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ON A GAUGE THEORY OF SELF-DUAL FIELD AND ITS QUANTIZATION

by

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ABSTRACT:

A gauge theory of self-dual field is constructed by adding a Wess-Zumino term to the recently studied formulation based on a second order scalar field Lagrangian carrying with it an auxiliary vector field to take care of the self-duality constraint in a linear fashion. The two versions are quantized using the BRST formulation following the BFV procedure. No violation of microcausality occurs and the action of ordinary scalar field may not be written as the sum of the actions of the self- and anti-self-dual fields.

Key-words: Chiral bosons; Gauge theory; Quantization.

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1. INTRODUCTION. CHIRAL BOSON:

In a recent study [1] on the quantized theory of self-dual field (chiral boson), it was pointed out that the following second order scalar field Lagrangian with an auxiliary field B_μ to take care of the self-duality constraint

$$\mathcal{L}^N = (1/2)(\partial_\mu\phi)(\partial^\mu\phi) + B_\mu(\eta^{\mu\nu} + \varepsilon^{\mu\nu})\,\partial_\nu\,\phi \qquad (1)$$

posed no problem. It is the natural one and should not be abandoned [2]. The action (1) is Lorentz invariant and contains a bilinear term in the (dynamical) scalar field ϕ contrary to the auxiliary vector field which appears only linearly. From the resulting Lagrange eqns. of motion we derive $\partial\mu\partial^\mu\,\phi = 0$, $(\eta^{\mu\nu} + \varepsilon^{\mu\nu})\,\partial_\nu B_\mu = 0$. We do not find the Klein-Gordon eqn. for *all* the components of the auxiliary vector field. In fact with a convenient notation we may obtain $\partial\mu\partial^\mu\ (B_0+B_1) = 0$ but no propagation equation ,however, results for (B_0-B_1). It is clear, therefore, that B_μ may not be declared , *a priori*, a dynamical field at this level. The situation is analogous to the case of the Lagrange multiplier field A^0 which enforces Gauss' law constraint [3] in Yang-Mills theory and the Lagrange eqns. do contain space-time derivatives of this field. It is only after we have implemented [3], say, by using the Dirac's procedure [4] for constrained dynamical systems, all the constraints and eliminated the superfluous auxiliary fields in order to obtain the reduced Hamiltonian that we may decide as to the real physical degrees of freedom in the classical theory over which a reduced phase space quantization may be based to start with. We may ,as is usually done [5] now a days, decide to enlarge the phase space by adding to the theory Lagrange multiplier fields along with the ghosts, which are treated as dynamical fields and satisfy the graded commtator algebra. The quantization, say, using functional integral, may then be conveniently done without the need to compute Dirac brackets.

For the action (1) it was shown [1] that following the Dirac's procedure [4] a self-consistent Hamiltonian formulation could be constructed. The field B_μ is eliminated due to the two non-trivial second class constraints in the theory ,which imply $\Pi_0-\Pi_1 = 0$ for the left-mover, leading to the reduced Lagrangian , $\Pi(\dot{\phi} - \phi')$,which shows that only ϕ is the dynamical field. It was also found that it is the field ϕ which satisfies the self-duality condition and that no violation of the micro-causality occurs on canonically quantizing the theory. This is in contrast to what is found for the dimension zero bosonic field formulation suggested by Floreanini and Jackiw [6] ,(inspired on the formulation of Siegel [2]), where ϕ' instead of ϕ is found to satisfy the self-duality constraint and the micro-causality is violated. It was also pointed out that the action of the ordinary scalar field may not in general be written as the sum of the actions of the self and anti-self-dual fields. In another paper we showed that the second order Lagrangian for a chiral bosonic particle [7] may also be written in the form analogous to (1). It was quantized following the BRST [8] formulation and its propagator [7] obtained following the BFV [9] procedure.

In the present paper we construct a Wess-Zumino [10] term corresponding to the self-dual field action (1). Its function is to cancel the undesired mode mentioned in the previous paragraph and it results in a gauge theory of self-dual field with only the first class constraints. Using the BRST [8] formulation we quantize the two versions following the BFV [9] procedure.

2. GAUGE THEORY OF CHIRAL BOSON. WESS-ZUMINO TERM:

The Lagrangian of the 'gauge invariant' action, $\mathcal{L}^I = \mathcal{L}^N + \mathcal{L}^{WZ}$,for the self-dual field, resulting in the first class constraints and which generate gauge transformations is constructed by adding to the action (1) the following term for the Wess-Zumino field θ with a coupling to the auxiliary vector field

$$\mathcal{L}^{WZ} = -(1/2)\,(\partial_\mu\theta)\,(\partial^\mu\theta) + \theta\,(\eta^{\mu\nu} + \varepsilon^{\mu\nu})\,\partial_\nu B_\mu \qquad (2)$$

Indicating by Π, p^μ, Π_θ the momenta canonical to ϕ, B_μ, θ respectively, the basic weak primary constraints may be written as $T_1 \equiv p^0+p^1-2\theta \approx 0$, $T_2 \equiv p^0-p^1 \approx 0$ and the canonical Hamiltonian is found to be, ($\varepsilon_{01} = 1$, $\eta_{00} = -\eta_{11} = 1$),

$$\mathcal{H}_c = \frac{1}{2} [\Pi - B_0 - B_1]^2 + \frac{1}{2} \phi'^2 + (B_0 + B_1) \phi' - \frac{1}{2} \Pi_\theta^2$$

$$- \frac{1}{2} \theta'^2 + \theta (B_1 + B_0)' \qquad (3)$$

where a prime indicates the space derivative. The primary Hamiltonion is $\mathcal{H}_1 = \mathcal{H}_c + u\, T_1 + v\, T_2$ where u, v are arbitrary functionals of the canonical variables. On requiring the persistency in time of the primary constraints it leads to the following secondary constraint

$$T_3 \equiv \Pi - B_0 - B_1 - \phi' + (\theta' + \Pi_\theta) \approx 0 \qquad (4)$$

The extended Hamiltonian is $\mathcal{H}_2 = \mathcal{H}_1 + \lambda\, T_3$ and we derive (an overdot indicating the time derivative)

$$\dot{T}_3 = \{ T_3, \mathcal{H}_2 \} = \{ T_3, \mathcal{H}_0 \} = -T_3' \approx 0 \qquad (5)$$

while $\dot{T}_1 = \dot{T}_2 = 0$. No more constraints are generated, T_1, T_2, T_3 are first class and T_3 satisfies the free field equation $\partial_\mu \partial^\mu\ T_3 = 0$. It is convenient, without any loss of generality, to remove the trivial constraints T_1 and T_2 by imposing the gauge-fixing conditions $B_0 \approx 0$, $B_1 \approx 0$. The Dirac brackets constructed with respect to the set

T_1, T_2, B_0, B_1 of the remaining variables ϕ, θ, Π, Π_θ coincide with the standard Poisson brackets. We may easily show that on requiring the persistency in time of this set of constraints we are lead to $u = v = 0$. The Hamiltonian is then given by

$$\mathcal{H} = \mathcal{H}_0 + \lambda\, T \tag{6}$$

where

$$\mathcal{H}_0 = \frac{1}{2}\,\Pi^2 + \frac{1}{2}\,\phi'^2 - \frac{1}{2}\,\Pi_\theta{}^2 - \frac{1}{2}\,\theta'^2 \tag{7}$$

and $T \equiv T_3 = \Pi - \phi' + \Pi_\theta + \theta' \approx 0$ with T satisfying the free field equation. Consequently, its decomposition into positive and negative frequencies makes sense. The canonical variables satisfy the standard Poisson bracket relations and the local gauge transformations generated by T are found to be $\delta\,\phi(x) = \delta\,\theta(x) = u(x)$, $\delta\,\Pi(x) = -\delta\,\Pi_\theta(x) = -u(x)'$. The first order action corresponding to the Hamiltonian (6), where λ is a Lagrange multiplier field, is invariant under these transformations if we assume $\delta\,\lambda(x) = \dot{u}$. The canonical quantization may be performed by the usual prescription of replacing the Dirac brackets by the (graded) commutators of the corresponding operators multiplied by $(-i)$,using a symmetrized form to avoid operator product ambiguities and ,following Gupta-Bleuler [11], by requiring that the physical states satisfy

$$(\,\Pi - \phi' + \Pi_\theta + \theta'\,)^{+}|\mathrm{Phys}\rangle = 0 \tag{8}$$

In the next Section we quantize both the gauge invariant version (8) and the gauge-non-invariant description (1) of the self-dual field using BRST [8] formalism.

3. BRST-BFV QUANTIZATION:

Consider first the gauge invariant formulation described by (8). We follow the procedure of ref.[9]. The Lagrange multiplier field λ along with its conjugate momentum Π_λ are now treated as dynamical fields over an extended phase space to which we add also the fermionic ghost fields η and $\bar{\eta}$ along with the corresponding canonical momenta $\bar{P}$ and P. The non-vanishing equal time graded brackets of these variables are

$$\{P \; , \; \bar{\eta}\} = \{\bar{P} \; , \; \eta\} = \{\Pi_\lambda \; , \; \lambda\} = \{\Pi \; , \; \phi\} = \{\Pi_\theta \; , \; \theta\} = -1 \qquad (9)$$

suppressing a delta function, $\delta(x-y)$, on the right hand side of (9). The nilpotent conserved BRST charge is simply $\Omega = P \Pi_\lambda + \eta T$ while the anti-BRST charge is $\bar{\Omega} = - \bar{P} \Pi_\lambda + \bar{\eta} T$. We construct the following effective action

$$S_{eff} = \int d^2x \; [\; \Pi \dot{\phi} + \Pi_\theta \dot{\theta} + \Pi_\lambda \dot{\lambda} + \dot{\eta} \bar{P} + \dot{\bar{\eta}} P - \mathcal{H}_0 - \{\Omega \; , \Psi\} \;] \qquad (10)$$

where Ψ is an arbitrary suitable gauge-fixing fermionic operator and $\overline{\mathcal{H}}_0 = \mathcal{H}_0 + \eta \; \partial_1 \overline{P}$ is the BRS improved Hamiltonian. The quantized theory is then obtained from the following functional integral

$$Z = N \int [d\,\mu] \;\; \exp\, (i\; S_{eff}) \tag{11}$$

where $[d\,\mu]$ is the Liouville measure of the extended phase space, $[d\,\mu] = [d\phi][d\Pi][d\theta][d\Pi_\theta][d\lambda][d\Pi_\lambda][d\eta][d\overline{P}][d\overline{\eta}][dP]$ and N stands for the normalization factor. A convenient choice for Ψ is found to be $\Psi = \frac{1}{\beta}\, \theta\, \overline{\eta} + \overline{P}\, \lambda$ where β is a parameter. We find

$$\{\Omega,\; \Psi\} = \frac{1}{\beta}\, \theta\, \Pi_\lambda - \lambda\, T + P\, \overline{P} + \frac{1}{\beta}\, \eta\, \overline{\eta} \tag{12}$$

The contribution of the ghosts to the functional integral is a field independent factor which is absorbed in the normalization. Making now a shift transformation $\Pi_\theta \to \Pi_\theta - \dot{\theta} - \lambda$ we obtain a Gaussian integral over Π_θ which is also absorbed in the normalization constant. The effective action then reduces to

$$S_{eff} = \int d^2x \; [\; \Pi\, \dot{\phi} + \Pi_\lambda\, \dot{\lambda} - \frac{1}{2}\, \Pi^2 - \frac{1}{2}\, \phi'^2 + \frac{1}{2}\, \theta'^2$$

$$- \frac{1}{2}\, (\dot{\theta} + \lambda)^2 + \lambda\, (\Pi - \phi' + \theta') - \frac{1}{\beta}\, \theta\, \Pi_\lambda \;] \tag{13}$$

A functional integration over Π_λ now brings in the integrand the delta functional $\delta\,(\dot{\lambda} - \frac{1}{\beta}\,\theta)$. Integrating over θ and making $\beta \to 0$ leaves us with the effective action

$$S_{eff} = \int d^2x \,[\, \Pi\,\dot{\phi} - \frac{1}{2}\,\Pi^2 - \frac{1}{2}\,\phi'^2 + \lambda\,(\Pi - \phi') - \frac{1}{2}\,\lambda^2 \,] \tag{14}$$

A further shift transformation $\lambda \to \lambda + \Pi - \phi'$ gives a Gaussian integral over λ which is absorbed in the normalization and we are left finally with the following action [1]

$$S_{eff} = \int d^2x \,[\, \Pi\,\dot{\phi} - \Pi\,\phi' \,] \tag{15}$$

The same result may also be derived on starting from the action given in (1). Following the Dirac procedure and removing the trivial first class constraint $T_2 \equiv p^0 - p^1 \approx 0$ by choosing the gauge condition $B_0 - B_1 \approx 0$, which in its turn determines $v = 0$ if we require its persistency in time, we are left with the following, after a convenient rearrangement, second class constraints in the theory [1]

$$G = \Pi - \phi' - \omega + \Pi'_\omega \approx 0 \qquad , \quad F = \Pi_\omega \approx 0 \tag{16}$$

Here $\omega = B_0 + B_1$ and $\Pi_\omega = \frac{1}{2}\,(p^0 + p^1)$ and the nonvanishing bracket is $\{\omega(x,t), \Pi_\omega(y,t)\} = \delta(x-y)$. For G and F we find

$$\{G,\,F\} = -1, \qquad \{G,\,G\} = \{F,\,F\} = 0 \tag{17}$$

The canonical Hamiltonian in the present case is given by

$$\mathcal{H}_c = \frac{1}{2}\,[(\Pi - \omega)^2 + \phi'^2] + \omega\,\phi' \tag{18}$$

It is tempting to look at G as a first class constraint and F as a gauge-fixing condition. However, we easily derive that the variation induced by G , $\delta\,\mathcal{H}_c \approx \delta\,(\Pi^2/2)$ is non-vanishing. In order to quantize the theory with second class constraints using functional integral, we may use the formulation [12,13] based on an Osp(1,1|2) invariant path integral. For the nilpotent BRST operator we take the symmetric form [12]

$$\Omega = \frac{1}{\sqrt{2}}\,[\,\eta\,(G + \lambda) + \mathcal{P}\,(F + \Pi_\lambda)\,] \tag{19}$$

Here we have rewritten as λ and Π_λ the Lagrange multiplier fields that enforce the pair of the second class constraints G,F. The extended phase space contains now ϕ, ω, λ, η, $\bar\eta$,Π, Π_ω, Π_λ, $\mathcal{P}$, $\bar{\mathcal{P}}$. They satisfy an algebra analogous to that in (8).

The effective action now is

$$S_{eff} = \int d^2x\,[\,\Pi\,\dot\phi + \Pi_\omega\,\dot\omega + \Pi_\lambda\,\dot\lambda + \dot\eta\,\bar{\mathcal{P}} + \dot{\bar\eta}\,\mathcal{P} - \mathcal{H}_o - \{\Omega\,,\Psi\}\,] \tag{20}$$

where $\mathcal{H}_o$ is given by (18) and the path integral (11) is now defined over the phase space under consideration here. It reproduces [12] the reduced phase space path integral by the Parisi-Sourlas [14] mechanism and we avoid constructing the Dirac brackets.

A convenient choice for Ψ in the present case is $\Psi = \sqrt{2}(\frac{1}{\beta} F \bar{\eta} + \bar{P} \lambda)$ where β is an arbitrary parameter. We find

$$\{\Omega, \Psi\} = \lambda (G + \lambda) + \frac{1}{\beta} F (F + \Pi_\lambda) + P \bar{P} + \frac{1}{\beta} \eta \bar{\eta} \tag{21}$$

The ghost contribution may be ignored as before. The integration over Π_λ brings down a delta functional $\delta(\frac{1}{\beta} \Pi_\omega - \dot{\lambda})$ which allows us to integrate over Π_ω and in the limit $\beta \to 0$ we obtain

$$S_{eff} = \int d^2x \, [\, \Pi \dot{\phi} - \lambda (\Pi - \phi' - \omega + \lambda) - \mathcal{H}_0 \,] \tag{22}$$

Performing a shift transformation $\omega \to \omega + \Pi - \phi' + \lambda$ the integral over ω gives a constant normalization factor and the action reduce to

$$S_{eff} = \int d^2x \, [\, \Pi \dot{\phi} - \frac{1}{2} (\Pi^2 + \phi'^2) + \frac{1}{2} (\Pi - \phi')^2 - \frac{1}{2} \lambda^2 \,] \tag{23}$$

which results in the same expression as in (15) after performing a Gaussian integral over λ.

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