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COMPARISON OF THE ANOMALOUS AND NON-ANOMALOUS GENERALIZED SCHWINGER MODELS VIA FUNCTIONAL FORMALISM

by

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Abstract: We calculate the Green functions of the two versions of the generalized Schwinger model, the anomalous and the non-anomalous one, in their higher order Lagrangian density form. Furthermore it is shown through a sequence of transformations that the bosonized Lagrangian density is equivalent to the former, at least for the bosonic correlation functions. The introduction of the sources from the beginning, leading to a gauge-invariant source term is also considered. It is verified that the two models have the same correlation functions only if the gauge-invariant sector is taken into account. Finally it is presented a generalization of the Wess-Zumino term, and its physical consequences are studied, in particular the appearance of gauge-dependent massive excitations.

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I) Introduction

Since long ago [1,2], quantum field theory has been widely investigated in two dimensional space-time, essentially with the hope of getting a deeper understanding of the quantum field mechanisms and that it may be translated in some way to higher dimensions. Indeed this has been done principally through the quest for models in which some types of interesting mechanisms work. This is exactly the case of the chiral Schwinger model (CSM) [3], that was introduced by Jackiw and Rajaraman as an anomalous gauge model that, despite of this feature, is unitary and renormalizable. The essential idea underlying the interest in this specific model was that of obtaining a model which dispenses with the Higgs field. This happens because the model presents a dynamical mass generation and concomitantly does not depend also of a "closing of family" in order to prevent the anomaly and preserving the consistency of the theory. This with the eyes pointing at the non-appearance of the top particle. Later it was discovered that the model could be thought as a gauge-fixed version of a non-anomalous model, provided that its functional measure were correctly manipulated [4,5]. After that, a lot of papers have appeared discussing a variety of its peculiarities and, in particular that the Wess-Zumino (WZ) term does not change the physical content of the model [6-13]. More recently, people have been interested in a model that contains as particular cases the old vector Schwinger model (VSM), the axial Schwinger model (ASM) and the chiral Schwinger model (CSM) [15-20].

This last model, the generalized Schwinger one (GSM), has been treated perturbatively by Chanowitz [15], who discussed some ambiguities in the specification of the anomaly, and the WZ term was also considered. Posteriorly Miyake and Shizuya [16] have worked on the fermion content of the model, and Boyanovsky, Schimidt and Golterman [9] used it as a starting point to do a comparison among the VSM and CSM properties. After that, Wotzasek and Naon [17] studied its bosonized version through the Dirac algorithm and also the case of massive fermions. Furthermore Shin Lee and Lee [18] leaned over the problem of their currents and energy-momentum tensor, and Alonso, Cortes and Rivas [19] used it to discuss the regularization ambiguity in Fujikawa's regularization [22]. More recently Dias and Linhares [20] presented the point-splitting computation of the fermionic determinant, and discussed the various classes of regularization. Furthermore this author [21], starting from a gauge principle, obtained gauged bosonized theories, applying it for the GSM and also in the case of 2 + 1 dimensions.

However, as far as we know, most of these papers discussed the models through their bosonized version, because in the functional formalism this is a possibility to rewrite locally the non-local gauge field effective Lagrangian density. Meanwhile, as we will see below, this non-local Lagrangian density can also be traded by a local one through the using of the decomposition property of the gauge field at two dimensions but, in this case, one is faced with a higher order theory. Here we intend to study the generalized model in this version, showing the equivalence with the bosonized one, and discussing both the anomalous and the non-anomalous one. In this last case we will introduce two types of gauge-fixing. They contain as particular cases the ones appearing in the literature. We will discuss also the gauge-invariant correlation functions, showing that they correspond to the anomalous ones as observed by Girotti and Rothe [12], this time using a modified source term in the generating functional as suggested by Linhares, Rothe and Rothe [10].

All the computations will be done in the functional integral formalism and, as the generalized Schwinger model (GSM), like the CSM, has a singular point for the regularization parameter ($M^2 = g_1^2$ in the GSM and $a = 1$ in the CSM), this case will be treated in a separated way. The arbitrary parameter case ($M^2 \neq g_1^2$) will be solved through field transformations that become singular when $M^2 = g_1^2$, enlightening, in this formalism, the origin of the problem with this parameter choice. Then, in the case $M^2 = g_1^2$, we will use another approach in order to solve it.

In the last section of the article, we will argue about the introduction of a novel and generalized WZ term. After its presentation by using the same method of Harada and Tsutsui [5], we will investigate the physical consequences of its postulation. In doing so, for the sake of simplicity, it will be considered the cases of the VSM, ASM and CSM. The first two are used to get some confidence in the mechanism because of their well-stated physical properties. Finally we construct one possible realization of the WZ term for the CSM and find out a surprising feature, that in this case the dynamically generated mass of the gauge field is no longer arbitrary, but is fixed when an appropriate gauge is used.

Another remarkable consequence of using generalized WZ terms is that, even when the gauge invariant correlation functions are used, some arbitrariness does persist in their definitions. This arbitrariness could be used, for example, to overcome the misbehaviour of the gauge propagator at low coupling constants, which has generated some controversy [7,12]. In fact it appears somewhat strange that, differently of all other gauge theories, the non-anomalous CSM and by extension the non-anomalous GSM, could be the only ones having their gauge propagators with the same form, no matter what gauge-fixing condition is being used.

The paper is organized as follows: In section II it is presented the model in their two versions, the anomalous and the non-anomalous one. Still in this section we prove the equivalence of the bosonized and higher-order versions of the model, at least for the bosonic correlation functions. In sections III and IV we study the cases of $M^2 \neq g_1^2$ and $M^2 = g_1^2$ respectively. Section V is devoted to the generalized WZ term and its peculiarities. Finally in section VI we do our final remarks.

II) The model and its versions:

The Lagrangian density for the generalized Schwinger model

(GSM) is given by

$$\mathfrak{L} = \overline{\psi}\ \gamma_\mu(i\partial^\mu + e_R P_+ A^\mu + e_L P_- A^\mu)\psi - (1/4)F_{\mu\nu}F^{\mu\nu} + J_\mu A^\mu + \overline{\psi}\zeta + \overline{\zeta}\psi, \tag{1}$$

with the source terms included and $P_\pm \equiv (1/2)(1 \pm \gamma_5)$. The fermion and gauge fields can be decoupled through a suitable transformation,

$$\psi = \exp\Big[-i\gamma_5(g_1\eta + g_2\chi) + i(g_2\eta + g_1\chi)\Big]\psi' \equiv U_5\psi', \tag{2a}$$

$$\overline{\psi} = \overline{\psi}'\ \exp\Big[-i\gamma_5(g_1\eta + g_2\chi) - i(g_2\eta + g_1\chi)\Big]\psi' \equiv \overline{\psi}'\overline{U}_5, \tag{2b}$$

where η and χ are identified with the longitudinal and transverse components of the gauge field A_μ, as we will see below. Using (2) and also the identity $\gamma_\mu\epsilon^{\mu\sigma} = -\gamma^\sigma\gamma_5$, valid in the two-dimensional Minkowski space-time, one decouples the classical Lagrangian density, giving

$$\mathfrak{L} = \overline{\psi}'\ i\gamma_\mu\partial^\mu\psi' - (1/4)F_{\mu\nu}F^{\mu\nu} + J_\mu A^\mu + \overline{\psi}'\overline{U}_5\zeta + \overline{\zeta}U_5\psi'. \tag{3}$$

Besides, as a consequence of the non-invariance of the functional measure under chiral transformations, there is an additional effective term in the Lagrangian density, coming from the regularized Jacobian [22]. Taking this into account, we have an effective higher-order Lagrangian density given by

$$\mathcal{L} = \bar{\psi}'(i\gamma_\mu\partial^\mu)\psi' + (1/2e^2)\chi\,\Box^2\chi + (1/2\pi e^2)(g_1^2 - M^2)\eta\Box\eta +$$

$$+ (1/2\pi e^2)(g_2^2 + M^2)\chi\Box\chi - (g_1 g_2/\pi e^2)\eta\Box\chi + (1/e)J_\mu(\partial^\mu\eta +$$

$$+ \epsilon^{\mu\sigma}\partial_\sigma\chi) + \bar{\psi}'\bar{U}_5\zeta + \bar{\zeta}U_5\psi'. \qquad (4)$$

where $M^2 \equiv a\ e^2$, a being the arbitrary regularization parameter, $g_1 \equiv (e_R - e_L)/2$, $g_2 \equiv (e_R + e_L)/2$, and $\eta(x)$ and $\chi(x)$ are, respectively, the longitudinal and transversal components of the gauge field, that in 1 + 1 dimensions can always be decomposed as

$$eA_\mu = \partial_\mu\eta + \epsilon^{\mu\sigma}\partial_\sigma\chi. \qquad (5)$$

From the above identity one can get two others,

$$\eta = e\partial.A/\Box,\ \chi = -e\bar{\partial}.A/\Box, \qquad (6)$$

where we defined: $\partial.A \equiv \partial_\mu A^\mu$ and $\bar{\partial}.A \equiv \epsilon^{\rho\sigma}\partial_\sigma A_\rho$. Using these identities and an additional one

$$\epsilon^{\mu\rho}\epsilon^{\nu\sigma} = g^{\mu\sigma}g^{\rho\nu} - g^{\mu\nu}g^{\rho\sigma}, \qquad (7)$$

all valid in two dimensions, we get for the effective Lagrangian density the expression [9]

$$\mathcal{L} = \bar{\psi}'(i\gamma_\mu\partial^\mu)\psi' + (1/2\pi)A_\mu\Big[(M^2 + g_2^2)g^{\mu\nu} - (g_1^2 + g_2^2)\partial^\mu\partial^\nu/\Box +$$

$$+ g_1 g_2(\partial^\mu\bar{\partial}^\nu + \bar{\partial}^\mu\partial^\nu)/\Box\Big]A_\nu + J_\mu A^\mu + \bar{\psi}'\bar{U}_5\zeta + \bar{\zeta}U_5\psi'. \qquad (8)$$

This non-local version can also be rewritten in another local form, by introducing an auxiliary field. Besides we can discard the sources and integrate out the fermions, obtaining the bosonized Lagrangian density [9]

$$\mathcal{L}_B = (1/2)\partial_\mu\phi\partial^\mu\phi - (1/\sqrt{\pi})(g_1 g^{\mu\nu} - g_2\epsilon^{\mu\nu})\partial_\nu\phi A_\mu + (M^2/2\pi)A_\mu A^\mu +$$

$$- (1/4)F_{\mu\nu}F^{\mu\nu}, \qquad (9)$$

or, in terms of both A_μ component fields,

$$\mathcal{L}_B = (1/2)\partial_\mu\phi\partial^\mu\phi + (1/2e^2)(\Box\chi)^2 - (1/e\sqrt{\pi})(g_1\partial^\mu\eta - g_2\partial^\mu\chi)\partial_\mu\phi +$$

$$+ (M^2/2\pi e^2)\Big[\partial_\mu\eta\partial^\mu\eta - \partial_\mu\chi\partial^\mu\chi\Big]. \qquad (10)$$

It is not difficult to see that these two local formulations, given by Eq.(4) and Eq.(9), for the GSM are equivalent. This can be seen by doing a transformation in the bosonization field $\phi(x)$,

$$\phi(x) = \phi'(x) - (g_1/e\sqrt{\pi})\eta(x) + (g_2/e\sqrt{\pi})\chi(x). \qquad (11)$$

Furthermore, this transformation has a trivial Jacobian for the functional measure. So, integrating over the $\phi'(x)$ field we get the same Lagrangian density as that in expression (4), apart from a trivial redefinition of the generating functional normalization. So we have shown that both local versions, one based in a higher order Lagrangian density (4) and the other with usual derivative order but with the introduction of an additional field (9), are equivalent, because in the transformation necessary to link these expressions, the fields $\eta(x)$ and $\chi(x)$ remains unchanged in form, and they are the ones which appear in the correlation functions, as can be seen from the generating functional.

As far as we know, almost all the works that discuss the chiral Schwinger model (CSM) [3-14] and GSM [16-21], are based in the bosonized version (9), the only exception is the work of Chanowitz [15], that works in the fermionic version through perturbative methods. Here, we will study the higher order version of the GSM (4), in its anomalous and non-anomalous formulations, and take the appropriate limits to check out our results against those in the literature.

The non-anomalous formulation of the Lagrangian density (4), can be easily obtained through the substitution of the longitudinal component of the photon field by a gauge-invariant combination. To do this we introduce a compensating field $\theta(x)$, the Wess-Zumino one, so that

$$A_\mu \to A_\mu + (1/e)\partial_\mu\Lambda \ , \ \eta \to \eta + \Lambda \ , \ \theta \to \theta - \Lambda \tag{13}$$

then substituting $\eta(x)$ by $\eta(x) + \theta(x)$ in expression (4), we get the non-anomalous higher-order formulation of the GSM model,

$$\mathcal{L} = \bar{\psi}'(i\gamma_\mu\partial^\mu)\psi' + (1/2e^2)\chi\,\Box^2\chi + (1/2\pi e^2)(g_1^2 - M^2)(\eta + \theta)\Box(\eta + \theta)$$

$$+ (1/2\pi e^2)(g_2^2 + M^2)\chi\Box\chi - (g_1 g_2/\pi e^2)(\eta + \theta)\Box\chi, \tag{14}$$

where we omit the source terms for future convenience.

III) The $M^2 \neq g_1^2$ case:

First of all we solve the anomalous GSM through the use of appropriate field transformations, to get a decoupled Lagrangian density. It is not difficult to see that doing the transformations below in Eq.(4),

$$\eta = \eta' + \left[\frac{g_1 g_2}{g_1^2 - M^2}\right]\chi', \ \chi = \chi', \tag{15}$$

one obtains

$$\mathcal{L}_{AGSM} = \bar{\psi}'(i\gamma_\mu\partial^\mu)\psi' + (1/2e^2)\chi'\Box(\Box + m^2)\chi' +$$

$$+ \ (1/2\pi e^2)(g_1^2 - M^2)\eta'\Box\eta' + (1/e)J^\mu\left[\partial_\mu\left(\eta' + \left[\frac{g_1 g_2}{g_1^2 - M^2}\right]\chi'\right) + \right.$$

$$\left. + \ \epsilon_{\mu\rho}\partial^\rho\chi'\right] + \overline{\psi}'\overline{U}_5'\zeta + \overline{\zeta}U_5'\psi' \tag{16}$$

where

$$m^2 = \frac{\left((g_1 g_2)^2 - (g_1^2 - M^2)(g_2^2 + M^2)\right)}{\pi(M^2 - g_1^2)}, \tag{17}$$

is the dynamically generated mass of this model, and we have done the transformation in the source terms also. As the sources generate the correlation functions of the original fields, all we need now is to do the usual functional derivatives with respect to these sources and, using the free propagators of the transformed fields, get the required Green functions. At this point one can see why the case $M^2 = g_1^2$ ($a = 1$ in the CSM) must be treated separately, as observed in the Dirac quantization for the CSM [6]; in the present approach this is easily seen because the transformation (16) becomes singular for this case, so one should substitute this condition in the Lagrangian density and, only then, search for the decoupling transformations. As an example of a Green function we compute the photon propagator, that comes from

$$D_{\mu\nu}(x-y) = - \delta^2 Z/\delta J_\mu(x)\delta J_\nu(y)\Big|_{J=0}. \tag{18}$$

This way we get the propagator in the momentum representation,

$$D_{\mu\nu}(k) = -(1/e^2)\left\{k_\mu k_\nu D_{\eta'} + \left[k_\mu k_\nu + \left[\frac{g_1 g_2}{g_1^2 - M^2}\right]^2 \bar{k}_\mu \bar{k}_\nu) D_{\chi'}\right]\right\}, \qquad (19)$$

from which, after the substitution of the free propagators of η' and χ', we obtain:

$$D_{\mu\nu}(k) = (i/(k^2 - m^2))\left\{-g_{\mu\nu} + \left[\frac{1}{(M^2 - g_1{}^2)}\right]\left[k_\mu k_\nu\left[\pi + \right.\right.\right.$$

$$\left.\left.\left. - (1/k^2)(g_1^2 + g_2^2)\right] + (g_1 g_2/k^2)(k_\mu \bar{k}_\nu + \bar{k}_\mu k_\nu)\right]\right\}, \qquad (20)$$

that is in agreement with that obtained in the literature for the anomalous GSM through the Dirac formalism [17].

Now we will solve the non-anomalous GSM, by performing decoupling field transformations. For this we need to break the gauge symmetry. This is done through the introduction of a gauge-fixing (GF) term in the Lagrangian density. In this work we will use the two GF given below:

$$\mathcal{L}_{GF}^{(I)} = -(1/2\alpha)(\partial.A + \beta\ \bar{\partial}.A)^2 = -(1/2\alpha e^2)(\Box\eta - \beta\ \Box\chi)^2, \qquad (21a)$$

and

$$\mathcal{L}_{GF}^{(II)} = -(1/2\alpha)\partial_\mu\theta\partial^\mu\theta, \qquad (21b)$$

where α and β are gauge parameters.

Using $\mathcal{L}_{GF}^{(1)}$, one decouples the Lagrangian density (14) performing the following transformations in the fields:

$$\eta = \eta' + \beta\chi',\ \chi = \chi',\ \theta = \theta' + \left[\frac{g_1 g_2}{g_1^2 - M^2} - \beta\right]\chi', \tag{22}$$

so that we obtain the decoupled Lagrangian density

$$\mathcal{L} = \mathcal{L}_{GSM} + \mathcal{L}_{GF}^{(1)} = \overline{\psi}'(i\gamma_\mu\partial^\mu)\psi' + (1/2e^2)\chi'\Box(\Box + m^2)\chi' +$$

$$+ (1/2\pi e^2)(g_1^2 - M^2)\theta'\Box\theta' - (1/2\alpha e^2)\eta'\Box^2\eta' +$$

$$+ (1/e)J^\mu\left[\partial_\mu(\eta' + \beta\chi') + \epsilon_{\mu\rho}\partial^\rho\chi'\right] + \overline{\psi}'\overline{U}_5'\zeta + \overline{\zeta}U_5'\psi'. \tag{23}$$

Taking the functional derivatives with respect to the source J_μ, we are led to the expression for the photon propagator,

$$D_{\mu\nu}(k) = -(1/e^2)\left[k_\mu k_\nu D_{\eta'} + (\beta k_\mu + \overline{k}_\mu)(\beta k_\nu + \overline{k}_\nu)D_{\chi'}\right], \tag{24}$$

where $\overline{k}_\mu \equiv \epsilon_{\mu\rho}k^\rho$. Using now the free propagators for the fields η' and χ' we get

$$D_{\mu\nu}(k) = -(i/(k^2 - m^2))\left[g_{\mu\nu} - \left(1 + \beta^2 - \alpha + \alpha m^2/k^2\right)k_\mu k_\nu/k^2 + \right.$$

$$- \beta(k_\mu \tilde{k}_\nu + \tilde{k}_\mu k_\nu)/k^2 \Big] . \tag{25}$$

This last expression, as far as we know, has not been obtained before for the GSM. It can be checked through its particular cases, specifically that of the CSM. For $\beta = 0$, $M^2 = a\ e^2$, $g_1 = -g_2 = e$, it corresponds to the *(ii)* case in the work of Harada and Tsutsui [13], and for $\beta = (a - 1)^{-1}$ to their case *(iii)*.

Now if we use the second GF condition $\mathcal{L}_{GF}^{(ii)}$, the decoupling transformations are

$$\eta = \eta' + \left[\frac{g_1 g_2}{(g_1^2 - M^2)}\right]\chi' - \theta',\ \chi = \chi'\ ,\ \theta = \theta', \tag{26}$$

also valid only for the $M^2 = g_1^2$ case as can be easily seen. The transformed Lagrangian density in this case is

$$\mathcal{L} = \mathcal{L}_{GSM} + \mathcal{L}_{GF}^{(ii)} = (1/2e^2)\chi'\Box(\Box + m^2)\chi' + (1/2\pi e^2)(g_1^2 - M^2)\eta'\Box\eta' +$$

$$+ (1/2\alpha)\theta'\Box\theta' + (1/e)J^\mu\left[\partial_\mu\left(\eta' + \left[\frac{g_1 g_2}{(g_1^2 - M^2)}\right]\chi' - \theta'\right) + \epsilon_{\mu\rho}\partial^\rho\chi'\right] +$$

$$+ \bar{\psi}'\bar{U}_5'\zeta + \bar{\zeta}U_5'\psi', \tag{27}$$

leading us to the photon propagator given by

$$D_{\mu\nu}(k) = (i/(k^2 - m^2))\left\{-g_{\mu\nu} + \left[\frac{1}{(M^2 - g_1{}^2)}\right]\left[k_\mu k_\nu\left[(1/e^2)(\pi e^2 + \right.\right.\right.$$

$$\left. - \alpha(M^2 - g_1^2)) + (1/k^2)\left((m^2\alpha/e^2)(M^2 - g_1{}^2) - (g_1^2 + g_2^2)\right)\right] +$$

$$\left.\left.(g_1 g_2/k^2)(k_\mu \bar{k}_\nu + \bar{k}_\mu k_\nu)\right]\right\}. \tag{28}$$

This case can also be compared with the results of the CSM by particularizing it. For the case $\alpha = 0$ the anomalous result is recovered, in agreement with the results for the CSM [7,13]. Besides it is possible to eliminate the misbehaved term in the coupling constant $(1/e^2)$.

IV) The $M^2 = g_1^2$ case:

Now we are going to treat the case where $M^2 = g_1^2$. For this we will develop an alternative way to calculate the correlation functions, without need to discover the decoupling transformations. In this approach we rewrite the bosonic part of the Lagrangian density for the non-anomalous GSM (14) in a matrix form

$$\mathcal{L} = (1/2)\rho^T \mathbb{M}\, \rho, \tag{29}$$

where $\rho = \begin{pmatrix} \chi \\ \eta \\ \theta \end{pmatrix}$, and $\mathbb{M}$ will differ from one gauge fixing to another. Then we can compute the correlation functions by inverting $\mathbb{M}$, obtaining

$$\mathbb{M}^{-1} = \begin{pmatrix} \langle\chi\ \chi\rangle & \langle\chi\ \eta\rangle & \langle\chi\ \theta\rangle \\ \langle\eta\ \chi\rangle & \langle\eta\ \eta\rangle & \langle\eta\ \theta\rangle \\ \langle\theta\ \chi\rangle & \langle\theta\ \eta\rangle & \langle\theta\ \theta\rangle \end{pmatrix}, \tag{30}$$

where the elements $\langle\chi\ \chi\rangle$, $\langle\chi\ \eta\rangle$, etc. , are the two-point correlation functions of their respective fields, from which one can write down the other ones as, for example, that for the photon propagator:

$$\langle A_\mu A_\nu\rangle = -(1/e^2)\left[k_\mu k_\nu\langle\eta\ \eta\rangle + k_\mu\bar{k}_\nu\langle\eta\ \chi\rangle + \bar{k}_\mu k_\nu\langle\chi\ \eta\rangle + \bar{k}_\mu\bar{k}_\nu\langle\chi\ \chi\rangle\right]. \tag{31}$$

It is important to remark that, in the construction of the matrix $\mathbb{M}$, we must to symmetrize it with respect to the principal diagonal, as a consequence of the hermicity requirement.

One more time we will start by solving the anomalous version. Here the matrix $\mathbb{M}$ is written as follows

$$\mathbb{M} = (\Box/e^2)\begin{pmatrix} \Box + (g_1^2 + g_2^2)/\pi & -\, g_1 g_2/\pi \\ -\, g_1 g_2/\pi & 0 \end{pmatrix}. \tag{32}$$

Inverting it, collecting the terms $\langle\eta\ \eta\rangle$, $\langle\chi\ \eta\rangle$, etc.., and substituting them in (31), we get

$$D_{\mu\nu}(k) = (\pi/g_1 g_2 k^2)\left[k_\mu k_\nu\left(\pi k^2/(g_1 g_2) - (g_1^2 + g_2^2)/(g_1 g_2)\right) + \right.$$

$$\left. - \left(k_\mu \bar{k}_\nu + \bar{k}_\mu k_\nu\right)\right], \tag{33}$$

in agreement with the result of the calculations on the anomalous GSM appearing in Ref.[17], where the Dirac quantization method was used. In particular it is seen that this case does not generate mass dynamically.

Now we apply the approach sketched above to the case of the non-anomalous model in the gauges imposed through $\mathcal{L}_{GF}^{(I)}$ and $\mathcal{L}_{GF}^{(II)}$ respectively. In the first case $\mathbb{M}$ is given by

$$\mathbb{M} = \frac{\Box}{e^2}\begin{pmatrix} (1-\beta^2/\alpha)\Box + (g_1^2 + g_2^2)/\pi & (\beta/\alpha)\Box - g_1 g_2/\pi & - g_1 g_2/\pi \\ (\beta/\alpha)\Box - g_1 g_2/\pi & -(1/\alpha)\Box & 0 \\ - g_1 g_2/\pi & 0 & 0 \end{pmatrix} \tag{34}$$

Substituting the correlation functions $<\eta\ \eta>$, $<\eta\ \chi>$, etc. in the expression (27) and using the relation $\bar{k}_\mu \bar{k}_\nu = - g_{\mu\nu}k^2 + k_\mu k_\nu$, we find

$$<A_\mu A_\nu> = \frac{\alpha\ k_\mu k_\nu}{k^4}\ . \tag{35}$$

Surprisingly this shows us that one could gauge away this correlation function in the Landau gauge (α =0). In a certain sense, this reinforces the idea of using gauge-independent correlation functions, as we will see more below.

In the second case the matrix $\mathbb{M}$ is

$$\mathbb{M} = (\Box/e^2)\begin{pmatrix} \Box + (g_1^2 + g_2^2) & -(g_1g_2/2\pi) & -(g_1g_2/2\pi) \\ -(g_1g_2/2\pi) & 0 & 0 \\ -(g_1g_2/2\pi) & 0 & (1/2\alpha) \end{pmatrix} \tag{36}$$

and the photon propagator is

$$\langle A_\mu A_\nu\rangle = \pi(k_\mu k_\nu/k^2)\left[k^2 - (g_2^2 + g_2^2) + \alpha(g_1g_2/\sqrt{\pi}\ e^2)\right] + \\ - (\pi/g_1g_2)\left[(k_\mu\bar{k}_\nu + \bar{k}_\mu k_\nu)/k^2\right]. \tag{37}$$

Again, we find a nasty behaviour in the coupling constant that can be eliminated when $\alpha = 0$. This time, however, the propagator cannot be eliminated through a convenient choice of the gauge parameter, as in the previous case.

From the results obtained above, one can observe that the gauge-dependent correlation function $\langle A_\mu A_\nu\rangle$ changes drastically from one gauge to another. Now we will discuss the case where the source terms include the WZ field [10]. In the bosonic case the source term becomes

$$J_\mu\left(A^\mu + (1/e)\partial^\mu\theta\right) \equiv J_\mu A_I^\mu\ , \tag{38}$$

where A^I_μ is a gauge-invariant field. Now, one can see that the propagators in the above gauges, or in any other one, will be the same as that of the anomalous model, as previously observed through different approaches in the CSM [12]. As an example we compute the gauge-invariant photon propagator in the $M^2 = g_1^2$ case, so obtaining

$$D^I_{\mu\nu}(k) = \langle A^I_\mu \, A^I_\nu \rangle = \langle A_\mu A_\nu \rangle - (1/e^2) k_\mu k_\nu \Big(\langle \eta \; \theta \rangle + \langle \theta \; \eta \rangle + \langle \theta \; \theta \rangle \Big) +$$

$$- (1/e^2) \Big(k_\mu \bar{k}_\nu \langle \theta \; \chi \rangle + \bar{k}_\mu k_\nu \langle \chi \; \theta \rangle \Big) . \qquad (39)$$

Then, after straightforward calculations we get the same propagator as in the anomalous case, appearing in (33). It is not difficult to convince oneself, after a little experimentation, that this will happen with every gauge-invariant correlation function.

V) On a generalized Wess-Zumino term:

In this section we intend to look for the possibility of the introduction of a generalized Wess-Zumino term. For this we will proceed analogously to Harada and Tsutsui [5]. In that paper they show that, from a careful dealing with the functional measure, that term appears quite naturally. Their starting point was the integral over only one representative of each class of the gauge field configurations in the vacuum-to-vacuum amplitude. This was done by using the Faddeev-Popov trick and then "unfixing" the gauge, going to an integral over all the gauge field configurations (equivalent or not). Let us see now how to do it in our case. Defining $\Delta_f[A]$ by

$$\Delta_f[A]\int dg\ \delta\left(f[A^{g^2}]\right) = 1, \tag{40}$$

where g is an invariant measure on the gauge symmetry group G. Then, inserting (40) in the vacuum-to-vacuum amplitude we find

$$Z = \int \mathcal{D}\psi\ \mathcal{D}\overline{\psi}\ dA_\mu \Delta_f[A] dg\ \delta\left(f[A^{g^2}]\right)\exp\left[i\ I(\psi,\overline{\psi},A)\right]. \tag{41}$$

Performing now a transformation in the integration field $A_\mu \to A_\mu^{g^{-1}}$, using the invariance of the measure dA_μ and of $\Delta_f[A]$, comes:

$$Z = \int \mathcal{D}\psi\ \mathcal{D}\overline{\psi}\ \mathcal{D}A_\mu dg\ \delta\left(f[A^g]\right)\exp\left[i\ I(\psi,\overline{\psi},A^{g^{-1}})\right], \tag{42}$$

where $\mathcal{D}A_\mu \equiv dA_\mu \Delta_f[A]$. Now, transforming the fermionic fields like

$$\psi_L \to \psi_L^g = g^{-1}\psi_L\ ,\ \overline{\psi}_L \to \overline{\psi}_L^g = \overline{\psi}_L g\ , \tag{43}$$

and remembering that the fermionic measure is not invariant (for anomalous models) under this transformation, we obtain finally that

$$Z = \int \mathcal{D}\psi\ \mathcal{D}\overline{\psi}\ \mathcal{D}A_\mu dg\ \delta\left(f[A^g]\right)\exp\left[i\ \left(I(\psi,\overline{\psi},A) + \alpha(A,g^{-1})\right)\right], \tag{44}$$

where we used that $I(\psi^g,\overline{\psi}^g,A^{g^{-1}}) = I(\psi,\overline{\psi},A)$. This expression, in contrast with (41), is totally gauge-invariant. This way we have "unfixed" the gauge defined by $f[A] = 0$. As an example we will treat the case of the Schwinger model in a Lorentz-type generalized WZ term. This is done to illustrate the differences between the conventional WZ term and the generalized one, in a very well established model.

Let us begin by writing down the Lagrangian density for the bosonized vector Schwinger model (VSM),

$$\mathcal{L}_{VSM} = (1/2)\partial_\mu\phi\partial^\mu\phi + e\,\epsilon^{\mu\nu}\partial_\mu\phi\,A_\nu + (a\,e^2/2)A_\mu A^\mu - (1/4)F_{\mu\nu}F^{\mu\nu}, \quad (45)$$

where we choose $g_1 = 0$, $g_2 = -\sqrt{\pi}\,e$ and $M^2 = \pi\,a\,e^2$ in the equation (9). This model is invariant under the gauge transformations $\delta\phi = 0$ and $\delta A_\mu = -(1/e)\partial_\mu\lambda$, provided that the arbitrary parameter be fixed such that $a = 0$. On the other hand this parameter can be kept arbitrary if it is introduced a conventional WZ term [5],

$$\mathcal{L}_{WZ} = (a/2)\partial_\mu\theta\partial^\mu\theta + a\,e\,\partial_\mu\theta\,A^\mu. \quad (46)$$

In this case the mass of the model is computed as $m^2 = (a + 1)e^2/\pi$, and this was done without loosing the gauge invariance. Furthermore, the usual mass e^2/π is recovered when a is chosen to be equal to zero. In this case, the WZ vanish, surviving an infinity that can be absorbed in the functional normalization.

Now we will introduce a generalized WZ term of Lorentz type, and will study its consequences,

$$\mathcal{L}_{GWZ} = -(1/2\alpha)(\partial_\mu A^\mu)^2 + (a/2)\partial_\mu\theta\partial^\mu\theta + a\ e\ \partial_\mu\theta\ A^\mu - (1/2\alpha e^2)(\Box\theta)^2 +$$
$$- (1/\alpha e)\Box\theta\ \partial_\mu A^\mu. \qquad (47)$$

Now imposing a gauge-fixing term of the type (*ii*) (with a gauge parameter γ), one gets:

$$\mathbb{M} = (\Box/e^2)\begin{pmatrix} \Box + e^2/\pi & 0 & 0 \\ 0 & -\Box/\alpha - ae^2/\pi & -\Box/\alpha - ae^2/\pi \\ 0 & -\Box/\alpha - ae^2/\pi & -\Box/\alpha - (a/\pi - \gamma^{-1})e^2 \end{pmatrix}, \qquad (48)$$

inverting the above matrix and computing the gauge-invariant photon propagator we find

$$D^I_{\mu\nu}(k) = \frac{i}{(k^2 - m_I^2)}\left\{ - g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \alpha\ \frac{(k^2 - m_I^2)}{(k^2 - m_D^2)}\right)\right\}, \qquad (49)$$

where $m_I^2 = e^2/\pi$ and $m_D^2 = \alpha\ a\ e^2/\pi$. As can be seen from above, the more remarkable feature of the generalized WZ term, is that of appearing gauge-dependent massive excitations like m_D. Besides we see that, in this case, the arbitrariness on the regularization parameter appears in the gauge-dependent mass, so that it disappears when this mass is gauged-away. This is done by working in the Landau gauge ($\alpha = 0$), leading to the correct propagator in this gauge, but now without any ambiguity in the mass ($m_I^2 = e^2/\pi$, $m_D^2 = 0$). Furthermore, the case $a = 0$ (the case where the usual WZ term vanishes) brings us to the usual propagator in a Lorentz-type gauge,

$$D^I_{\mu\nu}(k)\Big|_{a=0} = \frac{i}{(k^2 - e^2/\pi)}\left\{ - g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \alpha(k^2 - e^2/\pi)\right)\right\}. \tag{50}$$

Showing us that the generalized WZ term does not changes the photon propagator, when it is used its gauge-invariant version $D^I_{\mu\nu}$, as should be expected from a well-behaved WZ term.

A similar study can be made in the case of the axial Schwinger model, finding analogous results. Also in this model the generalized WZ term , contrary to the usual one, leads to the conclusion that the mass is not arbitrary but it is precisely equal to that of the VSM, when the gauge-dependent mass is eliminated.

Now we are going to treat the case of the chiral Schwinger model (CSM). As it is well known, this model is an anomalous one and, when it is added with an usual WZ, get its gauge-invariant counterpart. However this model in contrast with the VSM, does not have its mass fixed through the requirement of gauge invariance in both cases. In the anomalous case, there is none available choose of the parameter a such that the anomaly be canceled, and in the second case the gauge-invariance is kept for an arbitrary value of a.

The case of the CSM in a Lorentz type generalized WZ term, when written in the form (29), can be obtained from the GSM with the charges $g_1 = e$ and $g_2 = -e$, giving the matrix

$$\mathbb{M} = (\Box/e^2)\begin{pmatrix} \Box + 2e^2/\pi & e^2/\pi & e^2/\pi \\ e^2/\pi & -\Box/\alpha + (e^2-M^2)/\pi & -\Box/\alpha + (e^2-M^2)/\pi \\ e^2/\pi & -\Box/\alpha + (e^2-M^2)/\pi & -\Box/\alpha + ((1+\gamma^{-1})e^2-M^2)/\pi \end{pmatrix}, \tag{51}$$

whose inversion gives us the following elements in the momentum representation,

$$<\chi\ \chi> = \frac{e^2(\pi k^2 - \alpha(M^2 - e^2))}{\pi k^2(k^2 - m_I^2)(k^2 - m_D^2)}, \tag{52a}$$

$$<\chi\ \eta> = <\eta\ \chi) = \frac{\alpha e^4}{\pi k^2(k^2 - m_I^2)(k^2 - m_D^2)}\ , <\chi\ \theta> = <\theta\ \chi> = 0\ , \tag{52b}$$

$$<\eta\ \eta> = -\frac{1}{\pi k^2(k^2 - m_I^2)(k^2 - m_D^2)}\left\{\pi\gamma k^4 + \left[\alpha\left(\pi e^2 - \gamma(M^2 - e^2)\right) - 2\gamma e^2\right] + \right.$$

$$\left. + \alpha e^2\left(\gamma(2M^2 - e^2) - 2e^2\right)\right\}, \tag{52c}$$

$$<\eta\ \theta> = <\theta\ \eta> = \gamma/k^2,\ <\theta\ \theta> = -\gamma/k^2, \tag{52d}$$

where we defined that,

$$m_I^2 \equiv (1/2\pi)\left[\left(\alpha^2(M^2-e^2)^2-4e^2(\alpha M^2-e^2)\right)^{1/2}+\alpha(M^2-e^2)+2e^2\right],$$

(53a)

$$m_D^2 \equiv -(1/2\pi)\left[\left(\alpha^2(M^2-e^2)^2-4e^2(\alpha M^2-e^2)\right)^{1/2}-\alpha(M^2-e^2)-2e^2\right].$$

(53b)

From the above elements one can compute, for example, the gauge-dependent two-point Green function (31),

$$D_{\mu\nu}(k) = -\frac{1}{(k^2-m_I^2)(k^2-m_D^2)}\Bigg\{-g_{\mu\nu}\left(k^2-\alpha(M^2-e^2)/\pi\right)+ \\ +\alpha e^2(k_\mu\bar{k}_\nu+\bar{k}_\mu k_\nu)/\pi k^2-\frac{k_\mu k_\nu}{\pi e^2k^2}\left[\pi\gamma k^4+\left((\alpha-1)\pi e^2-\gamma\left(\alpha(M^2-e^2)+ \\ +2e^2\right)\right)k^2+\alpha e^2\left(M^2-3e^2+\gamma(2M^2-e^2)/\pi\right)\right]\Bigg\}. \quad (54)$$

Alternatively we can calculate the gauge-invariant correlation function (39) obtaining,

$$D^I_{\mu\nu}(k) = -\frac{1}{(k^2-m_I^2)(k^2-m_D^2)}\Bigg\{-g_{\mu\nu}\left(k^2-\alpha(M^2-e^2)/\pi\right)+ \\ +\alpha e^2(k_\mu\bar{k}_\nu+\bar{k}_\mu k_\nu)/\pi k^2-\frac{k_\mu k_\nu}{\pi k^2}\left((\alpha-1)\pi k^2+\alpha(M^2-3e^2)\right)\Bigg\}. \quad (55)$$

Once more the Landau gauge is the one in which one of the masses vanishes, for $\alpha = 0$ $m_D^2 = 0$ and $m_I^2 = 2e^2/\pi$. In this convenient gauge, the photon propagator $D_{\mu\nu}^I(k)$ is equal to that of the VSM but with a different mass for the gauge boson.

Now one can see that the generalized WZ term acts in such way that the massive excitation in the CSM is no more arbitrary. As can be obtained from (17) the anomalous (or even the non-anomalous with usual WZ) has an arbitrary mass $m^2= M^4/\pi(M^2- e^2)$. However the introduction of a generalized WZ leads to a fixed value of the mass for the gauge boson of the model ($m^2= 2e^2/\pi$), similarly to what happened in the cases of the VSM and ASM as we have seen above in this work. In fact it is easy to see that this value for the mass cannot be obtained through a real value of the arbitrary parameter M^2.

Another important observation is that now the gauge-invariant propagator $D_{\mu\nu}^I$, maintain some arbitrarity in the parameter α, although the parameter γ is eliminated. So, this propagator, differently to the case of the usual WZ [12], keeps its arbitrariness through the parameter α. This feature opens the possibility of using a generalized WZ term of θ-type (*ii*), fixing a gauge where the bad behavior of the CSM for vanishing coupling constant (the term $1/e^2$ in the propagator) [3] be gauged-away. Moreover these models have as a characteristic that the parameter α cannot be arbitrarily fixed, at least when one wishes to keep unitarity.

As in the case of usual WZ term, we must study it in a situation where $M^2 = g_1^2$, or $M^2 = e^2$, because we are now treating the special case of CSM. From the above discussion one could expect that this care would lead no more to a physically different content for the model (in the case of the usual WZ, it has not a massive excitation). This is so because now we see that the physical mass of the model, which appears when one works in the Landau gauge, does not depend on the regularizing parameter.

Let us see if this indeed happens. For this we take the matrix (51) and impose that $M^2 = e^2$. Then we invert it obtaining the two-point correlation functions $<\chi\ \chi>$, $<\chi\ \eta>$, etc. . It is easy to see that these expressions match with that in (52) when $M^2 = e^2$ is substituted in the latter. Consequently the masses are now written as,

$$m_I^2 = \frac{e^2}{\pi}\left[\sqrt{1 - \alpha} + 1\right], \tag{56a}$$

$$m_D^2 = -\frac{e^2}{\pi}\left[\sqrt{1 - \alpha} - 1\right], \tag{56b}$$

from which we see that when $\alpha = 0$, $m_D^2 = 0$ and $m_I^2 = 2\ e^2/\pi$ as supposed above. Moreover, in this simple case it becomes clearer that, in order to keep the theory unitary, one must restrict the range of validity for the parameter α $(0 \leq \alpha \leq 1)$.

VI) Final remarks

In this work we discussed the higher-order version of the GSM in its anomalous and non-anomalous versions. For this it was used the path-integral formalism for the quantization, and inside this framework two methods were used, one looking for decoupling transformations and then doing the functional derivatives with respect to the sources to get the correlation functions, and another where it was calculated the elementary two-point correlation functions and then compute the full correlation functions of the gauge field. This was performed both in the case where $M^2 \neq g_1^2$ as in the singular case with $M^2 = g_1^2$ (a = 1 for CSM). The corresponding Green functions were, when available, compared with that of the literature. Moreover it was verified the equality of the gauge-invariant correlation functions with that of the anomalous model as stated by Girotti and Rothe in the case of CSM [12].

Furthermore it was introduced a generalized WZ term in an analogous fashion of Harada and Tsutsui [5], whose consequences were studied in the particular cases of the VSM, ASM and CSM. It was observed that this term has as fundamental feature the introduction of further massive excitations, and that these have a gauge dependence through the gauge-parameters. Another interesting characteristic is that some of these gauge-dependent masses disappear in an appropriately chosen gauge, and that the resulting mass is precisely that of the physical model, without any dependence in the regularization parameter. This is well exemplified in the VSM and in the ASM.

As a consequence of applying this generalized WZ to the case of the CSM, we saw that it makes the mass renders unambiguous in a quite surprisingly result. If it is true, it would be a very good result because the ambiguity of a physical mass is not a pleasant thing. However, the confirmation or not of a such characteristic, deserves a further study on the constraint structure of the theory with the generalized WZ term. This should be done in order to conclude if this new WZ, as the usual one [8,9], does not alter the physical content of the model, or if it corresponds to a new model, in the sense that it has a completely different Hilbert space and particle spectrum. We expect to report on this in the near future.

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