from the correct description of a real baryon, and at the same time provide some insight about the formalism needed to describe a baryon and its internal consistency.

The main point is that for any quantity one arrives at an expression envolving as only dimensional parameters χ_0'' (see Eq.(10)) and f_π, and the rest are pure numbers, which only depend on the order of the Padé approximant used to approximate the solution for the non-linear classical sigma model lagrangean in the SU(2) hedgehog configuration. It is worth to notice that χ_0'' is dependent on the approximation as well.

Table I summarizes our results for the physical quantities we calculated. We shall now describe the calculation and comment on each column successively.

The first column contains the order of the approximant. It is not practical to go further than [12,11], which amounts to include (20) derivatives at the origin of which (10) are not zero. The coefficients become too small, and the problem of rounding errors is important. At the same time some numerical indications we discuss immediately don't push the need for improvement. The values for the chiral angle for different approximants are shown in Fig. 1.

The second column contains the value of the parameter χ_0'' at the minimum of the quantum energy, as given by Eq.(46) in units of f_π. Notice that it keeps growing steadily, though not as fast as in the first approximants. It is possible, analyzing the numerical values appearing in Eq. (46), and looking at the values of the coefficient $c_2[2K,2K-1]$ in the analogous of Eq. (63b), that this feature may come as a result of the inadequacy of our Padé approximants to represent the chiral angle near infinity (see Fig. 3). In fact, χ_0'' grows almost as the quantity b, defined in Eq. (44), that shows infinity certainly contribute heavily. In terms of $c_2[2K,2K-1]$, it turns out that this coefficient gets quite large with the order of the approximant. The next column expresses the parameter in units of GeV. The only remark to be made is that we use the value $f_\pi \doteq 67\text{MeV}$ and this is not clearly well understood. At what level of the dynamical input should f_π be taken as 67MeV?

The next two columns refer to the value of the quantum energy at the minimum for the state $J = I = \frac{1}{2}$. It turns out to be remarkably stable. Again, it happens that, though a in Eq. (45) increases much slower than b, its cubic power grows at almost the same rate with increasing order of the approximant. This is a nice feature, and stimulates to think that the χQB may be a good starting point for approximating a baryon. Again, if we were using the value of f_π from the fit by ANW we would have obtained more appealing figures for the mass of the state. Fig. 2 represents the mass of the state in terms of $\chi''(0)$.

The next column refers to the value of the weak axial coupling constant. In the χQB it results from the fact that the chiral angle goes like r^{-2} at infinity, with a coefficient known from the expansion of the Padé approximant used to approximate the exact solution (see Eqs. (54), (59), (60) and (62)).

$$F[2K,2K-1] = -\pi c_1[2K,2K-1]t^2 + O(t^3) \tag{65}$$

with

$$t = \frac{1}{s} = \frac{1}{\chi''(0)x} = \frac{2}{\chi''(0)r} \tag{66}$$

We have then for the hedgehog:

$$U_0 \mp 1 - \pi c_1[2K,2K-1]t^2\, \vec{\tau}.\vec{n} \tag{67}$$

Comparing with section 4, Eq. (33), of ANW.

$$B[2K,2K-1] = \frac{4\pi}{\chi_0''} c_1[2K,2K-1] \tag{68}$$

such that:

$$g_A = \frac{4}{3} \left(\frac{4\pi f_\pi}{\chi_0''}\right)^2 c_1[2K,2K-1] \tag{69}$$

Notice that there is a factor $2\sqrt{2}$ between the definitions of f_π in our work with respect to ANW [5]. It is also remarkable that this number independent of the choice of value for f_π is quite stable for higher approximants.

The remaining columns in Table I contain the results for electromagnetic and isotopic parameters of the χQB. The last, we shall show results in a pure number.

The isoscalar mean square radius is defined as in ANW [5]:

$$\begin{aligned}\langle r^2\rangle_{I=0} &= -\frac{2}{\pi}\int_0^\infty dr'\, r'^2 \sin^2 F(r') \frac{dF(r')}{dr'} \\ &= -\frac{8}{\pi}\frac{1}{\chi_0''^2}\int_0^\infty ds'\, s'^2 \sin^2 F(s') \frac{dF(s')}{ds'} \\ &= -\frac{8}{\pi}\frac{1}{\chi_0''^2}\, d\end{aligned} \tag{70}$$

The numbers are quite stable, though rather small (experimentally, for the nucleon, is 0,72 fm). This may be related to the fact that χQB rotates rather fast.

Again, the isoscalar magnetic moment root mean square radius is given by the expression from ANW [5]:

$$\langle r^2\rangle_{M,I=0} = \frac{4}{d}\frac{1}{\chi_0''^2}\int_0^\infty ds'\, s'^4 \sin^2 F(s') \frac{dF}{ds'} \tag{71}$$

We can make same remarks as before: it is stable, and surprisingly in rather good agreement with the nucleon (experimental value (0,81 fm).

The next two columns contain the figures for the isoscalar and isovector gyromagnetic factors. Starting from the definitions in ANW[5] we arrive at the following expressions ($M = E_0$, Eq. (47)):

$$g_{I=0} = \frac{1}{3\pi} \frac{M\chi_0''^3}{bf_\pi^2} \langle r^2 \rangle_{I=0} \tag{72}$$

$$g_{I=1} = \frac{8}{3} \pi \frac{f_\pi^2 M}{\chi_0''^3} b \tag{73}$$

With the help of Eqs. (46) and (47), we obtain:

$$\frac{\chi_0''^3}{\pi f_\pi^2 b} = \frac{M}{\vec{J}^2} \tag{74}$$

so finally:

$$g_{I=0} = \frac{M^2 \langle r^2 \rangle_{I=0}}{3\vec{J}^2} \tag{75}$$

$$g_{I=1} = \frac{8}{3} \vec{J}^2 \tag{76}$$

For the state $J = I = \frac{1}{2}$,

$$g_{I=0} \ (I=J=\tfrac{1}{2}) = \frac{4}{9} M^2 \langle r^2 \rangle_{I=0} \tag{77}$$

$$g_{I=1} \ (I=J=\tfrac{1}{2}) = 2 \ . \tag{78}$$

One can test with the experimental values for the nucleon the validity of Eq. (77), and find 5.22, whereas the experimental value is 9.4.

Besides, eliminating $\vec{J}^2$ between Eqs. (75) and (76), we find:

$$g_{I=0} = \frac{8}{9} \frac{M^2 \langle r^2 \rangle_{I=0}}{g_{I=1}} \tag{79}$$

The same expression is arrived at from the work by ANW[5]. Trying to test the expression with the experimental values for the nucleon, they don't fit. One gets $g_{I=0} = 1{,}11$ (Exp.: 1,76) and $g_{I=1} = 5{,}93$ (Exp.: 9,4).

Somehow, then, the expressions used up to now need improvement. Notice that the χQB satisfies all these expressions consistently.

A point can be made with respect to the consistency of expressions involving the masses of the states 1/2 and 3/2. In the model including the Skyrme term, ANW[5] used the fact that

$$M_\Delta - M_N = \frac{3}{2\theta} = 294\text{MeV}/C^2$$

and replaces it into the expression for the gyromagnetic ratios:

$$g_{I=0} = \frac{2}{3} \frac{M}{\theta} \langle r^2 \rangle = \frac{4}{9} \left[4M_N (M_\Delta - M_N) - (M_\Delta - M_N)^2\right] \langle r^2 \rangle_{I=0} \tag{90}$$

$$g_{I=1} = \frac{4}{3} \theta M = \frac{2M_N}{M_\Delta - M_N} - \frac{1}{2} \tag{91}$$

The resulting numerical values are:

$$g_{I=0} = 2{,}910 \langle r^2 \rangle_{I=0} = 1{,}01$$

$$g_{I=1} = 5{,}88$$

where in the first expression the calculated value for $\langle r^2 \rangle_{I=0}$ was used. Since only purely model information was used, this shows that the complete Skyrme model needs improvements of the same kind as those required by the χQB.

6 DISCUSSION, TENTATIVE CONCLUSIONS AND POSSIBLE PATHS

In the preceding sections we have expounded rather widely on the main properties of the chiral quantum baryon. Let us summarize in some sentences. We have shown that the outstanding feature of the solution for the hedgehog configuration in the non-linear SU(2) sigma model is the invariance under scale transformations.

This translates into the need for dimensional parameters to be completely undetermined at the classical level. The addition of the Skyrme term allows to find a value for this parameter.

At the quantum level, this feature allows a stable solution, the χQB, for the pure sigma model.

We have shown that numerically many physical parameters of the χQB show a remarkable stability. At the same time, their values are not quite far but either not quite approximate to the corresponding ones for the lowest baryon state.

This is not necessarily a fault. The dynamical content "en jeu" is so scarce that a better agreement with the numerical values for the nucleon would raise many questions. It is a matter of discussion, but nonetheless a matter of wide agreement, that the flavour group should be enlarged to SU(3) for a realistic description of the baryons(17).

It has long been recognized, besides, the need of pion exchange currents to describe low energy nucleon physics. We are now looking for the introduction of both ingredients into the picture.

A subject for further study concerns the applicability of expressions like Eqs. (79), (80) and (81) to real baryons. As they stand, they may be of restricted value, only for the hedgehog configuration. But how far real nucleons are from the hedgehog deserves better understanding.

Another point we have been able to show is that improvement is needed for the representation of the solution of the non-linear sigma model at the classical level. Special care should be given to the asymptotic regime. Physically, this could be translated in the need of a nicer description for the nuclear surface.

We believe that the chiral quantum baryon may be considered an inciting possibility towards a self consistent treatment of hadron dynamics through pure chiral dynamical input. The economy in the number of parameters, and the fact that the non-linear sigma model lagrangean and the Wess-Zumino term may possibly be obtained as an effective lagrangean from a gauge theory, makes it quite appealing.

The traditional Skyrme lagrangean, however, seems to share an attractive feature, namely, the scale invariance[12]. It is broken, however, already at the classical level, and further work should be devoted to it at the quantum level. The quantum solution again will have a stable minimum, which, to our knowledge, has not been explored.

In conclusion, it seems that many interesting questions need to be cleared up, but there is solid ground to believe that a consistent, realistic picture for baryons, is within range.

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APPENDIX A

ON THE STABILITY OF THE CLASSICAL HEDGEHOG SKYRME SOLITON

The lagrangean of the complete Skyrme model for the chiral angle of the hedgehog is:

$$L = -4\pi f_\pi^2 \int_0^\infty dr'\, r'^2 \left\{ \left(\frac{dF}{dr'}\right)^2 + \frac{2}{r'} \sin^2 F \right.$$
$$\left. + \frac{4\sin^2 F}{e^2 f_\pi^2 r'^2} \left[2\left(\frac{dF}{dr'}\right)^2 + \frac{\sin^2 F}{r'^2} \right] \right\} \tag{A.1}$$

Notice that the pure non-linear sigma model results in the limit $e \to \infty$.

We apply the usual procedure. If F_0 is a classical solution of (A.1), i.e., a minimum for its functional derivative, write any function in the vicinity of it as:

$$F(r) = F_0(r) + \varepsilon\, u_1(r) + \frac{1}{2}\, \varepsilon^2 u_2(r) \tag{A.2}$$

Substituting it into (A.1), after several integrations by parts we can write L in (A.1) as:

$$L = L_0 + \varepsilon L_1 + \frac{1}{2}\, \varepsilon^2 L_2$$
$$= L_0 + \frac{1}{2}\, \varepsilon^2 L_2 \tag{A.3}$$

Since $F_0(r)$ is such that the first functional derivative of (A.1) vanishes, L_2 itself results in two pieces, one depending entirely in u_2 and the other on u_1 and its first derivative. It happens that the term proportional to u_2 is just L_1, so, finally:

$$L_2 = -4\pi f_\pi^2 \int_0^\infty dr' \left\{ 2\cos 2F_o\, u_1^2 \right.$$

$$2\left[r'^2 + \frac{8}{e^2 f_\pi^2}\sin^2 F_o\right]\left(\frac{du_1}{dr'}\right)^2 - 8\,\frac{1}{e^2 f_\pi^2}\left[\sin 2F_o \left(\frac{d^2F_o}{dr'^2}\right) + \right.$$

$$\left. \left. + \cos 2F_o \left(\frac{dF_o}{dr'}\right)^2 - \frac{1}{r'^2}\sin^2 F_o \cos 2F_o - \frac{1}{r'^2}\sin^2 2F_o\right] u_1^2 \right\} \qquad \text{(A.4)}$$

We see that the term is of not definite sign, and it suggests that one may have troubles in interpreting the classical hedgehog solution as equation for the soliton.

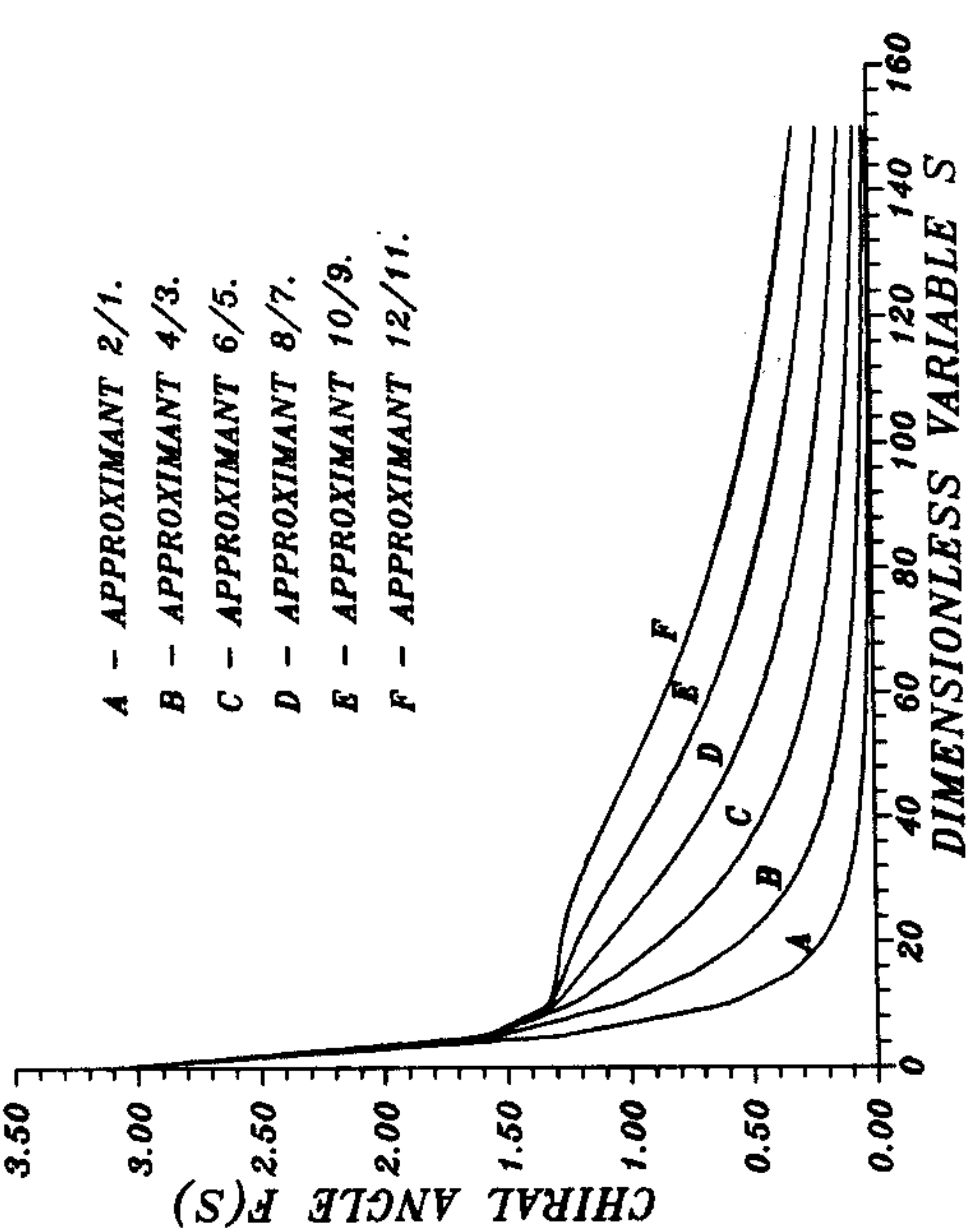

Fig.1 - The Chiral Angle for several Pade' Approximants.

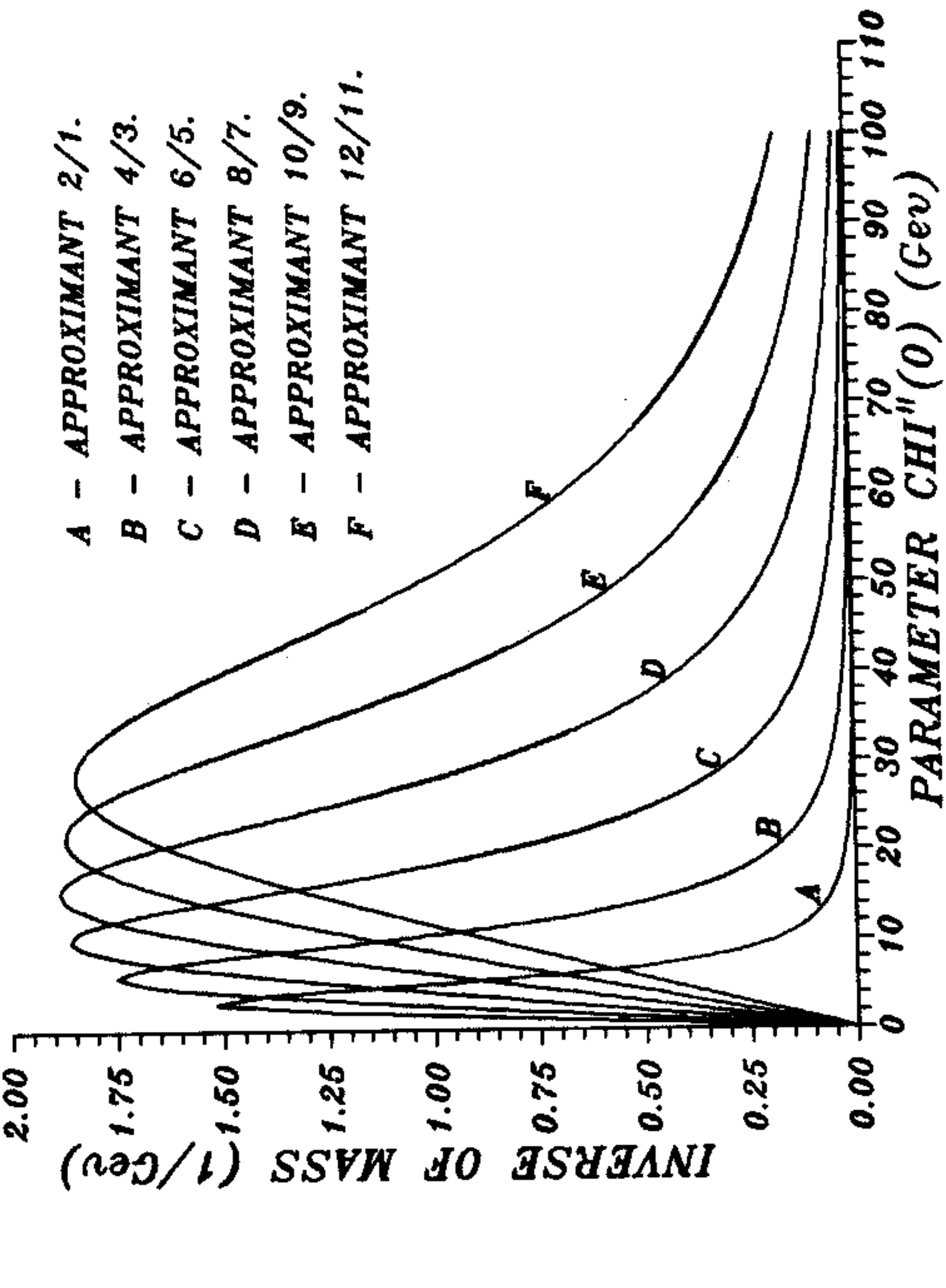

Fig.2 - The Mass of Quantum I=J=1/2 Baryon.

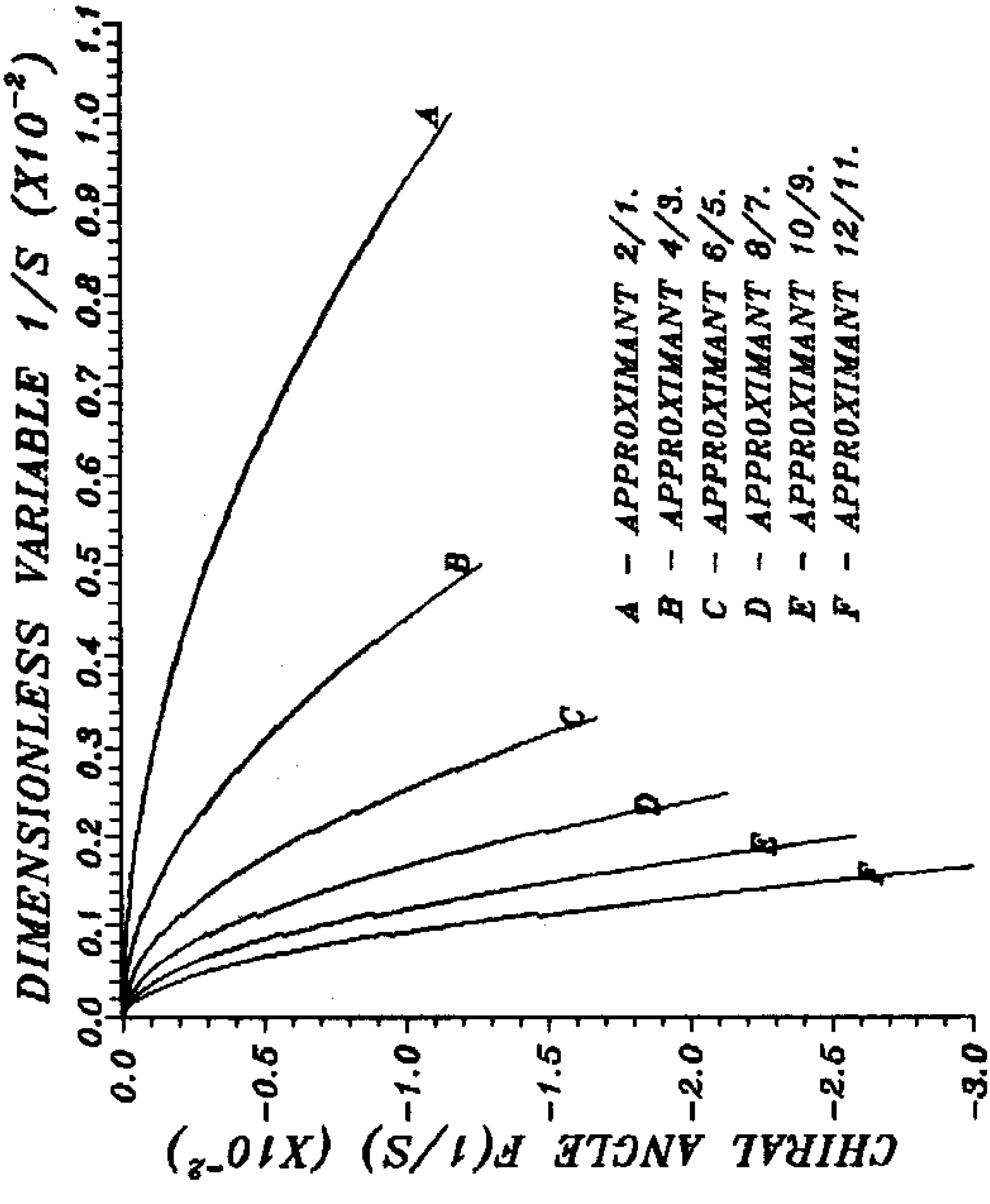

Fig.3 The Chiral Angle at infinity

Table I: Physical properties of the chiral quantum baryon (f_π = 0.067GeV)

Order	χ_o'' (f_π)	χ_o'' (GeV)	E_o (f_π)	E_o (GeV)	g_A	$\langle r^2\rangle^{1/2}_{I=0}$ (fm)	$\langle r^2\rangle^{1/2}_{M,I=0}$ (fm)	$g_{I=0}$	$g_{I=1}$
[2,1]	193.73	12.98	9.985	0.669	0.891	0.324	0.676	0.538	2.0
[4,3]	371.19	24.87	8.701	0.583	1.088	0.280	0.790	0.304	2.0
[6,5]	625.97	41.94	8.179	0.548	1.161	0.206	0.872	0.229	2.0
[8,7]	946.86	63.44	8.089	0.542	1.166	0.293	0.900	0.214	2.0
[10,9]	1729.8	84.44	8.149	0.546	1.149	0.253	0.899	0.218	2.0
[12,11]	2222.4	115.90	8.418	0.554	1.128	0.255	0.888	0.228	2.0

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