ISSN 0029 - 3865

CBPF-NF-030/90

CHARMONIUM DECAYS INTO PROTON-ANTIPROTON AND A QUARK-DIQUARK MODEL FOR THE NUCLEON

by

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ABSTRACT

A quark-diquark model of the nucleon is applied to a perturbative QCD description of several decays of the charmonium family: $\eta_c, \chi_{c0,c1,c2}, \to p\bar{p}$. Both experimental data and theoretical considerations are used to fix the parameters of the model. Decay rates for the χ's in good agreement with the existing experimental results may be obtained. The values for the decay of the η_c are found instead to be much smaller than the data. Our formalism provides a general framework for the computation of the decay amplitudes of any ${}^{2S+1}L_J, C = +1$, heavy quarkonium state into hadron-antihadron. The explicit expression for the decay into two photons is also given.

Key-words: Diquark model; Perturbative QCD; Charmonium decays.

Introduction

The presence of diquarks as constituents inside nucleons has been extensively discussed in the literature [1] and seems to be well supported both by theoretical and experimental arguments. In a previous paper [2] we have computed the $\eta_c \to p\bar{p}$ decay, at the tree level in perturbative QCD, modeling the proton with a quark-diquark (qQ) system. Contrary to pure quark models [3], this approach allows to obtain a value for the decay rate which differs from zero. Its actual numerical value, however, still depends on several poorly known parameters, some of which have been fixed by comparison with other processes computed in a simplified version of our model [4,5].

We extend here the discussion of Ref.[2] to other decays of the charmonium family, in order to be able to fix the parameters and to provide a consistent check of our scheme. That is, we consider a full set of exclusive processes, in the same energy range and in the same framework, and we see if our model can give a good description for all of them. The energy range is that of the masses of the $c\bar{c}$ mesons, where diquarks are supposed to act as quasi-elementary objects, and the framework is the modified Brodsky-Farrar-Lepage scheme [3], already used in Ref.[2] and, with scalar diquarks only, in Ref.[5].

We consider the η_c and $\chi_{0,1,2}$ decays into $p\bar{p}$. We fix the values of the charmed meson wave functions in the origin by computing the decay rates of η_c and $\chi_{0,2}$ into two photons and comparing with the experimental data. We derive some of the remaining parameters by fitting our result for the decay rate of $\chi_2 \to p\bar{p}$ to the experimental data which, in this case, are well established. Other parameters are fixed using theoretical considerations.

We obtain a reasonable agreement with the known data on the decay rates of $\chi_1 \to p\bar{p}$; we can also get a result of the same order of magnitude for the decay rate of $\chi_0 \to p\bar{p}$, in agreement with an existing upper bound. We obtain, instead, much smaller values for the decay rate of $\eta_c \to p\bar{p}$, to be compared, unfortunately, with a seemingly very large experimental result. If such a disagreement should persist, even with more precise data, it would be a problem for the application of our quark-diquark model to the description of exclusive reactions.

The plan of the work is as follows. In Sect.1 we present our scheme and give the explicit formulae for the computation of the helicity amplitudes for the decay of any ${}^{2S+1}L_J$ heavy $q\bar{q}$ state into baryon-antibaryon. In Sect.2 we compute the elementary helicity amplitudes for the process $c\bar{c} \to qQ\bar{q}\bar{Q}$ and give the helicity amplitudes for the considered charmonium state decays into $p\bar{p}$. In Sect.3 we give the general expressions to compute the decay rate of any ${}^{2S+1}L_J, C = +1$ heavy $q\bar{q}$ state into two photons: in particular we obtain, in the non relativistic limit, the decay rate for $\eta_c, \chi_{0,2} \to \gamma\gamma$, in agreement with Ref. [6], and use such results to fix the values of the charmed meson wave functions in the origin. We also give the decay rate of the expected f_{c2} state into two photons. In Sect.4 we discuss the diquark form factors and give numerical results for the decays into $p\bar{p}$. In Sect.5 we draw our conclusions and make some comments.

1 - General formalism

In analogy with the QCD scheme of Ref.[3] we describe exclusive interactions by the convolution of a hard elementary process, involving free hadronic constituents, with a soft part, the hadronic wave function which models the hadronization of the constituents into the observed particles.

In the intermediate energy region we are considering (energy transfers of the order of few GeV) non perturbative or higher twist effects are still important. Following the program explained in Refs.[2,4,5,7] we model some of these effects by considering diquarks, bound states of two quarks, as active constituents. Such an assumption is supported by a large amount of experimental and theoretical information [1].

In our scheme, the helicity amplitudes for the decay into two baryons ($B\bar{B}$) of a ${}^{2S+1}L_J\,(c\bar{c})$ state are

$$
\begin{aligned}
A^{\lambda_B,\lambda_{\bar{B}}}_{J,M,L,S}(\theta) = \sum_{\lambda_q\lambda_{\bar{q}};\lambda_Q\lambda_{\bar{Q}};\lambda_c\lambda_{\bar{c}}} \int dx dy\, d^3k\, \{ &< B\lambda_B|h_B|y;qQ;\lambda_q\lambda_Q > \\
&< \bar{B}\lambda_{\bar{B}}|h_{\bar{B}}|x;\bar{q}\bar{Q};\lambda_{\bar{q}}\lambda_{\bar{Q}} > T^{(Q)}_{\lambda_q\lambda_{\bar{q}};\lambda_Q\lambda_{\bar{Q}};\lambda_c\lambda_{\bar{c}}}(\vec{k},\theta,x,y) \\
&< \vec{k};c\bar{c};\lambda_c\lambda_{\bar{c}}|h_C|\vec{k};J,M,L,S > \}
\end{aligned}
\tag{1.1}
$$

where $T^{(Q)}_{\{\}}$ is the center of mass helicity amplitude which describes the elementary annihilation of c and $\bar{c}$ into quark-antiquark diquark-antidiquark pairs ($c\bar{c} \rightarrow qQ\bar{q}\bar{Q}$); the operators h describe the hadronization process of the elementary constituents into mesons and baryons. By assuming, as usual, that qQ and $\bar{q}\bar{Q}$ are collinear, the baryonic wave functions $< B|h_B|qQ >$ and $< \bar{B}|h_{\bar{B}}|\bar{q}\bar{Q} >$ depend only on the fraction of the baryonic momentum $y(x)$ carried by the diquark (antidiquark). The amplitudes depend on the quantum numbers J, M, L, S of the initial charmonium state, on the helicities $\lambda_B, \lambda_{\bar{B}}$ of the final particles $B\bar{B}$ and on the decay angle θ between the baryon momentum and the quantization axis of the spin of the decaying particle (chosen as the z-axis). The initial wave functions are defined in the momentum space and $\vec{k}$ is the $c\bar{c}$ relative momentum. All the sums over the flavours and colours of the constituents are not explicitely written for simplicity of notation.

The hadronization operators are supposed to be diagonal in angular momentum space; that implies $\lambda_B = \lambda_q + \lambda_Q$, $\lambda_{\bar{B}} = \lambda_{\bar{q}} + \lambda_{\bar{Q}}$. The transformation from the canonical base $|JMLS>$ to the helicity base $|\lambda_c\lambda_{\bar{c}}>$ is given by the usual Clebsch-Gordan coefficients. By inserting the explicit expression for the $^{2S+1}L_J(c\bar{c})$ state wave function and after some algebra Eq.(1.1) can be rewritten as [8]

$$
\begin{aligned}
A^{\lambda_B\lambda_{\bar{B}}}_{JMLS} = \sum_{\lambda_c\lambda_{\bar{c}}} \left(\frac{2L+1}{4\pi}\right)^{\frac{1}{2}} & C^{\frac{1}{2}\frac{1}{2}S}_{\lambda_c-\lambda_{\bar{c}}\lambda}\, C^{LSJ}_{0\lambda\lambda} \times \\
\times \int & d^3k\, M_{\lambda_B\lambda_{\bar{B}};\lambda_c\lambda_{\bar{c}}}(\theta;\alpha,\beta,k)\, D^{J*}_{M\lambda}(\beta,\alpha,0)\, \psi_c(k)
\end{aligned}
\tag{1.2}
$$

where ψ_c is the charmonium wave function, $\lambda = \lambda_c - \lambda_{\bar{c}}$,

$$M_{\lambda_B,\lambda_{\bar{B}};\lambda_c,\lambda_{\bar{c}}} = \sum_{\lambda_q\lambda_{\bar{q}};\lambda_Q\lambda_{\bar{Q}};Q} \int dx dy\, \psi^*_{B,\lambda_B}(y)\psi^*_{\bar{B},\lambda_{\bar{B}}}(x)\, \delta_{\lambda_B;\lambda_q+\lambda_Q}\, \delta_{\lambda_{\bar{B}};\lambda_{\bar{q}}+\lambda_{\bar{Q}}} \times \times T^{(Q)}_{\lambda_q\lambda_{\bar{q}};\lambda_Q\lambda_{\bar{Q}};\lambda_c\lambda_{\bar{c}}}, \tag{1.3}$$

and the ψ_{B,λ_B} are the baryon wave functions. The relative momentum of the $c\bar{c}$ system, $\vec{k}$, has been expressed in spherical coordinates in terms of the polar and azymuthal angles α and β. After integration over α and β, Eq.(1.2) should give the correct angular distribution for the decay of a particle with quantum numbers J and M into $B\bar{B}$, i.e., the angular dependence of the helicity amplitude A must be given by the rotation matrix element $d^J_{M,\lambda_B-\lambda_{\bar{B}}}(\theta)$.

The formalism defined through Eqs.(1.1-3) is quite general and applies to the decay of any ${}^{2S+1}L_J$ heavy $(q\bar{q})$ state into baryon-antibaryon.

2 - Decay amplitudes for $\eta_c, \chi_{0,1,2}, f_2 \to p\bar{p}$

We will consider the decays of charmonium states with $C = +1$. The corresponding elementary processes are given by the two gluon exchange diagrams of Fig. 2.1. These diagrams contain only vertices with one gluon line attached to a diquark line. This allows us to use in our computation the most general couplings of scalar (S) and vector (V) diquarks to gluons (*), given by:

$$\begin{aligned} S^\mu &\equiv -ig_s T^a_{ij}(Q-\bar{Q})^\mu F_s \\ V^\mu &\equiv ig_s T^a_{ij}\{(\epsilon^*_Q \cdot \epsilon^*_{\bar{Q}})(Q-\bar{Q})^\mu G_1 \\ &\quad - [(Q\cdot\epsilon^*_{\bar{Q}})(\epsilon^*_Q)^\mu - (\bar{Q}\cdot\epsilon^*_Q)(\epsilon^*_{\bar{Q}})^\mu]G_2 \\ &\quad - (\epsilon^*_Q\cdot\bar{Q})(\epsilon^*_{\bar{Q}}\cdot Q)(Q-\bar{Q})^\mu G_3\} \end{aligned} \tag{2.1}$$

(*) For couplings with two or more gluons attached to a diquark line the most general form (allowed by Lorentz, gauge invariance, etc.) would be much more complicated.

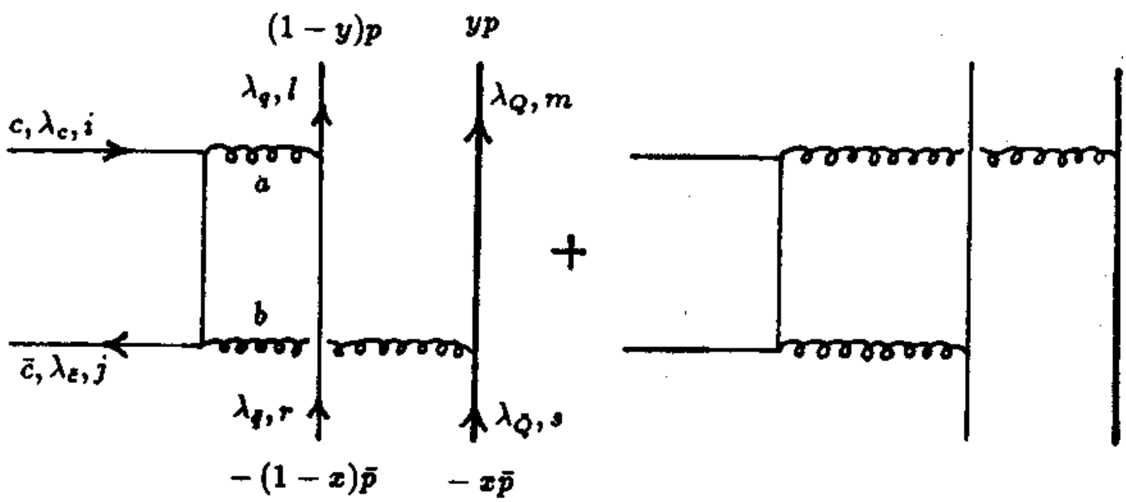

Fig. 2.1 *Feynman diagrams for the elementary process $c\bar{c} \to qQ\bar{q}\bar{Q}$. Here $p^\mu = (E, p\sin\theta, 0, p\cos\theta)$, $c^\mu = (E; \frac{k}{2}\sin\alpha\cos\beta, \frac{k}{2}\sin\alpha\sin\beta, \frac{k}{2}\cos\alpha)$; i, j, l, m, r, s, a, b are colour indices. λ_q, $\lambda_{\bar{q}}$, λ_Q, $\lambda_{\bar{Q}}$, λ_c and $\lambda_{\bar{c}}$, label helicities*

where the T^a are Gell-Mann colour matrices; Q e $\bar{Q}$ are defined in Fig. 2.1, F_S, G_1, G_2 e G_3 are form factors which will be discussed in Sect. 4, and ϵ_Q, $\epsilon_{\bar{Q}}$ are the diquark polarization vectors.

We can now compute the elementary amplitudes corresponding to the diagrams of Fig. 2.1, where the kinematics is defined. Throughout our calculation we use the naive parton model, neglecting the Fermi motion of the constituents inside the baryons; we must then assign to quarks, antiquarks, diquarks and antidiquarks a running mass $m_q = (1-y)\,m_p$, $m_{\bar{q}} = (1-x)\,m_p$, $m_Q = y\,m_p$, $m_{\bar{Q}} = x\,m_p$ respectively.

We do not give here all the details of the lengthy calculation; the interested reader can find them in Ref.[8]. Once we have the full expression for the elementary amplitudes $T^{(Q)}$, we can use them in Eqs.(1.3) and (1.2) to obtain the desired decay amplitudes. We list in Table 2.1 the $(c\bar{c})$ meson states which we shall study, together with their quantum numbers.

Meson	${}^{2S+1}L_J$	J^{PC}	L	S
η_c	${}^1S_0$	0^{-+}	0	0
χ_{c0}	${}^3P_0$	0^{++}	1	1
χ_{c1}	${}^3P_1$	1^{++}	1	1
χ_{c2}	${}^3P_2$	2^{++}	1	1
f_{c2}	${}^1D_2$	2^{-+}	2	0

Table 2.1 *Quantum numbers of some charmonium states with $C = +1$.*

We consider as final states only protons, for which we take the $SU(6)$ type wave functions [2,5]:

$$\begin{aligned}\varphi_{p,\lambda_p=\pm\frac{1}{2}}(x) =& \frac{\pm F_N}{\sqrt{18}}\Big\{\phi_2(x)\left[\sqrt{2}V_{\pm1}(ud)u_{\mp} - 2V_{\pm1}(uu)d_{\mp}\right] \\ &+ \phi_3(x)\left[\sqrt{2}V_0(uu)d_{\pm} - V_0(ud)u_{\pm}\right] \\ &\mp \left[2\phi_1(x) + \phi_3(x)\right] S(ud)u_{\pm}\Big\}\end{aligned} \tag{2.2}$$

The $\phi_i(x)$ $(i = 1,2,3)$ are the diquark momentum density distributions normalized as $\int_0^1 dx\, \phi_i(x) = 1$; $V_\lambda(ud)$ stands for a vector (ud) diquark with helicity λ and so on. F_N is the hadronization constant, with the dimension of [mass], somewhat analogous to the pion decay constant F_π. We also introduce a certain amount of $SU(6)$ violation [9]:

$$\begin{aligned}\phi_2(x) = \phi_3(x) &= \sqrt{2}\,\phi_V(x)\,\sin\Omega \\ 2\phi_1(x) + \phi_3(x) &= 3\sqrt{2}\,\phi_S(x)\,\cos\Omega\end{aligned} \tag{2.3}$$

By varying the value of the angle Ω we can give different weights to the vector and scalar components (for $\Omega = \pi/4$ we recover the $SU(6)$ wave function).

Using the wave functions (2.2) into the helicity amplitudes (1.3) and (1.2) and carrying out the α and β integrations we get the decay amplitudes $A_{\lambda_p,\lambda_{\bar{p}};M}$, for the charmonium states listed in Table 2.1:

$$A_{\pm\pm}(\eta_c) = \mp\sqrt{\frac{\pi}{2}}m_p \int dxdy \int dkk^2\, \psi_{\eta_c}(k)G(k)\times \tag{2.4}$$
$$12\frac{p^2E^2}{m_p^2}\varphi_{23}G_2y(x-y)\,c_0$$
$$A_{\pm\mp}(\eta_c) = 0 \tag{2.4'}$$

$$A_{\pm\pm}(\chi_0) = \frac{\sqrt{2\pi}}{12}\int dxdy \int dkk^2\, \psi_{\chi_0}(k)G(k)m_p\Big\{pk(x+y)\times$$
$$\Big\{-9\varphi_S F_S + 3\varphi_3\frac{1}{m_p^2}\left[(p^2+E^2)G_1 - 4xyp^2E^2G_3 - 2E^2G_2\right]$$
$$-6\varphi_2G_1\Big\}\left(c_0+\frac{2}{5}c_2\right) - 24\frac{E^2}{m_p^2}pk\varphi_{23}G_2y\left(c_0-\frac{1}{5}c_2\right) \tag{2.5}$$
$$+(x-y)\Big\{\left(2p^2+m_p^2(2-x-y)\right)\times$$
$$\left[-9\varphi_SF_S + 3\varphi_3\frac{1}{m_p^2}\left[(p^2+E^2)G_1 - 4xyp^2E^2G_3\right] - 6\varphi_2G_1\right]$$
$$-12\varphi_3\,\frac{p^2}{m_p^2}E^2G_2 + 24\frac{p^2}{m_p^2}E^2G_2\varphi_{23}y\Big\}\,c_1\Big\}$$
$$A_{\pm\mp}(\chi_0) = 0 \tag{2.5'}$$

$$A_{\pm\pm;M}(\chi_1) = 0 \tag{2.6}$$
$$A_{\pm\mp;M}(\chi_1) = \mp\frac{1}{2}\sqrt{\frac{\pi}{6}}d^1_{M,\pm1}(\theta)\int dxdy \int dkk^2\, \psi_{\chi_1}(k)G(k)\frac{E^2}{m_c}\Big\{pk\times$$
$$\Big\{(x+y-4xy)\Big[9\varphi_SF_S + 3\varphi_3\frac{1}{m_p^2}\left[(p^2+E^2)G_1 - 4xyp^2E^2G_3\right.$$
$$\left.-2E^2G_2\right]\Big] + 6\varphi_3\,(x-y)^2G_2 - 12(1-x-y)\varphi_{23}G_2y\Big\}\,c_0$$
$$+pk\Big\{2(x+y-xy)\Big[9\varphi_SF_S + 3\varphi_3\frac{1}{m_p^2}\left[(p^2+E^2)G_1\right. \tag{2.6'}$$
$$\left.-4xyp^2E^2G_3 - 2E^2G_2\right]\Big] + 3\varphi_3(x-y)^2G_2 + (1+\frac{1}{2}(x+y))\times$$
$$12\varphi_{23}G_2y\Big\}\,\frac{1}{5}c_2 + (x-y)\Big\{(2p^2+m_p^2(2-x-y))\times$$
$$\left[9\varphi_SF_S + 3\varphi_3\frac{1}{m_p^2}\left[(p^2+E^2)G_1 - 4xyp^2E^2G_3\right]\right]$$

$$- 12\varphi_3 \frac{p^2}{m_p^2} E^2 G_2 + 12p^2 G_2 \varphi_{23} y \Big\} c_1 \Big\}$$

$$A_{\pm\pm;M}(\chi_2) = -\frac{\sqrt{\pi}}{210} d^2_{M,0}(\theta) \int dxdy \int dkk^2 \, \psi_{\chi_2}(k) G(k) \frac{m_p}{m_c} \times$$
$$\Big\{ pk(3E + 2m_c) \Big\{ (x + y) \Big[-9\varphi_S F_S + 3\varphi_3 \frac{1}{m_p^2} \left[(p^2 + E^2) G_1 \right.$$
$$\left. -4xyp^2 E^2 G_3 - 2E^2 G_2 \right] - 6\varphi_2 G_1 \Big] + 12 \frac{E^2}{m_p^2} G_2 \varphi_{23} y \Big\} 7c_0$$
$$+ pk \Big\{ (3E + 11m_c)(x + y) \Big[-9\varphi_S F_S + 3\varphi_3 \frac{1}{m_p^2} \left[(p^2 + E^2) G_1 \right.$$
$$\left. -4xyp^2 E^2 G_3 - 2E^2 G_2 \right] - 6\varphi_2 G_1 \Big] \tag{2.7}$$
$$+ (3E - 10m_c) 12 \frac{E^2}{m_p^2} G_2 \varphi_{23} y \Big\} c_2 + pk(m_c - E) \Big\{ (x + y) \times$$
$$\Big[-9\varphi_S F_S + 3\varphi_3 \frac{1}{m_p^2} \left[(p^2 + E^2) G_1 - 4xyp^2 E^2 G_3 - 2E^2 G_2 \right] - 6\varphi_2 G_1 \Big]$$
$$+ 12 \frac{E^2}{m_p^2} G_2 \varphi_{23} y \Big\} 4c_4 + (x - y) \Big\{ (2p^2 + m_p^2(2 - x - y)) \times$$
$$\Big[-9\varphi_S F_S + 3\varphi_3 \frac{1}{m_p^2} \left[(p^2 + E^2) G_1 - 4xyp^2 E^2 G_3 \right] - 6\varphi_2 G_1 \Big] +$$
$$- 12\varphi_3 \frac{p^2}{m_p^2} E^2 G_2 + 24 \frac{p^2}{m_p^2} E^2 G_2 \varphi_{23} y \Big\} \times$$
$$[7(3E + 2m_c) c_1 - 9(E - m_c) c_3] \Big\}$$

$$A_{\pm\mp;M}(\chi_2) = \frac{1}{210} \sqrt{\frac{\pi}{6}} d^2_{M,\pm 1}(\theta) \int dxdy \int dkk^2 \, \psi_{\chi_2}(k) \times$$
$$G(k) \frac{E}{m_c} \Big\{ pk(3E + 2m_c) \Big\{ (x + y) \times$$
$$\Big[9\varphi_S F_S + 3\varphi_3 \frac{1}{m_p^2} \left[(p^2 + E^2) G_1 - 4xyp^2 E^2 G_3 - 2E^2 G_2 \right] \Big]$$
$$+ 12 G_2 \varphi_{23} y \Big\} 21c_0 + pk \Big\{ (2m_c - 9E + 21E(x + y - 2xy)) \times$$
$$\Big[9\varphi_S F_S + 3\varphi_3 \frac{1}{m_p^2} \left[(p^2 + E^2) G_1 - 4xyp^2 E^2 G_3 - 2E^2 G_2 \right] \Big]$$

$$+63\varphi_3\,(x-y)^2G_2E+\left[2m_c-9E+\frac{21}{2}E(x+y)\right]\times \tag{2.7'}$$
$$12\varphi_{23}G_2y\Big\}\,3c_2+pk(E-m_c)\Big\{(x+y)\times$$
$$\left[9\varphi_SF_S+3\varphi_3\frac{1}{m_p^2}\left[(p^2+E^2)G_1-4xyp^2E^2G_3-2E^2G_2\right]\right]$$
$$+12\varphi_{23}G_2y\Big\}\,8c_4+(x-y)\Big\{(2p^2+m_p^2(2-x-y))\times$$
$$\left[9\varphi_SF_S+3\varphi_3\frac{1}{m_p^2}\left[(p^2+E^2)G_1-4xyp^2E^2G_3\right]\right]$$
$$-12\varphi_3\,\frac{p^2}{m_p^2}E^2G_2+12p^2G_2\varphi_{23}y\Big\}\times$$
$$[21(3E+2m_c)\,c_1+18(E-m_c)\,c_3]\Big\}$$

$$A_{\pm\pm;M}(f_2)=\mp\sqrt{\frac{\pi}{10}}m_pd^2_{M,0}(\theta)\int dxdy\int dkk^2\,\psi_{f_2}(k)G(k)\times \tag{2.8}$$
$$12\varphi_{23}\frac{p^2}{m_p^2}E^2G_2y(x-y)\,c_2$$
$$A_{\pm\mp;M}(f_2)=0 \tag{2.8'}$$

where M is the z-component of the spin of the decaying particle and

$$\begin{aligned}
G(k)&=-i\frac{2^9\pi^2\sqrt{3}}{81g_1^2g_2^2}\,F_N^2\,\alpha_s^2\\
c_0&=\frac{z^2}{d^2}\,\frac{1}{2z}\ln\left|\frac{1+z}{1-z}\right|\\
c_1&=\frac{z}{d^2}\,3\left[\frac{z}{2}\ln\left|\frac{1+z}{1-z}\right|-1\right]\\
c_2&=\frac{z^2}{d^2}\,\frac{5}{2}\left[\frac{1}{2z}(3z^2-1)\ln\left|\frac{1+z}{1-z}\right|-3\right]\\
c_3&=\frac{z}{d^2}\,7\left[\frac{1}{4}(5z^3-3z)\ln\left|\frac{1+z}{1-z}\right|-\frac{5}{2}z^2+\frac{2}{3}\right]\\
c_4&=\frac{z^2}{d^2}\,9\left[\frac{1}{16z}(35z^4-30z^2+3)\ln\left|\frac{1+z}{1-z}\right|-\frac{35}{8}z^2+\frac{55}{24}\right]\\
g_1^2&=(x-y)^2m_p^2+4xyE^2\\
g_2^2&=(x-y)^2m_p^2+4(1-x)(1-y)E^2\\
d^2(k)&=(x-y)^2m_p^2+2(2xy-x-y)E^2
\end{aligned} \tag{2.9}$$

with $z = d^2/[kp(x-y)]$; $\alpha_s = g_s^2/4\pi$ is the usual strong coupling constant. The different terms coming from the proton and antiproton wave functions, Eq.(2.2), always appear in Eqs.(2.4-8) in the following combinations:

$$\begin{aligned} \varphi_S &= \frac{1}{9}(2\phi_1(x)+\phi_3(x))(2\phi_1(y)+\phi_3(y)) \\ \varphi_2 &= \phi_2(x)\phi_2(y) \\ \varphi_3 &= \phi_3(x)\phi_3(y) \\ \varphi_{23} &= \phi_2(y)\phi_3(x) \end{aligned} \tag{2.10}$$

Finally, we introduce the usual [6] non relativistic, small k limit for the charmonium wave functions $\psi_c(k)$. We get [8], according to the values of L:

$$\begin{aligned} (L=0) \qquad & \psi_{\eta_c}(k) = \sqrt{\frac{\pi}{2}}R(0)\frac{1}{k^2}\delta(k) \\ (L=1) \qquad & \psi_{\chi}(k) = -3i\sqrt{2\pi}R'(0)\frac{1}{k^2}\frac{d}{dk}\delta(k) \\ (L=2) \qquad & \psi_{f_2}(k) = -\frac{15}{2}\sqrt{\frac{\pi}{2}}R''(0)\frac{1}{k^2}\frac{d^2}{dk^2}\delta(k) \end{aligned} \tag{2.11}$$

where $R(0), R'(0), R''(0)$ are the radial wave function and its derivatives, computed in the origin.

If we use the wave functions (2.11) in the amplitudes (2.4-8) we find, performing the dk integration

$$\begin{aligned} A_{\pm\pm}(\eta_c) = \pm i\frac{2^{10}\pi^3\sqrt{3}}{3^3 m_p}R(0)F_N^2\alpha_s^2(m_c^2-m_p^2)m_c^2\int dxdy\,\times \\ \varphi_{23}G_2 y(x-y)\left(\frac{1}{g_1^2g_2^2d^2}\right)_{k=0} \end{aligned} \tag{2.12}$$

$$\begin{aligned} A_{\pm\pm}(\chi_0) = \frac{2^8\sqrt{3}}{3^4}\pi^3 R'(0)F_N^2\alpha_s^2 m_p\sqrt{m_c^2-m_p^2}\int dxdy\,\times \\ \Bigg\{\Bigg\{-9\varphi_S F_S + 3\varphi_3\frac{1}{m_p^2}\left[(2m_c^2-m_p^2)G_1 - 4xym_c^2(m_c^2-m_p^2)G_3\right] \\ -6\varphi_2 G_1\Bigg\}4xy(x+y-2)m_c^2 - 6\frac{m_c^2}{m_p^2}\varphi_3 G_2(x+y-2)\,\times \\ \left[(x-y)^2m_p^2+4xym_c^2\right] - 24\frac{m_c^2}{m_p^2}\varphi_{23}G_2 y\left[2(x-y)^2m_p^2\right. \\ \left.+\left[2(2xy-x-y)-(x-y)^2\right]m_c^2\right]\Bigg\}\left(\frac{1}{g_1^2g_2^2d^4}\right)_{k=0} \end{aligned} \tag{2.13}$$

$$A_{\pm\mp;M}(\chi_1) = -\lambda\frac{2^8}{3^3}\pi^3 R'(0)F_N^2\alpha_s^2\, m_c\sqrt{m_c^2-m_p^2}\, d^1_{M\lambda}(\theta)\int dxdy$$
$$\left\{\left\{9\varphi_S F_S + 3\varphi_3\frac{1}{m_p^2}\left[(2m_c^2-m_p^2)G_1 - 4xym_c^2(m_c^2-m_p^2)G_3\right]\right\}\times\right.$$
$$4xy\left[(3(x+y)-4xy-2)m_c^2-(x-y)^2m_p^2)\right] - 6\frac{m_c^2}{m_p^2}\varphi_3 G_2\times$$
$$\left\{4xy(3(x+y)-4xy-2)m_c^2+(x+y-4xy-2)(x-y)^2m_p^2\right. \quad (2.14)$$
$$\left.-\frac{m_p^2}{m_c^2}(x-y)^2\left[2(2xy-x-y)m_c^2+(x-y)^2m_p^2\right]\right\}$$
$$-12\varphi_{23}G_2 y\left\{\left[2(1-x-y)(2xy-x-y)-(x-y)^2\right]m_c^2\right.$$
$$\left.\left.+(2-x-y)(x-y)^2m_p^2\right\}\right\}\left(\frac{1}{g_1^2g_2^2d^4}\right)_{k=0}$$

$$A_{\pm\pm;M}(\chi_2) = -\frac{2^8\sqrt{6}}{3^4}\pi^3 R'(0)F_N^2\alpha_s^2 m_p\sqrt{m_c^2-m_p^2}\, d^2_{M\lambda}(\theta)\int dxdy$$
$$\left\{\left\{-9\varphi_S F_S + 3\varphi_3\frac{1}{m_p^2}\left[(2m_c^2-m_p^2)G_1 - 4xym_c^2(m_c^2-m_p^2)G_3\right]\right.\right.$$
$$\left.-6\varphi_2 G_1\right\}4xy(x+y-2)m_c^2 - 6\frac{m_c^2}{m_p^2}\varphi_3 G_2(x+y-2)\times \qquad (2.15)$$
$$\left[(x-y)^2m_p^2+4xym_c^2\right]+12\frac{m_c^2}{m_p^2}\varphi_{23}G_2 y\times$$
$$\left.\left\{2\left[2xy-x-y+(x-y)^2\right]m_c^2-(x-y)^2m_p^2\right\}\right\}\left(\frac{1}{g_1^2g_2^2d^4}\right)_{k=0}$$

$$A_{\pm\mp;M}(\chi_2) = \frac{2^8}{3^3}\pi^3 R'(0)F_N^2\alpha_s^2\, m_c\sqrt{m_c^2-m_p^2}\, d^2_{M\lambda}(\theta)\int dxdy$$
$$\left\{\left\{9\varphi_S F_S + 3\varphi_3\frac{1}{m_p^2}\left[(2m_c^2-m_p^2)G_1 - 4xym_c^2(m_c^2-m_p^2)G_3\right]\right\}\times\right.$$
$$4xy(x+y-2)m_c^2 - 6\frac{m_c^2}{m_p^2}\varphi_3 G_2(x+y-2)\times \qquad (2.15')$$
$$\left.\left[(x-y)^2m_p^2+4xym_c^2\right]+12\varphi_{23}m_c^2 G_2 y(x+y)(x+y-2)\right\}\times$$
$$\left(\frac{1}{g_1^2g_2^2d^4}\right)_{k=0}$$

$$A_{\pm\pm;M}(f_2) = \mp i \frac{2^9\sqrt{15}}{3^4} \pi^3 \alpha_s^2 F_N^2 R''(0) \frac{m_c^2}{m_p} (m_c^2 - m_p^2)^2 d^2_{M\lambda}(\theta) \int dx dy \times$$
$$12\varphi_{23} G_2 y (x-y)^3 \left(\frac{1}{g_1^2 g_2^2 d^6}\right)_{k=0} \tag{2.16}$$

Finally, from the explicit expressions of the decay amplitudes, Eqs.(2.12-16), we can compute the unpolarized decay rates for the spin J charmonium states:

$$\Gamma = \frac{1}{8(2\pi)^5} \frac{(m_c^2 - m_p^2)^{\frac{1}{2}}}{m_c} \sum_{\lambda_p,\lambda_{\bar{p}},M} \frac{1}{2J+1} \int d\Omega \, |A_{\lambda_p,\lambda_{\bar{p}},M}|^2 \tag{2.17}$$

3 - Charmonium decays into two photons

Each charmonium wave function, Eq.(2.11), still contains one unknown parameter, $R(0), R'(0)$ or $R''(0)$. In order to fix them we study the decays of $\eta_c, \chi_{0,2}$ and f_2 into two photons (the decay of χ_1 into $\gamma\gamma$ is forbidden and, indeed, we find it to be zero). The scheme is the same as in Eqs.(1.2-3) except that now we do not have any hadronization process and M and T in Eq.(1.3) coincide. The elementary subprocess is directly $c\bar{c} \to \gamma\gamma$ and it is described by the diagrams of Fig.3.1, where we also define the kinematics.

By computing the amplitudes for the elementary process, inserting them into Eq.(1.2) and integrating over α and β we find the decay amplitudes $A'_{\lambda_1\lambda_2;M}$:

$$A'_{\pm\pm}(\eta_c) = \mp 4\sqrt{2\pi} \int dk k^2 \psi_{\eta_c}(k) G'(k) c'_0 E$$
$$A'_{\pm\mp}(\eta_c) = 0 \tag{3.1}$$

$$A'_{\pm\pm}(\chi_0) = \frac{4\sqrt{2\pi}}{3} \int dk k^2 \psi_{\chi_0}(k) \left[G'(k)\left(c'_0 - \frac{1}{5}c'_2\right) k + G''(k) c'_1 E\right]$$
$$A'_{\pm\mp}(\chi_0) = 0 \tag{3.2}$$

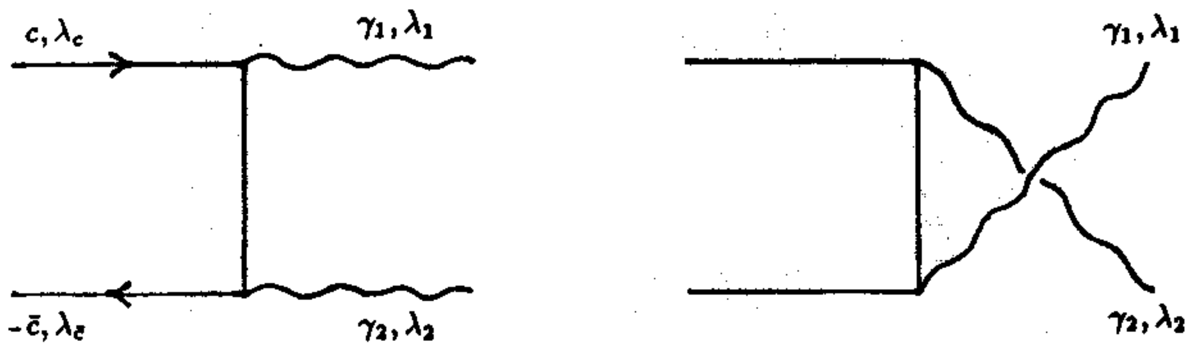

Fig. 3.1 *Feynman diagrams for the elementary process $c\bar{c} \to \gamma\gamma$. We compute them in the $c\bar{c}$ center of mass frame, where the independent four-vectors are given by $c^\mu = (E; \frac{k}{2}\sin\alpha\cos\beta, \frac{k}{2}\sin\alpha\sin\beta, \frac{k}{2}\cos\alpha)$ and $\gamma_1^\mu = (E; \vec{\gamma}_1)$ with $\vec{\gamma}_1 \equiv (E\sin\theta, 0, E\cos\theta)$; λ_1 e λ_2 are the helicities of the photons.*

$$\begin{aligned}
A'_{\pm\pm;M}(\chi_2) = {} & \frac{4\sqrt{\pi}}{35m_c} d^2_{M\lambda}(\theta) \int dk k^2 \psi_{\chi_2}(k) \Big\{ G'(k) \left[7\left(\frac{2}{3}m_c + E\right) c'_0 \right. \\
& \left. + \left(-\frac{10}{3}m_c + E\right) c'_2 + \frac{4}{3}(m_c - E)c'_4 \right] k + \\
& - 2G''(k) \left[7\left(\frac{2}{3}m_c + E\right) c'_1 + 3(m_c - E)c'_3 \right] E \Big\} \qquad (3.3) \\
A'_{\pm\mp;M}(\chi_2) = {} & -\frac{4\sqrt{6\pi}}{35m_c} d^2_{M\lambda}(\theta) \int dk k^2 \psi_{\chi_2}(k) G'(k) \Big\{ -7\left(\frac{2}{3}m_c + E\right) c'_0 \\
& + \left(\frac{4}{3}m_c + E\right) c'_2 + \frac{2}{9}(E - m_c)c'_4 \Big\} k
\end{aligned}$$

$$\begin{aligned}
A'_{\pm\pm;M}(f_2) &= \mp 4\sqrt{\frac{2\pi}{5}} d^2_{M,\lambda}(\theta) \int dk k^2 \psi_{f_2}(k) G'(k) c'_2 E \\
A'_{\pm\mp;M}(f_2) &= 0 \qquad (3.4)
\end{aligned}$$

where $G'(k) = (i32\pi\sqrt{3}\alpha)/(9k^2)$, $G''(k) = (i16\pi\sqrt{3}\alpha)/(9kE)$ and the coefficients c' are defined by:

$$c'_0 = \frac{1}{2z} \ln \left| \frac{1+z}{1-z} \right|$$

$$c'_1 = 3\left[\frac{z}{2}\ln\left|\frac{1+z}{1-z}\right| - 1\right]$$
$$c'_2 = \frac{5}{2}\left[\frac{1}{2z}(3z^2-1)\ln\left|\frac{1+z}{1-z}\right| - 3\right]$$
$$c'_3 = 7\left[\frac{1}{4}(5z^3-3z)\ln\left|\frac{1+z}{1-z}\right| - \frac{5}{2}z^2 + \frac{2}{3}\right]$$
$$c'_4 = 9\left[\frac{1}{16z}(35z^4-30z^2+3)\ln\left|\frac{1+z}{1-z}\right| - \frac{35}{8}z^2 + \frac{55}{24}\right] \tag{3.5}$$

with $z = 2E/k$.

The decay rates are then given in terms of these amplitudes, for unpolarized spin J states, by:

$$\Gamma = \frac{1}{16(2\pi)^5}\frac{1}{2J+1}\sum_{M,\lambda_1,\lambda_2}\int d\Omega\,|A'_{\lambda_1\lambda_2;M}|^2. \tag{3.6}$$

In the non relativistic, small k approximation, Eqs.(2.11), we recover the results of Ref.[6], for states with $L = 0, 1$, that is:

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{16}{27}\frac{\alpha^2}{m_c^2}|R(0)|^2 \tag{3.7}$$
$$\Gamma(\chi_0 \to \gamma\gamma) = \frac{16}{3}\frac{\alpha^2}{m_c^4}|R'(0)|^2 \tag{3.8}$$
$$\Gamma(\chi_2 \to \gamma\gamma) = \frac{4}{15}\Gamma(\chi_0 \to \gamma\gamma) \tag{3.9}$$

while for f_2 $(L = 2)$ we have:

$$\Gamma(f_2 \to \gamma\gamma) = \frac{4}{27}\frac{\alpha^2}{m_c^6}|R''(0)|^2 \tag{3.10}$$

Eq.(3.7) agrees also with the value given in Ref.[2], where $F_{\eta_c} = R(0)/(\sqrt{4\pi m_c})$.

The known experimental values for the decay rates into two photons are [10]

$$\Gamma(\eta_c \to \gamma\gamma) = 5.7 \pm 2.6 \pm 3.7\,KeV \tag{3.11}$$
$$\Gamma(\chi_0 \to \gamma\gamma) = 4.0 \pm 2.8\,KeV \tag{3.12}$$
$$\Gamma(\chi_2 \to \gamma\gamma) = 2.9^{+1.3}_{-1.0} \pm 1.7\,KeV \tag{3.13}$$

By comparing Eqs.(3.7-9) and Eqs.(3.11-13) we get

$$|R(0)| = 0.63 \pm 0.25\,(GeV)^{3/2} \tag{3.14}$$

$$|R'_{\chi_0}(0)| = 0.35 \pm 0.12\,(GeV)^{5/2} \tag{3.15}$$

$$|R'_{\chi_2}(0)| = 0.61 \pm 0.22\,(GeV)^{5/2} \tag{3.16}$$

We have combined quadratically the statistical and systematic errors in Eqs.(3.11-13) and we have assumed, consistently with our scheme and the zero binding energy approximation, the mass of the c quark to be one half the corresponding $(c\bar{c})$ meson mass. The two determinations of $R'(0)$, coming from the χ_0 and χ_2 experimental data, are, within errors, in agreement with each other. When computing the $\chi_{0,1,2} \rightarrow p\bar{p}$ decay rates we shall use the corresponding $R'(0)$ values; for χ_1 we shall take the average value

$$|R'_{\chi_1}(0)| = 0.48 \pm 0.17\,(GeV)^{5/2}. \tag{3.17}$$

An alternative way of fixing the values of $R(0)$ and $R'(0)$ would be that of assuming the total decay rates into hadrons to be given by the decay rates into two gluons, for which we have [6]

$$\Gamma(\eta_c, \chi_{0,2} \rightarrow gg) = \frac{9}{8}\,\Gamma(\eta_c, \chi_{0,2} \rightarrow \gamma\gamma)\left(\frac{\alpha_s}{\alpha}\right)^2 \tag{3.18}$$

This procedure leads to results which, within errors, agree with those given in Eqs.(3.14-16).

4 - Numerical results for $\eta_c, \chi_{0,1,2} \rightarrow p\bar{p}$ decay rates.

After fixing the parameters which characterize the charmonium wave function, $R(0)$ and $R'(0)$, we still remain with those related to the diquark form factors and the hadronic wave functions. The latter have the general form (2.2-3) with

$$\begin{aligned} \phi_S &= N_1 x^{\alpha_1}(1-x)^{\beta_1} \\ \phi_V &= N_2 x^{\alpha_2}(1-x)^{\beta_2} \end{aligned} \tag{4.1}$$

where $N_{1,2}$ are the normalization constants such that $\int_0^1 dx\, \phi_{V,S}(x) = 1$. By varying α and β we get wave functions with different "average" values of x, the fraction of the mass and the momentum of the proton carried by the diquark:

$$< x >_{S,V} \equiv N_{1,2} \int_0^1 dx\, x^{\alpha_{1,2}+1}(1-x)^{\beta_{1,2}} = \frac{\alpha_{1,2}+1}{\alpha_{1,2}+\beta_{1,2}+2} \qquad (4.2)$$

We expect the average mass of scalar diquarks to be smaller than the average mass of vector diquarks: this is supported by the analogy with the $q\bar{q}$ bound states (the π mass versus the ρ mass) and by explicit calculations [11] which indicate $m_S < m_V \lesssim 2m_S$. A similar conclusion, $< x >_S << x >_V \lesssim 2 < x >_S$, has been reached by studying the contribution of diquarks to deep inelastic scattering [12]. We shall use in our computations four different sets of wave functions:

$$\alpha_1 = 1 \quad \beta_1 = 3 \quad \alpha_2 = 3 \quad \beta_2 = 1 \qquad (4.3a)$$

$$\alpha_1 = 1 \quad \beta_1 = 2.5 \quad \alpha_2 = 2.5 \quad \beta_2 = 1 \qquad (4.3b)$$

$$\alpha_1 = 1 \quad \beta_1 = 1 \quad \alpha_2 = 5 \quad \beta_2 = 1 \qquad (4.3c)$$

$$\alpha_1 = 1 \quad \beta_1 = 1 \quad \alpha_2 = 4 \quad \beta_2 = 1 \qquad (4.3d)$$

These are consistent with the above requirement $< x >_S << x >_V \lesssim 2 < x >_S$, and are representative of the dependence of the numerical results on α and β. Such dependence will turn out to be very weak. We have checked that more elaborate kinds of wave functions [2,5] do not improve the numerical results.

The mixing angle Ω, which weighs differently the vector and scalar diquark components, and the hadronization constant F_N will be discussed in the sequel.

Let us now consider the diquark form factors. We know what their pointlike limits ($Q^2 \to 0$) are:

$$F_S(0) = 1 \quad G_1(0) = 1 \quad G_2(0) = 1 + \kappa \quad G_3(0) = 0 \qquad (4.4)$$

where κ is the vector diquark anomalous magnetic moment. We could then get some ideas on their large Q^2 behaviour from perturbative QCD, resolving the diquarks in two quarks [4]. Rather than following this procedure which, to be exact, would require the knowledge of the diquark wave functions in terms of two quarks, we prefer to fix the large Q^2 behaviour of the form factors by looking at the consequences, caused by the presence of diquarks inside nucleons, on deep inelastic scattering (DIS) [7]. We also assume the diquark strong and electromagnetic form factors to be the same, up to colour factors.

Diquarks as constituents generate scaling violations in DIS; in order for these violations to be compatible with the observed ones, we must have, at large Q^2 [7]:

$$\begin{aligned} F_S(Q^2) &\sim \frac{1}{Q^2} \\ G_1(Q^2) = G_2(Q^2) &\sim \frac{1}{Q^2} \\ G_3(Q^2) &\sim \frac{1}{Q^6} \end{aligned} \tag{4.5}$$

We then parametrize the diquark form factors as

$$\begin{aligned} F_S &= \frac{Q_S^2}{Q_S^2 + Q^2} \\ G_1 = G_2 &= \frac{Q_V^2}{Q_V^2 + Q^2} \\ G_3 &= 0 \end{aligned} \tag{4.6}$$

The values of $Q^2_{S,V}$ set the scale for the transition from the small Q^2 region, where diquarks act as elementary objects, to the large Q^2 one, where they start being resolved in two quarks. It is generally agreed [1,12] that scalar diquarks are more pointlike than vector diquarks; accordingly we take $Q_S^2 = 10\,(GeV)^2$ and $Q_V^2 = 2\,(GeV)^2$. Small variations of these values do not lead to relevant changes in the numerical results.

We take for the strong coupling constant the usual expression $\alpha_s(m^2_{(c\bar{c})}) = 12\pi/(25\,ln(m^2_{(c\bar{c})}/\Lambda^2))$, $\Lambda = 0.2\,GeV$.

At this point we still have two free parameters, Ω and F_N. The available experimental information is the set of decay rates [13]

$$\Gamma(\eta_c \to p\bar{p}) = 12.1 \pm 7.9\,KeV \tag{4.7}$$

$$\Gamma(\chi_1 \to p\bar{p}) = 57^{+13}_{-11} \pm 11\,eV \tag{4.8}$$

$$\Gamma(\chi_2 \to p\bar{p}) = 233^{+51}_{-45} \pm 48\,eV \tag{4.9}$$

The above results are based on very limited numbers of events and certainly need further confirmation. In particular the value of the η_c decay rate is surprisingly large. The only piece of data available on $\Gamma(\chi_0 \to p\bar{p})$ is the upper limit

$$\Gamma(\chi_0 \to p\bar{p}) < 12\,KeV \tag{4.10}$$

obtained by combining the total decay rate [14]

$$\Gamma_{\chi_0} = 13.5 \pm 3.3 \pm 4.2\,MeV \tag{4.11}$$

with the branching ratio bound [15]

$$BR(\chi_0 \to p\bar{p}) < 9.0 \times 10^{-4} \tag{4.12}$$

We have fixed the value of F_N, for different values of Ω, by fitting the data on χ_2, Eq.(4.9), which seem to be the most reliable ones [13]. We find, in MeV

Ω	$45°$	$30°$	$0°$	
F_N	67 ± 13	62 ± 12	55 ± 11	(4.13*a*)
F_N	72 ± 14	64 ± 12	57 ± 11	(4.13*b*)
F_N	58 ± 11	55 ± 11	50 ± 10	(4.13*c*)
F_N	52 ± 10	53 ± 10	50 ± 10	(4.13*d*)

Eqs.(4.13*a,b,c,d*) refer, respectively, to the wave functions (4.1) and (4.3*a,b,c,d*).

The above sets of values give (all results are in eV)

Ω	$\Gamma(\chi_1 \to p\bar{p})$	$\Gamma(\chi_0 \to p\bar{p})$	$\Gamma(\eta_c \to p\bar{p})$	
$45°$	218^{+229}_{-218}	1942^{+2011}_{-1942}	2^{+2}_{-2}	(4.14*a*)
$30°$	108^{+113}_{-108}	538^{+556}_{-538}	$0.5^{+0.6}_{-0.5}$	(4.15*a*)
$0°$	41^{+44}_{-41}	46^{+48}_{-46}	0	(4.16*a*)
$45°$	152^{+160}_{-152}	1769^{+1834}_{-1769}	4^{+4}_{-4}	(4.14*b*)
$30°$	64^{+66}_{-64}	437^{+444}_{-437}	$0.6^{+0.7}_{-0.6}$	(4.15*b*)
$0°$	25^{+26}_{-25}	46^{+48}_{-46}	0	(4.16*b*)

45°	95^{+99}_{-95}	2382^{+2436}_{-2382}	1^{+1}_{-1}	(4.14*c*)
30°	2^{+2}_{-2}	685^{+722}_{-685}	$0.2^{+0.2}_{-0.2}$	(4.15*c*)
0°	22^{+24}_{-22}	48^{+51}_{-48}	0	(4.16*c*)

45°	190^{+199}_{-190}	3105^{+3200}_{-3105}	$0.8^{+0.9}_{-0.8}$	(4.14*d*)
30°	20^{+21}_{-20}	1084^{+1105}_{-1084}	$0.2^{+0.2}_{-0.2}$	(4.15*d*)
0°	22^{+24}_{-22}	49^{+52}_{-49}	0	(4.16*d*)

where, again, (a,b,c,d) refers respectively to Eqs.(4.3*a*,*b*,*c*,*d*).

Eqs.(4.14-16) have to be compared with Eqs.(4.7-10). First we notice that, as anticipated, the dependence of the above results on the wave function exponents α and β is very weak. This is to be contrasted to similar computations in the pure quark model [16,17] where the amplitudes vary by several orders of magnitude with analogous changes in the wave function. Although we may still tune two parameters (Ω and F_N) these are strongly correlated, as demonstrated by the fact that it is not possible to reproduce an arbitrary pair of decay rates out of the above (4.7-9). In particular, no choice of Ω and F_N leads to the very large experimental value (4.7) of $\Gamma(\eta_c \to p\bar{p})$ while keeping the values of the three $\Gamma(\chi \to p\bar{p})$ within reasonable bounds. It is not difficult instead to get a good agreement with the experimental information on the decays $\chi_{0,1,2} \to p\bar{p}$, at the price of a value of $\Gamma(\eta_c \to p\bar{p})$ which is much smaller (by a factor $\simeq 10^{-4}$) than the observed one. We will comment on the η_c problem in the next Section.

We do not present here any result for the decay rate of f_2, due to the lack of experimental information on $\Gamma(f_2 \to \gamma\gamma)$, from which we could deduce the value of $R''(0)$. Should such data become available one could easily compute also the value of $\Gamma(f_2 \to p\bar{p})$.

5 - Comments and conclusions

We have consistently applied a quark-diquark model for the nucleon, previously introduced [2,5], to several intermediate energy exclusive reactions, in order to fix all the parameters and to provide a full test of our scheme. We have considered η_c and $\chi_{0,1,2}$ decays into $p\bar{p}$, in a natural modification of the Brodsky-Farrar-Lepage scheme for exclusive reactions [3], modeling the proton as a quark-diquark system.

After fixing most of the parameters using both theoretical considerations and comparison with experimental results, we still remain with two of them which, however, are strongly correlated. It emerges that our picture can give a good description for the decays of the $\chi_{0,1,2}(c\bar{c})$ meson states. The vector diquark component of the proton wave function seems to be smaller than the scalar one, but not necessarily zero. The same picture, however, fails to describe the $\eta_c \to p\bar{p}$ decay, in that it gives a result which is by a factor $\simeq 10^{-4}$ smaller than the experimental one. The main reason for such a failure is the combination of the facts that only vector diquarks can contribute to the η_c decay and that the known experimental value for $\Gamma(\eta_c \to p\bar{p})$, Eq.(4.7) is surprisingly large, i.e., much larger than the analogous decay rates for $\chi_{1,2} \to p\bar{p}$.

Amongst the decays considered here only the $\chi_2 \to p\bar{p}$ decay rate has been computed in the framework of the pure quark model [16]. A value of the branching ratio in reasonable agreement with the experimental one can be obtained; however, the normalization of the amplitudes (i.e., the hadronization constant) shows a very strong dependence on the proton wave function. Moreover, in a pure quark approach, the amplitudes for the other decays that we discussed either vanish [2,16], or are ill-defined due to collinear divergences [16].

The η_c decay into $p\bar{p}$, strictly forbidden in the pure quark model of Ref.[3], is one out of many spin effects, most of which cannot be explained [2,4] in perturbative QCD massless quark schemes; the introduction of vector diquarks could, in principle, offer a solution to these problems and it would be very unfortunate if their contribution turned out to be too small. While the observation of the $\eta_c \to p\bar{p}$ decay cannnot be doubted, the actual decay rate value is based on very few events and indeed needs a confirmation; if the strong disagreement between our result and the experimental data should persist, it would be a serious problem for the quark-diquark model of the nucleon, or, at least, for its application to the description of exclusive reactions at intermediate energies.

The treatment of non perturbative effects by the introduction of diquarks in an overall QCD perturbative scheme might be too drastic or simplistic; higher order corrections might still be much too large. Another possible source of uncertainties is the neglect, throughout all our calculations, of the scalar-vector diquark transition, which would introduce one extra coupling (to be added to Eqs.(2.1)), $\sim \epsilon_{\mu\nu\rho\sigma} Q^\nu \bar{Q}^\rho (\epsilon^*)^\sigma$. We have checked that such a coupling could increase the value of $\Gamma(\eta_c \to p\bar{p})$, but not by such a large factor as needed [18].

Let us add that the $p\bar{p}$ channel is not the only "weird" decay of the η_c; its decays into vector particles, $\eta_c \to \rho\rho, K\bar{K}^*, \phi\phi$, are in fact forbidden in the BFL scheme and one still gets a zero result for all amplitudes even when taking into account quark mass effects [19]. All these decays have been observed experimentally. It might be that the η_c decays receive a strong, leading contribution from other mechanisms not taken into account either in the BFL scheme or in its quark-diquark generalization (glueballs?).

Waiting for clarification of the η_c puzzle, there are still some other tests of our model left, since all parameters have now been fixed; of particular interest is the computation of the decay rate $\Gamma(J/\psi \to \gamma p\bar{p})$ which is in progress [2,20].

Acknowledgements

We thank Prof. E. Predazzi for many useful discussions and for his continuous interest in this work. M.A. and F.C. are very grateful to the Dipartimento di Fisica Teorica of Torino University, where most of this work was done, for the kind hospitality; M.A. also thanks the Centro Brasileiro de Pesquisas Físicas for the same reason.

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