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CRITICAL SURFACE OF THE QUENCHED BOND-DILUTE CUBIC MODEL IN SELF-DUAL LATTICE: RENORMALISATION GROUP APPROACH

by

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ABSTRACT

We propose a quite simple real space renormalisation group which enables us to calculate (for the first time as far as we know,and presumably with high precision) the critical surface of the quenched bond-diluted discrete N-vector ferromagnet.

Key-words:Cubic model;Criticality;Renormalisation group;Quenched ferromagnet

The quenched site-diluted problem of an extended cubic ferromagnet (or discrete N-vector ferromagnet) was studied in detail[1] in what concerns its criticality (phase diagram, universality classes, critical exponents). The technique was a variational renormalisation group one, and enabled the exhibition of a rich phase diagram as well as of a non trivial fixed line associated with the Ashkin-Teller model. However, the precision of the numerical results (particularly those corresponding to the critical surface) tends to be rather rough.

In the present letter we focus the critical surface of the quenched bond-diluted problem of the same system. As we shall verify later on, the calculation yields results whose precision is presumably quite high, in spite of the simplicity of the method.

The quenched bond-diluted (extended) cubic model is defined by the dimensionless Hamiltonian

$$-\beta\mathcal{H} = NK \sum_{\langle i,j\rangle} \mathbf{S}_i \cdot \mathbf{S}_j + N^2 L \sum_{\langle i,j\rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 \qquad (1)$$

where $\beta \equiv 1/k_B T$, $\langle i,j\rangle$ runs over all the couples of first-neighbouring sites, the spin $\mathbf{S}_i$ at any given site is an N-component unitary vector which can point only along the 2N positive or negative orthogonal coordinate directions (i.e., $\mathbf{S}_i=(\pm1,0,0,\ldots,0)$ or $(0,\pm1,0,\ldots,0)$ or ...or $(0,0,0,\ldots,\pm1)$); K and L are coupled random variables satisfying the probability law

$$\mathcal{P}(K\ ,L\ ) = (1-p)\delta(K\ )\delta(L\ ) + p\delta(K\ -K_0)\delta(L\ -L_0) \qquad (2)$$

with $K_0>0$ and $K_0+NL_0>0$ in order to guarantee that the fundamental state is the ferromagnetic one. At this point, let us introduce the following convenient variables (referred to as thermal transmissivities[2]):

$$t_1 = \frac{1-\exp(-2NK)}{1+2(N-1)\exp[-N(K+NL)]+\exp(-2NK)} \qquad (3a)$$

$$t_2 = \frac{1-2\exp[-N(K+NL)]+\exp(-2NK)}{1+2(N-1)\exp[-N(K+NL)]+\exp(-2NK)} \qquad (3b)$$

Consequently, Eq.(2) can be re-written as follows:

$$\mathcal{P}(t_1,t_2) = (1-p)\delta(t_1)\delta(t_2) + p\delta(t_1-t_1^0)\delta(t_2-t_2^0) \qquad (4)$$

To discuss the criticality of this system we shall now construct a real space renormalisation group (RG) by following along the lines of Refs.(3). This type of approach has proved fruitful in a variety of similar systems. The approximation consists in replacing the square lattice (our primary interest) by the self-dual Wheatstone-bridge hierarchical lattice (see Fig.1). It is this self-duality which, together with other ingredients of the procedure, will guarantee high precision results for the phase diagram.

If we associate with each one of the five bonds of the graph of Fig.1 the distribution given by Eq.(4),we shall obtain an overall distribution which we note $P_W(t_1,t_2)$.This distribution is more complex than a binary one.At this point we introduce a second approximation,namely we shall use,instead of $P_W(t_1,t_2)$,the following renormalised binary one

$$\mathcal{P}'(t_1,t_2) = (1-p')\delta(t_1)\delta(t_2) + p'\delta(t_1-t_1^{0\prime})\delta(t_2-t_2^{0\prime}) \tag{6}$$

To obtain $(p',t_1^{0\prime},t_2^{0\prime})$ as function of $(p\ ,t_1^0,t_2^0\ )$ we impose the following set of equations:

$$p' = 2p^5 - 5p^4 + 2p^3 + 2p^2 \tag{6a}$$

$$< t_1 >_{\mathcal{P}'} = < t_1 >_{\mathcal{P}_W} \tag{6b}$$

$$< t_2 >_{\mathcal{P}'} = < t_2 >_{\mathcal{P}_W} \tag{6c}$$

where <...> denotes the standard mean value.Notice that Eq.(6a) is nothing but the standard bond percolation recursive relation associated with the Wheatstone-bridge graph.Eqs.(6b) and (6.c) can be <u>explicitly</u> solved thus yielding

$$t_1^{0\prime} = F_1(p,t_1^0,t_2^0) \tag{7a}$$

$$t_2^{0\prime} = F_2(p,t_1^0,t_2^0) \tag{7b}$$

where F_1 and F_2 are relatively simple functions.

Through recurrent use of Eqs.(6a),(7a) and (7b) we obtain the

RG flow in the (p,t_1^0,t_2^0) space.We verify the existence of four relevant fully stable fixed points,namely at $(p,t_1^0,t_2^0)=(1,0,0)$ and (0,0,0) characterising the <u>paramagnetic</u> (P) phase,as well as (1,1,1) and (1,0,1) respectively characterising the <u>ferromagnetic</u> (F) phase and the <u>intermediate</u> (I) phase (we recall that in the intermediate phase the system has chosen one of N axis,but not a direction within this axis).The unstable fixed points (t_1,t_2) on the p=1 plane are: Ising $(\sqrt{2}-1,1)$, N-state Potts $(0,(\sqrt{N}+1)^{-1})$, 2N-state Potts $(t_1=t_2=(\sqrt{2N}+1)^{-1})$ and N cubic(which for N=1 is (0.41,0.03), for N=2 is (0.41,0.17), and for N=3 is (0,34,0.21)).The critical surfaces for typical values of N are indicated in Figs.(2) and (3).The linearisation of the RG equations in the neighbourhood of the fixed points yields nonvanishing Jacobian matrix which typically enable the calculation of the slopes indicated in Table 1.

The fact that,within the present approximation scheme,the RG recurrence for p is decoupled from (t_1^0,t_2^0) makes this tool not appropriate for discussing the universality classes (for a discussion of this point for p=1,see Refs.(2) and (4)).Furthermore it gives rise to an artificial bump in the phase diagram near the percolation point p=1/2 (this kind of difficulty has been already encountered in similar approaches[5]).

Let us conclude by saying that,in spite of its mathematical simplicity,the present treatment yields critical surfaces which we believe to be quite good approximations for the square lattice (whenever the transition is a second order one) and the

Wheatstone-bridge hierarchical lattice (for which the transition seems to always be a second order one).

Caption for Figures and Tables

Figure 1:Iteration associated with the Wheatstone-bridge(● and 0 respectively denote the internal and terminal sites of the graph).

Figure 2 :Phase diagram of the bond-diluted N=2 cubic model. [P], [F], [I] denote the Paramagnetic,Ferromagnetic and Intermediate phase respectively. Δ, ●, ■ denote respectivelythe fully unstable, semi-stable and fully stable fixed points.

The region below the plane determined by the point $(p,t_1^0,t_2^0)=(1,1,1)$ and the straight line joining the points (1,1/2,0) and (1/2,1,1), is a forbidden one since the coupling constants K and L become imaginary there

Figure 3:Typical p-planes:(a) for N=2,(b)for N=3.

Table 1:Values for some fixed points and related derivatives on the p=1 plane.

TABLE 1

SLOPE	PRESENT RG	EXACT
$- \left. \frac{\partial t_1}{\partial p} \right\|_{t_1^0 = 2 - 1,\ t_2^0 = 1}$	0.45	$6\sqrt{2} - 8$ $\cong 0.48$
$- \left. \frac{\partial t_1}{\partial p} \right\|_{t_1^0 = 0.41,\ t_2^0 = 0.17}$	0.50	1/2
$- \left. \frac{\partial t_1}{\partial p} \right\|_{t_1^0 = 0.34,\ t_2^0 = 0.21}$	0.30	?
$- \left. \frac{\partial t_1}{\partial p} \right\|_{t_1^0 = t_2^0 = (1+ 2)^{-1}}$	0.45	$2\sqrt{2} / (1+\sqrt{2})^2$ $\cong 0.48$
$- \left. \frac{\partial t_1}{\partial p} \right\|_{t_1^0 = t_2^0 = 1/3}$	0.39	4/9

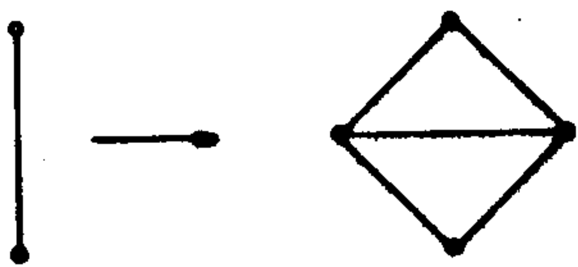

FIGURE 1

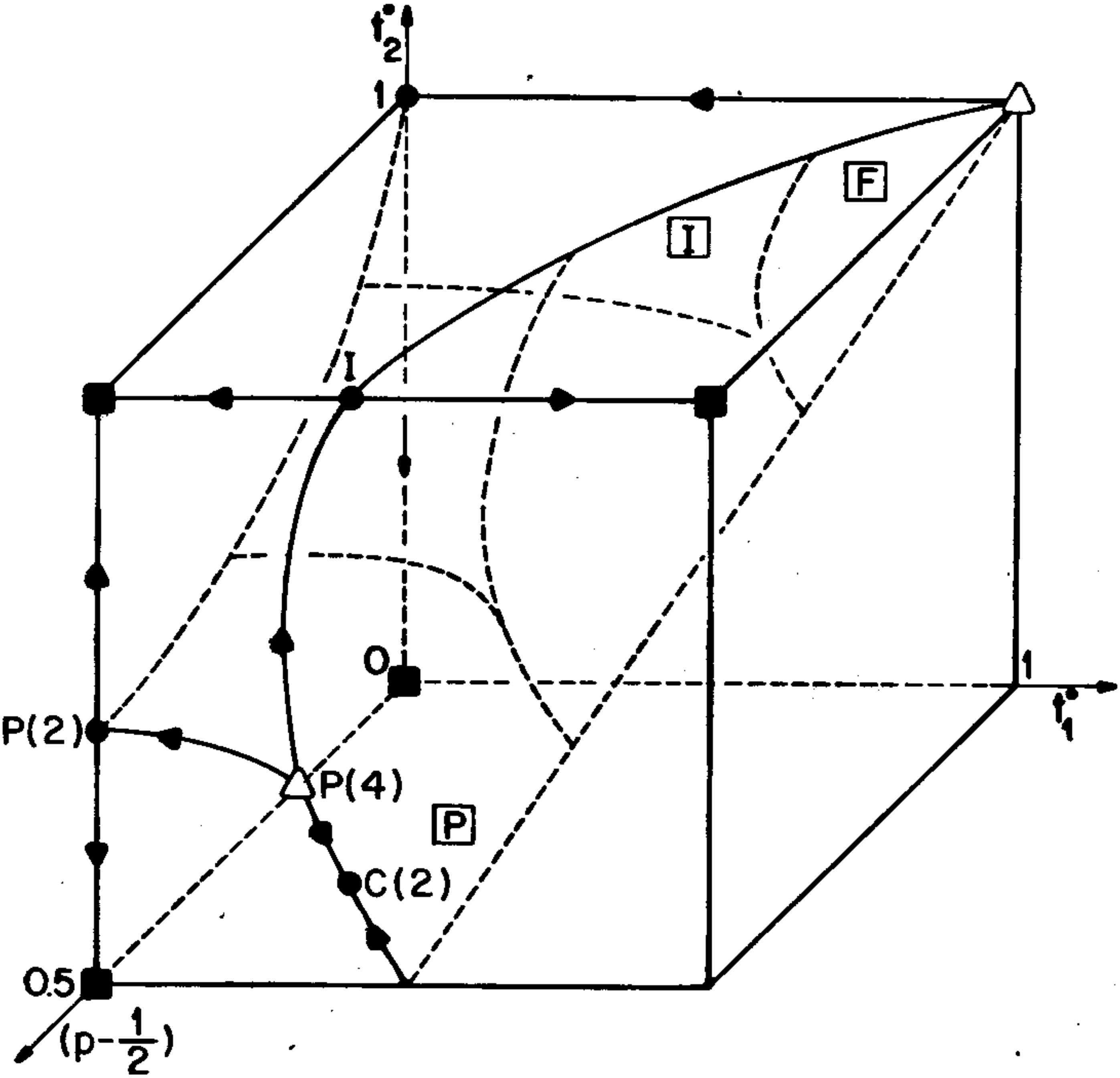

FIGURE 2

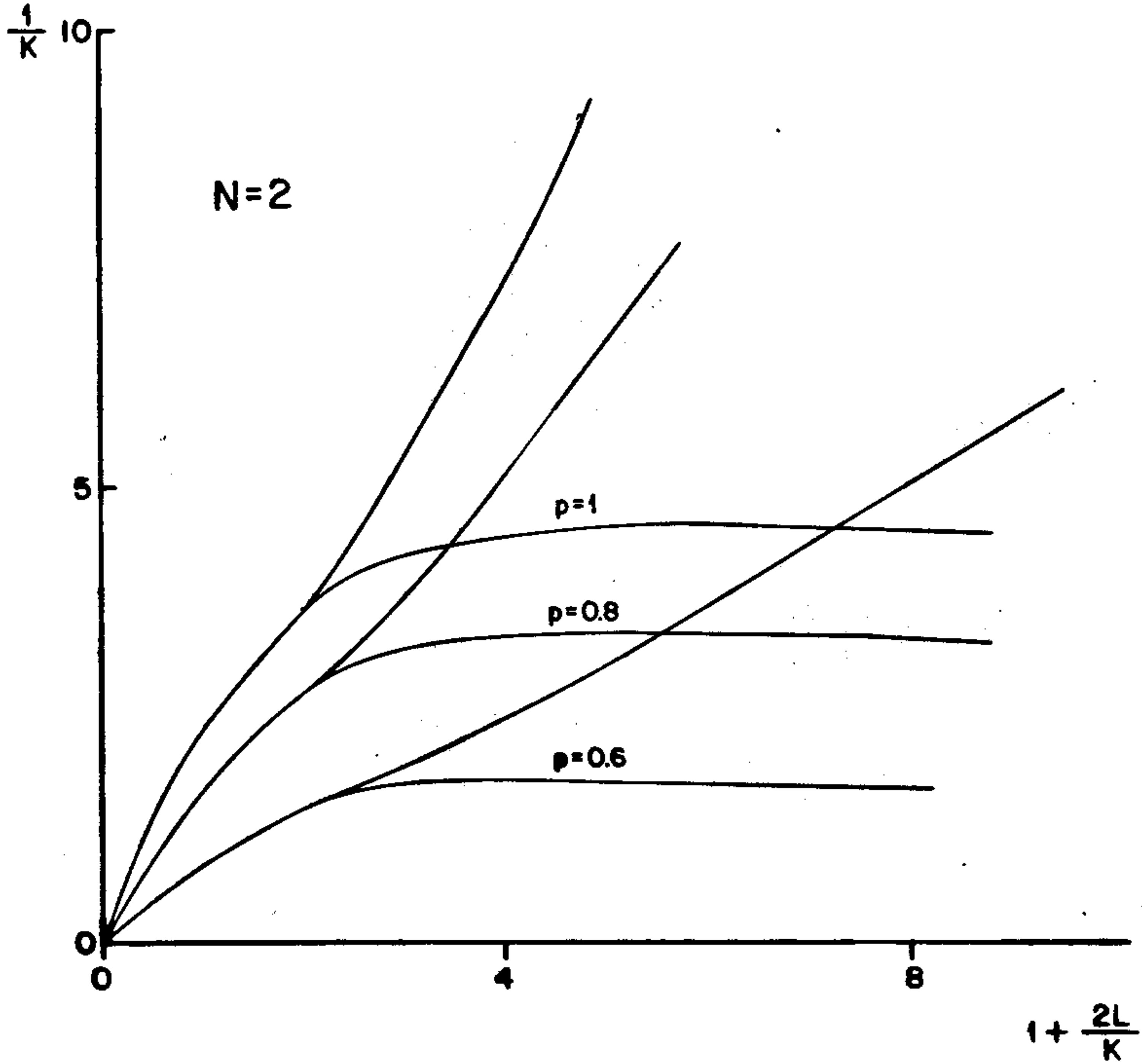

FIGURE 5a

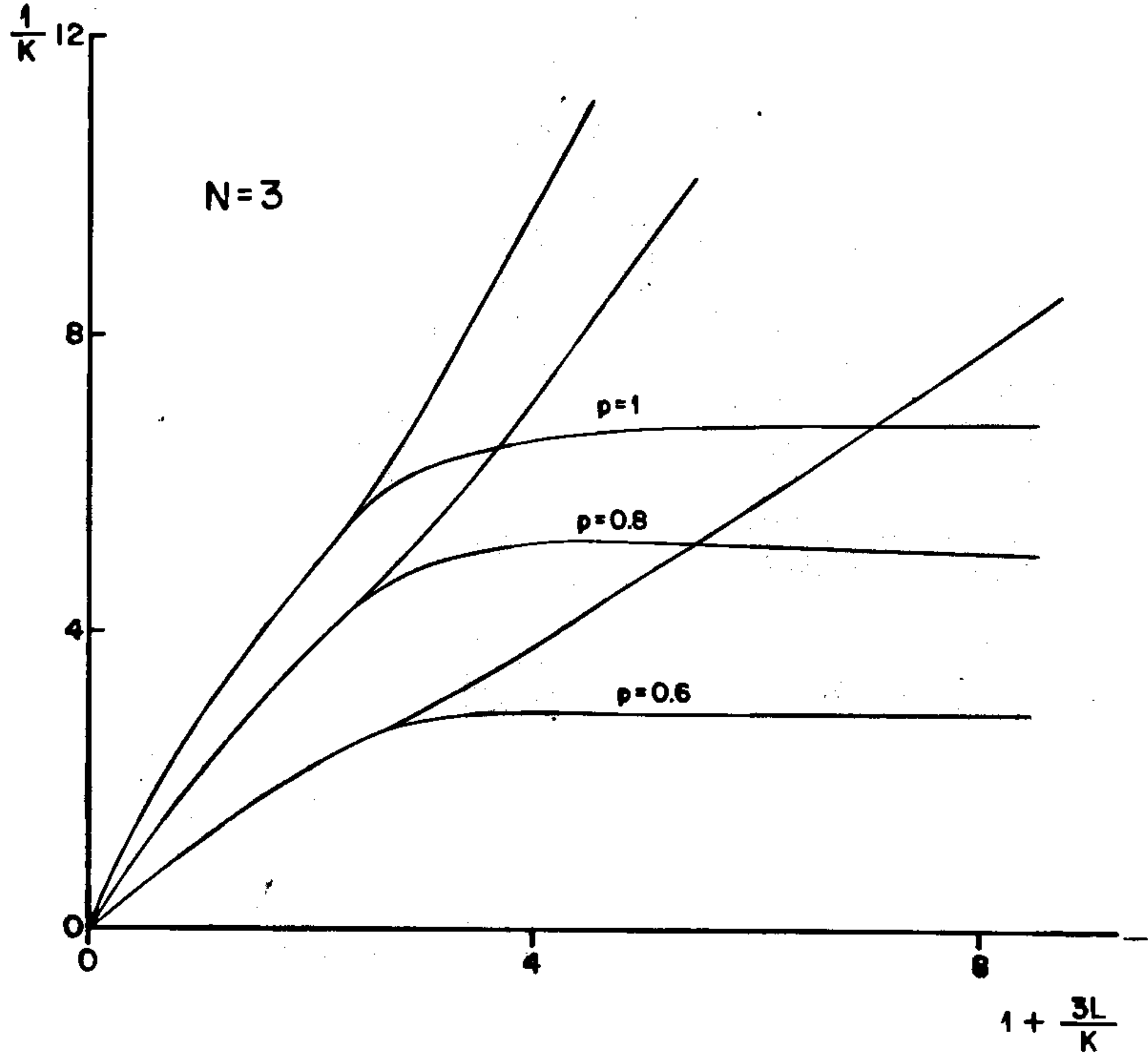

FIGURE 3b

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