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Some Quantum Aspects of Complex Vector Fields with Chern-Simons Term

by

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Abstract

Complex vector fields with Maxwell, Chern-Simons and Proca terms are minimally coupled to an Abelian gauge field. The consistency of the spectrum is analysed and 1-loop quantum corrections to the self-energy are computed and discussed.

Key-words: Chern-Simons-Maxwell; Field; Self-energy.

1 Introduction

One of the central problems in the framework of gauge field theories is the issue of gauge field mass. Gauge symmetry is not, in principle, conflicting with the presence of a massive gauge boson. In 2 space-time dimensions, the well-known Schwinger model puts in evidence the presence of a massive photon without the breaking of gauge symmetry [1]: a dynamical mass generation takes place by virtue of fermion 1-loop corrections to the Maxwell field polarization tensor.

Another evidence for the compatibility between gauge symmetry and massive vector fields comes from the study of 3-dimensional gauge theories [2, 3]. A topological mass term referred to as the Chern-Simons Lagrangian, once added to the Maxwell kinetic term, shifts the photon mass to a non-vanishing value without breaking gauge invariance at all [2, 3]. Even if the Chern-Simons term, which is gauge invariant, is not written down at tree-level, it may be generated by 1-loop corrections whenever massive fermions are minimally coupled to an Abelian gauge field [4, 5, 6]. Again, a dynamical mass generation mechanism takes place. Also, in 3 space-time dimensions, there occurs a dynamical fermionic mass generation if massless fermions are minimally coupled to a Chern-Simons field [4, 5, 6, 7].

In the more realistic case of 4 space-time dimensions, the best mechanism known, up to now, to solve the problem of intermediate boson masses is the spontaneous symmetry breaking mechanism [8, 9]. It is not known any 4-dimensional counterpart of the dynamical mechanism to generate gauge field masses along the lines previously mentionned. However, in 4 dimensions, one should quote the dynamical breaking of chiral symmetry which takes place through a dynamical mass generation mechanism for fermions [10, 11].

Since, over the past years, 3-dimensional field theories have been shown to play a central rôle in connection with the behaviour of 4-dimensional theories at finite temperature [12] and in the description of a number of problems in Condensed Matter Physics [13], it seems reasonable to concentrate efforts in trying to understand some peculiar features of gauge field dynamics in 3 dimensions.

The main propose of this paper is to consider 3-dimensional models built up in terms of complex vector fields with Chern-Simons terms and to which one minimally couples a Maxwell field. At tree-level, we study the Chern-Simons-Maxwell (CSM*) and the Chern-Simons-Maxwell-Proca (CSMP*) cases, in order to analyse the conditions to be set on the free parameters of the Lagrangians so as to avoid the presence of tachyons and ghosts in the spectrum. This is carried out in Sections 2 and 3, respectively. In Section 4, we study the Abelian CSM* model and show that, upon the incorporation of 1-loop corrections to the CSM*-field self-energy, a finite Proca mass term is generated. The analysis of Section 2, in combination with the latter result, ensures that the generated Proca-like term does not plug the theory with tachyons or ghosts. Finally, in Section 5, we draw our general conclusions and present our prospects for future work. Two appendices follow: the results for 1-loop self-energy diagrams are listed in the Appendix A. In the Appendix B, the explicit results for the momentum-space 1-loop integrals are collected. The metric adopted throughout this work is $\eta_{\mu\nu} = (+;-,-)$.

2 The Complex Chern-Simons-Maxwell Field (CSM*)

The CSM* model is described by the Lagrangian

$$\mathcal{L}^0_{CSM} = \frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu} - \frac{1}{2M}G^*_{\mu\nu}G^{\mu\nu} \ , \qquad (1)$$

where $G_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ and $G^*_{\mu\nu} \equiv \partial_\mu B^*_\nu - \partial_\nu B^*_\mu$ are the field-strengths, and M is a real parameter with dimension of mass.

There are two kinds of $U(1)$ symmetries that may be observed in (1). A global $U_\alpha(1)$ given by

$$B'_\mu(x) = e^{i\alpha}B_\mu(x) \ , \qquad (2)$$

where α is a real parameter, and a local $U_\beta(1)$ that reads

$$B'_\mu(x) = B_\mu(x) + \partial_\mu\beta(x) \ , \qquad (3)$$

where $\beta(x)$ is an arbitrary C^∞ complex function. The question involving gauge symmetries with complex parameters has already been contemplated in the context of spontaneously broken symmetries in supersymmetric gauge models [14].

To minimally couple the CSM* fields, B_μ and B^*_μ, to the Maxwell field, A_μ, we define the following $U_\alpha(1)$-covariant derivatives :

$$D_\mu \equiv \partial_\mu + i\omega A_\mu \ \text{ and } \ D^*_\mu \equiv \partial_\mu - i\omega A_\mu \ , \qquad (4)$$

where ω is a coupling constant with dimension of $(\text{mass})^{\frac{1}{2}}$. Then, the total Lagrangian becomes

$$\begin{aligned}\mathcal{L}_{CSM}(B,B^*,\partial B,\partial B^*,A) &= \mathcal{L}^0_{CSM}(B,B^*,DB,D^*B^*)-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\ &= \frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha\tilde{G}_{\mu\nu}-\frac{1}{2M}\tilde{G}^*_{\mu\nu}\tilde{G}^{\mu\nu}-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (5)\end{aligned}$$

where $\tilde{G}_{\mu\nu}\equiv D_\mu B_\nu - D_\nu B_\mu$ and $F_{\mu\nu}$ is the field-strength for A_μ. By replacing the covariant derivatives as given in eq.(4), the total Lagrangian reads :

$$\begin{aligned}\mathcal{L}_{CSM} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu}-\frac{1}{2M}G^*_{\mu\nu}G^{\mu\nu}+i\omega\epsilon^{\alpha\mu\nu}B^*_\alpha A_\mu B_\nu+\\ &-i\frac{\omega}{M}(G^*_{\mu\nu}A^\mu B^\nu - G_{\mu\nu}A^\mu B^{*\nu})-\frac{\omega^2}{M}(A_\mu B_\nu - A_\nu B_\mu)A^\mu B^{*\nu}\,. \quad (6)\end{aligned}$$

It can be noticed that the local $U_\beta(1)$-symmetry (3) is explicitly broken by the interaction terms in (6).

In order to perform the analysis of the spectral consistency of this model, it is necessary to obtain the propagator for the fields B and B^*. Since the local $U_\beta(1)$-symmetry is broken only at the interaction level, we need a gauge-fixing term to be able to read off the propagators. So, for the sake of extracting them, we consider the Lagrangian below :

$$\hat{\mathcal{L}}^0_{CSM}=\frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu}-\frac{1}{2M}G^*_{\mu\nu}G^{\mu\nu}+\frac{1}{\hat{\alpha}}(\partial_\mu B^{*\mu})(\partial_\nu B^\nu)\,, \quad (7)$$

where $\hat{\alpha}$ is the gauge-fixing parameter.

The field equations coming from (7) are given by

$$\mathcal{O}^{\epsilon\alpha}B^*_\alpha = 0\,, \quad (8)$$

with

$$\mathcal{O}^{\epsilon\alpha}\equiv -\epsilon^{\epsilon k\alpha}\partial_k - \frac{\Box}{M}\left(\eta^{\epsilon\alpha}-\frac{\partial^\epsilon\partial^\alpha}{\Box}\right)+\frac{\Box}{\hat{\alpha}}\left(\frac{\partial^\epsilon\partial^\alpha}{\Box}\right)\,, \quad (9)$$

where

$$\Theta^{\mu\nu}\equiv\eta^{\mu\nu}-\frac{\partial^\mu\partial^\nu}{\Box}\,,\ \ S^{\mu\nu}\equiv\epsilon^{\mu\alpha\nu}\partial_\alpha \ \text{ and } \ \Omega^{\mu\nu}\equiv\frac{\partial^\mu\partial^\nu}{\Box} \quad (10)$$

are spin operators that fulfil the algebra displayed in Table 1.

	Ω	Θ	S
Ω	Ω	0	0
Θ	0	Θ	S
S	0	S	$-\Box\,\Theta$

Table 1: Operator algebra fulfilled by Ω,Θ and S.

Inverting the operator $\mathcal{O}$ with the help of the Table 1, we obtain the following operators in the transverse and longitudinal subspaces :

$$(\mathcal{O}_L^{-1})^{\mu\nu} = \frac{\hat{\alpha}}{\Box}\,\Omega^{\mu\nu} \tag{11.a}$$

and

$$(\mathcal{O}_T^{-1})^{\mu\nu} = \frac{M^2}{\Box(\Box+M^2)}\left[S^{\mu\nu} - \frac{\Box}{M}\,\Theta^{\mu\nu}\right]. \tag{11.b}$$

As

$$< T[B^{*\mu}(y)B^{\nu}(x)] >_L = -i\,(\mathcal{O}_L^{-1})^{\mu\nu}\,\delta^3(x-y) \tag{12.a}$$

and

$$< T[B^{*\mu}(y)B^{\nu}(x)] >_T = -i\,(\mathcal{O}_T^{-1})^{\mu\nu}\,\delta^3(x-y)\,, \tag{12.b}$$

then, in momentum-space, we have

$$\Delta_L^{\mu\nu}(k) = i\,\frac{\hat{\alpha}}{k^2}\left(\frac{k^\mu k^\nu}{k^2}\right) \tag{13.a}$$

and

$$\Delta_T^{\mu\nu}(k) = -i\,\frac{M^2}{k^2(k^2-M^2)}\left[i\,\epsilon^{\mu k\nu}k_k + \frac{k^2}{M}\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right], \tag{13.b}$$

where $\Delta_L^{\mu\nu}(k)$ and $\Delta_T^{\mu\nu}(k)$ are the CSM* propagators in the longitudinal and transverse subspaces, respectively.

By saturating the propagators with external conserved currents, J^μ and $J^{\mu *}$, we get the following expressions for the imaginary parts of the residues of the amplitudes at the poles :

$$ImRes\{J_\mu\, \Delta_L^{\mu\nu}(k)\, J_\nu^*\}\,|_{k^2=0} = 0 \,, \tag{14.a}$$

$$ImRes\{J_\mu\, \Delta_T^{\mu\nu}(k)\, J_\nu^*\}\,|_{k^2=0} = 0 \tag{14.b}$$

and

$$ImRes\{J_\mu\, \Delta_T^{\mu\nu}(k)\, J_\nu^*\}\,|_{k^2=M^2} = 2M\ J_1 J_1^* \,. \tag{14.c}$$

Then, the following result on the spectrum can be stated :

$$L-sector \longrightarrow \text{pole at } k^2 = 0 \text{ non-dynamical (on/off-shell)} \tag{15.a}$$

$$T-sector \longrightarrow \begin{cases} \text{pole at } k^2 = 0 \text{ non-dynamical (on-shell)} \\ \text{pole at } k^2 = M^2 \begin{cases} \text{dynamical} \\ \text{no tachyons, no ghosts } if\ \ M > 0 \end{cases} \end{cases} . \tag{15.b}$$

Thus, we may conclude that, once the mass parameter, M, is taken to be positive, the CSM* model describes a free physical dynamical excitation of mass $k^2 = M^2$. Nevertheless, to have full control of the unitarity at tree-level, it is still necessary to study the behaviour of the scattering cross sections in the limit of very high center-of-mass energies [15].

3 The Complex Chern-Simons-Maxwell-Proca Field (CSMP*)

The CSMP* model is described by a Lagrangian obtained from (1) by the addition of a Proca term, $\hat{\mu} B_\mu^* B^\mu$. Then,

$$\mathcal{L}^o_{CSMP} = \frac{1}{2}\epsilon^{\alpha\mu\nu} B_\alpha^* G_{\mu\nu} - \frac{1}{2M} G_{\mu\nu}^* G^{\mu\nu} + \hat{\mu} B_\mu^* B^\mu \,, \tag{16}$$

where $\hat{\mu}$ is a real parameter with mass dimension.

It may be observed that the Lagrangian of eq.(16) exhibits only one global symmetry, $U_\alpha(1)$:

$$B'_\mu(x) = e^{i\alpha} B_\mu(x) \ , \tag{17}$$

where α is a real parameter. The local symmetry $U_\beta(1)$ (3) is explicitly broken by the Proca term.

Carrying out the minimal coupling of the CSMP* fields, B_μ and B^*_μ, to the Maxwell field A_μ, one gets the Lagrangian

$$\begin{aligned} \mathcal{L}_{CSMP} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\alpha\mu\nu}B^*_\alpha G_{\mu\nu} - \frac{1}{2M}G^*_{\mu\nu}G^{\mu\nu} + \hat{\mu}B^*_\mu B^\mu + \\ & +i\omega\epsilon^{\alpha\mu\nu}B^*_\alpha A_\mu B_\nu - i\frac{\omega}{M}(G^*_{\mu\nu}A^\mu B^\nu - G_{\mu\nu}A^\mu B^{*\nu}) + \\ & -\frac{\omega^2}{M}(A_\mu B_\nu - A_\nu B_\mu)A^\mu B^{*\nu} \, . \end{aligned} \tag{18}$$

To pursue our investigation on the consistency of the spectrum, we shall now derive the propagators for the CSMP* fields and then analyse their poles and associated residues.

The field equations following from (16) read

$$\bar{\mathcal{O}}^{\epsilon\alpha} B^*_\alpha = 0 \, , \tag{19}$$

with

$$\bar{\mathcal{O}}^{\epsilon\alpha} \equiv -\,\epsilon^{\epsilon\kappa\alpha}\partial_\kappa - \frac{\Box}{M}\left(\eta^{\epsilon\alpha} - \frac{\partial^\epsilon\partial^\alpha}{\Box}\right) - \hat{\mu}\,\eta^{\epsilon\alpha} \, . \tag{20}$$

Again, Table 1 is used in the task of inverting the operator $\bar{\mathcal{O}}$ (20). We find :

$$(\bar{\mathcal{O}}_L^{-1})^{\mu\nu} = -\frac{1}{\hat{\mu}}\,\Omega^{\mu\nu} \tag{21.a}$$

and

$$(\bar{\mathcal{O}}_T^{-1})^{\mu\nu} = \frac{M}{[(\Box + \hat{\mu}M)^2 + M^2\Box]}\left[M\,S^{\mu\nu} - (\Box + \hat{\mu}M)\,\Theta^{\mu\nu}\right] \, . \tag{21.b}$$

The momentum-space expressions for the propagators are :

$$\bar{\Delta}_L^{\mu\nu}(k) = i\,\frac{1}{\hat{\mu}}\left(\frac{k^\mu k^\nu}{k^2}\right) \qquad (22.a)$$

and

$$\begin{aligned}\bar{\Delta}_T^{\mu\nu}(k) &= -i\,\frac{M}{[(k^2-\hat{\mu}M)^2 - M^2k^2]}\left[iM\epsilon^{\mu\kappa\nu}k_\kappa + (k^2-\hat{\mu}M)\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right] \\ &= -i\,\frac{M}{(k^2-m_+^2)(k^2-m_-^2)}\left[iM\epsilon^{\mu\kappa\nu}k_\kappa + (k^2-\hat{\mu}M)\left(\eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\right],\end{aligned} \qquad (22.b)$$

where

$$m_+^2 = \frac{M}{2}\,[M + 2\hat{\mu} + \sqrt{M(M+4\hat{\mu})}\,] \qquad (23.a)$$

and

$$m_-^2 = \frac{M}{2}\,[M + 2\hat{\mu} - \sqrt{M(M+4\hat{\mu})}\,], \qquad (23.b)$$

where $M(M+4\hat{\mu}) \geq 0$, in order to avoid unphysical complex roots.

To avoid the apperance of a double pole, $m_+^2 = m_-^2$, (which would certainly lead to a ghost) we must actually have $M(M+4\hat{\mu}) > 0$.

Again, by saturating the propagators with external conserved currents, J^μ and $J^{\mu *}$, we get the following expressions for the imaginary parts of the residues of the amplitudes at the different poles :

$$ImRes\{J_\mu\,\bar{\Delta}_L^{\mu\nu}(k)\,J_\nu^*\}\,|_{k^2=0} = 0\,, \qquad (24.a)$$

$$ImRes\{J_\mu\,\bar{\Delta}_T^{\mu\nu}(k)\,J_\nu^*\}\Big|_{k^2=m_+^2} = \frac{M[M + \sqrt{M(M+4\hat{\mu})}\,]}{\sqrt{M(M+4\hat{\mu})}} J_1 J_1^* \qquad (24.b)$$

and

$$ImRes\{J_\mu\,\bar{\Delta}_T^{\mu\nu}(k)\,J_\nu^*\}\Big|_{k^2=m_-^2} = -\frac{M[M - \sqrt{M(M+4\hat{\mu})}\,]}{\sqrt{M(M+4\hat{\mu})}} J_1 J_1^* \qquad (24.c)$$

The summary of our analysis is therefore :

$$T\text{-}sector \longrightarrow \begin{cases} \text{pole at } k^2 = m_+^2 \begin{cases} \text{dynamical} \\ \text{no tachyons, no ghosts } \; if \;\; M \text{ and } \hat{\mu} > 0 \end{cases} \\ \text{pole at } k^2 = m_-^2 \begin{cases} \text{dynamical} \\ \text{no tachyons, no ghosts } \; if \;\; M \text{ and } \hat{\mu} > 0 \end{cases} \end{cases} . \quad (25)$$

The analysis of the residues shows that the T-sector is free from tachyons and ghosts whenever $\hat{\mu} > 0$ and $M > 0$.

The conditions $\hat{\mu} > 0$ and $M > 0$ automatically avoid a double pole. Then, the CSMP* model is perfectly physical, as long as the spectrum is concerned, if these two conditions are set. Nevertheless, as in the case of the CSM*-field, to have full control of the unitarity at tree-level, it is necessary to study the behaviour of the scattering cross sections in the limit of very high center-of-mass energies [15].

A peculiar feature concerns the presence of two different simple poles in the transverse sector. This is also a characteristic of a real CSMP-field. They are to be interpreted as two distinct excitations whose spins have to be fixed in terms of the masses, after a detailed analysis of the Lorentz group generators as functionals of the fields is carried out, in the same way it is done for a topologically massive theory [3]. However, each of the masses has a definite value for the spin (there is no room for different polarization states in $D = 3$), so that the 2 degrees of freedom of the real CSMP-field correspond to the 2 possible mass states. In the complex case, the 4 degrees of freedom are associated to the 2 different states of charge that each massive pole may present.

4 Dynamical Mass Generation in the CSM* Model

By reconsidering the Lagrangian (6), the following interaction vertices (see Fig.1) come out :

$$\mathcal{L}_{CSM}^{(1)\,int} = i\,\omega\epsilon^{\alpha\mu\nu}\, B_\alpha^* A_\mu B_\nu \longrightarrow V_3\,, \quad (26.a)$$

$$\mathcal{L}_{CSM}^{(2)\,int} = -i\,\frac{\omega}{M}\,(G_{\mu\nu}^* A^\mu B^\nu - G_{\mu\nu} A^\mu B^{*\nu}) \longrightarrow \tilde{V}_3 \quad (26.b)$$

and

$$\mathcal{L}_{CSM}^{(3)\,int} \;=\; -\frac{\omega^2}{M}\,(A_\mu B_\nu - A_\nu B_\mu)A^\mu B^{*\nu} \;\longrightarrow\; V_4\,, \tag{26.c}$$

Before the calculation of the Feynman graphs relevant for our analysis on the mass generation, we present the expression we get for the superficial degree of divergence of the primitively divergent graphs.

Analysing the CSM* propagator in the high energy limit, and taking into account the interaction vertices above, we find the following expression for the superficial degree of divergence, δ_{CSM} :

$$\delta_{CSM} = 3 - \frac{3}{2}v_3 - \frac{1}{2}\bar{v}_3 - v_4 - \frac{1}{2}E_A - \frac{1}{2}E_B\,, \tag{27}$$

where v_3, $\bar{v}_3$ and v_4 are the numbers of vertices V_3, $\bar{V}_3$ and V_4 respectively, E_A are the external lines of A_μ and E_B are the external lines of B_μ and B_μ^*. Therefore, the CSM* is a super-renormalizable model: ultraviolet divergences appear only up to 2-loops. Now, since in 3 space-time dimensions no 1-loop divergences show up, all renormalizations have to be performed at 2-loops.

The vertex Feynman rules of the model are as below :

$$(V_3)_{\alpha\mu\nu} \;=\; \omega\epsilon_{\alpha\mu\nu}\;, \tag{28.a}$$

$$(\bar{V}_3)_{\alpha\mu\nu} \;=\; i\frac{\omega}{M}(\eta_{\nu\alpha}k_\mu - \eta_{\mu\alpha}k_\nu + \eta_{\nu\alpha}m_\mu - \eta_{\mu\nu}m_\alpha) \tag{28.b}$$

and

$$(V_4)_{\alpha\nu\beta\mu} \;=\; i\frac{2\omega^2}{M}(\eta_{\alpha\beta}\eta_{\mu\nu} - \eta_{\alpha\nu}\eta_{\beta\mu})\,. \tag{28.c}$$

In Fig.2, we list the 1-loop diagrams (Σ, Λ, Ξ^R, Ξ^L, and Γ) which contribute to the CSM*-field self-energy. The explicit results for these diagrams are presented in Appendix A.

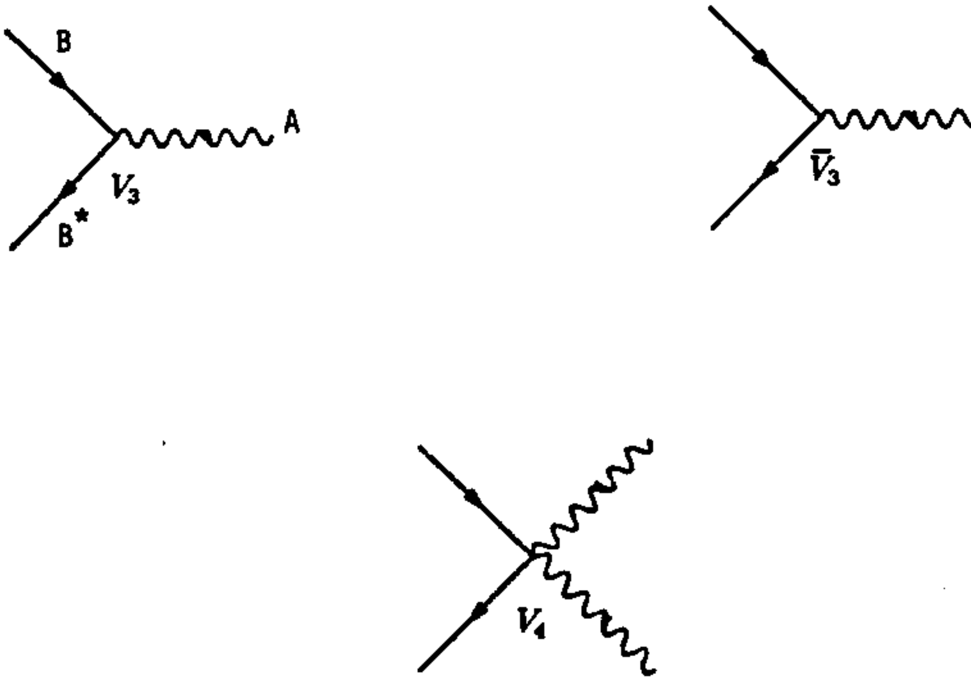

Figure 1: Interaction vertices.

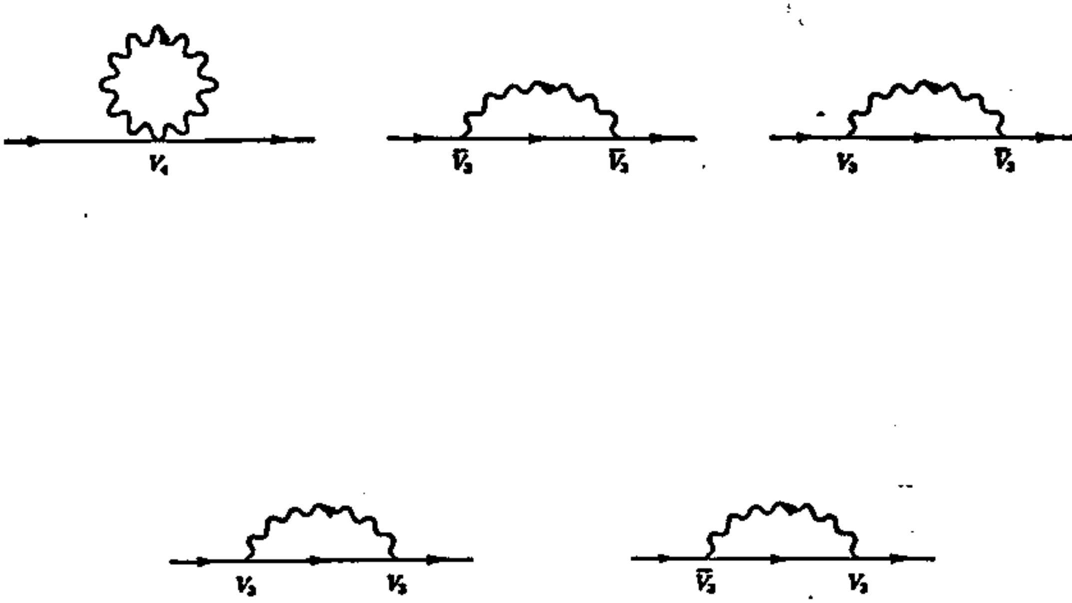

Figure 2: 1-loop CSM*-field self-energy diagrams.

Bearing in mind that we are concerned with the possibility of inducing a 1-loop (finite) mass contribution of the Proca type, we can select only those terms that do not exhibit any dependence on the external momenta and are moreover symmetric on the free indices of the external lines. Therefore, the only terms that potentially contribute a finite Proca mass term have been found to be :

$$(I_1)_{\alpha\beta} = \frac{\omega^2}{M}\int\frac{d^3k}{(2\pi)^3}\frac{k_\alpha k_\beta}{(k-p)^2(k^2-M^2)} \tag{29.a}$$

$$= \frac{\omega^2}{M}\int_0^1 dx\int\frac{d^3k}{(2\pi)^3}\frac{k_\alpha k_\beta}{[k^2-2k.px+(p^2+M^2)x-M^2]^2},$$

$$(I_2)_{\alpha\beta} = \frac{\omega^2}{M}\eta_{\alpha\beta}\int\frac{d^3k}{(2\pi)^3}\frac{k^2}{(k-p)^2(k^2-M^2)} \tag{29.b}$$

$$= \frac{\omega^2}{M}\eta_{\alpha\beta}\int_0^1 dx\int\frac{d^3k}{(2\pi)^3}\frac{k_\mu k^\mu}{[k^2-2k.px+(p^2+M^2)x-M^2]^2},$$

$$(I_3)_{\alpha\beta} = M\omega^2\eta_{\alpha\beta}\int\frac{d^3k}{(2\pi)^3}\frac{1}{(k-p)^2(k^2-M^2)} \tag{29.c}$$

$$= M\omega^2\eta_{\alpha\beta}\int_0^1 dx\int\frac{d^3k}{(2\pi)^3}\frac{1}{[k^2-2k.px+(p^2+M^2)x-M^2]^2}$$

and

$$(I_4)_{\alpha\beta} = M\omega^2\int\frac{d^3k}{(2\pi)^3}\frac{k_\alpha k_\beta}{(k-p)^2(k^2-M^2)k^2} \tag{29.d}$$

$$= \frac{\omega^2}{M}\int_0^1 dx\int\frac{d^3k}{(2\pi)^3}\frac{k_\alpha k_\beta}{[k^2-2k.px+(p^2+M^2)x-M^2]^2} +$$

$$-\frac{\omega^2}{M}\int_0^1 dx\int\frac{d^3k}{(2\pi)^3}\frac{k_\alpha k_\beta}{(k^2-2k.px+p^2x)^2}.$$

With the help of the momentum integrals (42), (43) and (44) in Appendix B, expressions (29.a), (29.b), (29.c) and (29.d) can be written as the following parametric integrals :

$$(I_1)_{\alpha\beta} = i\frac{\omega^2}{8\pi M}\left\{p_\alpha p_\beta\int_0^1 dx\frac{x^2}{[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}} + \right.$$

$$\left. + \eta_{\alpha\beta}\int_0^1 dx[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}\right\}, \tag{30.a}$$

$$(I_2)_{\alpha\beta} = i\frac{\omega^2}{8\pi M}\eta_{\alpha\beta}\Big\{p^2\int_0^1 dx\frac{x^2}{[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}}+ + 3\int_0^1 dx[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}\Big\}\,, \quad (30.b)$$

$$(I_3)_{\alpha\beta} = i\frac{\omega^2 M}{8\pi}\eta_{\alpha\beta}\int_0^1 dx\frac{1}{[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}} \quad (30.c)$$

and

$$(I_4)_{\alpha\beta} = i\frac{\omega^2}{8\pi M}\Big\{p_\alpha p_\beta\int_0^1 dx\frac{x^2}{[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}}+ + \eta_{\alpha\beta}\int_0^1 dx[p^2x^2-(p^2+M^2)x+M^2]^{\frac{1}{2}}+ - p_\alpha p_\beta\int_0^1 dx\frac{x^2}{(p^2x^2-p^2x)^{\frac{1}{2}}}-\eta_{\alpha\beta}\int_0^1 dx(p^2x^2-p^2x)^{\frac{1}{2}}\Big\} \quad (30.d)$$

The explicit results of the remaining integrals, (30.a), (30.b), (30.d) and (30.d), are presented in Appendix B. By observing these results (see 45, 46, 47 and 48), we conclude that a 1-loop term given by $i\frac{\omega^2}{32\pi}\eta_{\alpha\beta}$, coming from I_1 and I_4, will lead to the generation of the Proca term.

The whole 1-loop CSM* self-energy diagram, $\Omega^{(1)}$, is the sum of the diagrams Σ, Λ, Ξ^R, Ξ^L and Γ of Fig.2 :

$$\Omega^{(1)} = \Sigma + \Lambda + \Xi^R + \Xi^L + \Gamma\,. \quad (31)$$

By summing up all these pieces, we finally get that the 1-loop induced Proca term comes from the contribution

$$\Omega_{\hat{\mu}}^{(1)\alpha\beta} = i\,\frac{\omega^2}{8\pi}\,\eta^{\alpha\beta} = i\,\hat{\mu}\,\eta^{\alpha\beta}\,, \quad (32)$$

from which we can readily read the Proca mass :

$$\hat{\mu} = \frac{\omega^2}{8\pi} > 0\,. \quad (33)$$

It is interesting to emphasize that this term, $\Omega_{\hat{\mu}}^{(1)}$, generated by the 1-loop quantum corrections to the CSM* self-energy, is a finite one, therefore it will not be necessary to add any counter-term to the Lagrangian (6). Such a finite term amounts to the contribution

$$\mathcal{L}_{\hat{\mu}}^{(1)} = \hat{\mu} B_\mu^* B^\mu \tag{34}$$

to the classical Lagrangian. Since the parameter $\hat{\mu}$ in $\mathcal{L}_{\hat{\mu}}^{(1)}$ automatically satisfies the condition $\hat{\mu} > 0$, the espectral consistency discussed in the previous section is not jeopardized.

5 General Conclusions

Our basic proposal in this paper was to understand a number of features concerning the dynamics of complex vector fields in $D = 1 + 2$.

The first step of our study consisted in establishing conditions under which a general CSMP complex vector field describes physically acceptable excitations. It was obtained that such a complex vector field describes, in principle, two distinct massive excitations, each of them appearing of course in two states with opposite charges. The value of the spin for each of those massive states has not been calculated here [16].

Having understood how to control the physical character of the quanta of the model, we proposed to study the dynamics of a CSM*-field minimally coupled to an Abelian vector field (Maxwell field). The explicit calculation of 1-loop corrections revealed the generation of a (finite) Proca term that was not present at tree-level, respecting the spectral conditions set on the study of the propagation of the CSMP*-field. We then concluded that the 1-loop Proca mass generation does not introduce neither tachyons nor ghosts in the spectrum. As for the unitarity, it still remains to be investigated the asymptotic behaviour of scattering amplitudes for very high (much higher than the masses of the quanta) center-of-mass energies.

Also, another delicate point should be discussed. The CSM* model presents divergences at the 2-loop level. Therefore, it is crucial to check whether or not a ultraviolet divergent term of the form $|(\partial_\mu B^\mu)|^2$ appears as a 2-loop contribution to the CSM*-field self-energy. In view of this result, one may have to add, for the sake of renormalization, the term $|(\partial_\mu B^\mu)|^2$ already at the classical level, and ghosts will show up that spoil the spectrum. This matter is now under investigation [17]. However, in the Abelian case, these undesirable states do not harm the spectrum whenever the CSM*-field couples to conserved currents [18].

A 1-loop self-energy diagrams

The 1-loop CSM* self-energy diagrams, Σ, Λ, Ξ^R, Ξ^L and Γ, were calculated by using the Landau gauge, $\hat{\alpha} = 0$, for the CSM* propagator, Δ, and the Feynman gauge, $\xi = 1$, for the Maxwell propagator, D :

$$\Delta^{\alpha\beta}(k) = -i\,\frac{M^2}{k^2(k^2-M^2)}\left[i\,\epsilon^{\alpha\kappa\beta}k_\kappa + \frac{k^2}{M}\left(\eta^{\alpha\beta} - \frac{k^\alpha k^\beta}{k^2}\right)\right], \tag{35}$$

$$D^{\mu\nu}(k) = -i\,\frac{1}{k^2}\,\eta^{\mu\nu}\,. \tag{36}$$

The explicit results for the diagrams presented in Fig.2 are listed below :

$$\begin{aligned}\Sigma_{\alpha\beta} &= \int \frac{d^3k}{(2\pi)^3}\,(V_4)_{\alpha\beta\mu\nu}\,D^{\nu\mu}(k)\\ &= \int \frac{d^3k}{(2\pi)^3}\left\{-\frac{4\omega^2}{M}\,\frac{1}{k^2}\left[\eta_{\alpha\beta}\right]\right\} = 0\end{aligned} \tag{37}$$

$$\begin{aligned}\Lambda_{\alpha\beta} &= \int \frac{d^3k}{(2\pi)^3}\,(\tilde{V}_3)_{\alpha\mu\nu}\,D^{\mu\sigma}(k-p)\,\Delta^{\rho\nu}(k)\,(\tilde{V}_3)_{\rho\sigma\beta}\\ &= \int \frac{d^3k}{(2\pi)^3}\Big\{i\omega^2\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[3\epsilon_{\mu\nu\alpha}k^\mu p^\nu k_\beta - 3\epsilon_{\mu\nu\beta}k^\mu p^\nu k_\alpha +\\ &+ \epsilon_{\mu\nu\alpha}k^\mu p^\nu p_\beta - \epsilon_{\mu\nu\beta}k^\mu p^\nu p_\alpha + \epsilon_{\mu\alpha\beta}k^\mu k^2 + 2\epsilon_{\mu\alpha\beta}k^\mu(k.p) + \epsilon_{\mu\alpha\beta}k^\mu p^2\Big] +\\ &+ \frac{\omega^2}{M}\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[-k_\alpha k_\beta p^2 + k_\alpha p_\beta(k.p) + k_\beta p_\alpha(k.p) +\\ &- \eta_{\alpha\beta}(k.p)^2\Big] + \frac{\omega^2}{M}\frac{1}{(k-p)^2(k^2-M^2)}\Big[k_\alpha k_\beta - k_\alpha p_\beta - k_\beta p_\alpha - 2p_\alpha p_\beta +\\ &+ \eta_{\alpha\beta}k^2 + 2\eta_{\alpha\beta}(k.p) + 2\eta_{\alpha\beta}p^2\Big]\Big\}\end{aligned} \tag{38}$$

$$\begin{aligned}
\Xi^R_{\alpha\beta} &= \int \frac{d^3k}{(2\pi)^3}\,(V_3)_{\alpha\mu\nu}\,D^{\mu\sigma}(k-p)\,\Delta^{\rho\nu}(k)\,(\bar{V}_3)_{\rho\sigma\beta} \\
&= \int \frac{d^3k}{(2\pi)^3}\Big\{-i\omega^2\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[\epsilon_{\mu\nu\alpha}k^\mu p^\nu k_\beta + \epsilon_{\mu\alpha\beta}k^\mu(k.p)\Big] + \\
&+ i\omega^2\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[\epsilon_{\mu\alpha\beta}k^\mu k^2 + 2\epsilon_{\mu\alpha\beta}p^\mu k^2\Big] + \\
&+ M\omega^2\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[k_\alpha k_\beta - k_\beta p_\alpha + \eta_{\alpha\beta}k^2 + \eta_{\alpha\beta}(k.p)\Big]\Big\} \quad (39)
\end{aligned}$$

$$\begin{aligned}
\Xi^L_{\alpha\beta} &= \int \frac{d^3k}{(2\pi)^3}\,(\bar{V}_3)_{\alpha\mu\nu}\,D^{\mu\sigma}(k-p)\,\Delta^{\rho\nu}(k)\,(V_3)_{\rho\sigma\beta} \\
&= \int \frac{d^3k}{(2\pi)^3}\Big\{i\omega^2\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[\epsilon_{\mu\nu\beta}k^\mu p^\nu k_\alpha - \epsilon_{\mu\alpha\beta}k^\mu(k.p)\Big] + \\
&+ i\omega^2\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[\epsilon_{\mu\alpha\beta}k^\mu k^2 + 2\epsilon_{\mu\alpha\beta}p^\mu k^2\Big] + \\
&+ M\omega^2\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[k_\alpha k_\beta - k_\alpha p_\beta + \eta_{\alpha\beta}k^2 + \eta_{\alpha\beta}(k.p)\Big]\Big\} \quad (40)
\end{aligned}$$

$$\begin{aligned}
\Gamma_{\alpha\beta} &= \int \frac{d^3k}{(2\pi)^3}\,(V_3)_{\alpha\mu\nu}\,D^{\mu\sigma}(k-p)\,\Delta^{\rho\nu}(k)\,(V_3)_{\rho\sigma\beta} \\
&= \int \frac{d^3k}{(2\pi)^3}\Big\{-iM^2\omega^2\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[\epsilon_{\alpha\lambda\beta}k^\lambda\Big] + \\
&+ M\omega^2\frac{1}{(k-p)^2(k^2-M^2)}\Big[\eta_{\alpha\beta}\Big] + \\
&+ M\omega^2\frac{1}{(k-p)^2(k^2-M^2)k^2}\Big[k_\alpha k_\beta\Big]\Big\} \quad (41)
\end{aligned}$$

B 1-loop integrals for the CSM*-field self-energy

To solve the integrals I_1, I_2, I_3 and I_4 in Section 4, use has been made of the following well-known results [19] :

$$J_0 = \int \frac{d^Dk}{(2\pi)^D} \frac{1}{(k^2+2p.k-c)^\alpha} = i(-1)^\alpha \frac{\pi^{\frac{D}{2}}}{(2\pi)^D}(c+p^2)^{\frac{D}{2}-\alpha} \times \times \frac{\Gamma(\alpha-\frac{D}{2})}{\Gamma(\alpha)}, \tag{42}$$

$$J_1^\mu = \int \frac{d^Dk}{(2\pi)^D} \frac{k^\mu}{(k^2+2p.k-c)^\alpha} = i(-1)^{\alpha+1} \frac{\pi^{\frac{D}{2}}}{(2\pi)^D}(c+p^2)^{\frac{D}{2}-\alpha} \times \times p^\mu \frac{\Gamma(\alpha-\frac{D}{2})}{\Gamma(\alpha)} \tag{43}$$

and

$$J_2^{\mu\nu} = \int \frac{d^Dk}{(2\pi)^D} \frac{k^\mu k^\nu}{(k^2+2p.k-c)^\alpha} = i(-1)^\alpha \frac{\pi^{\frac{D}{2}}}{(2\pi)^D}(c+p^2)^{\frac{D}{2}-\alpha} \times \times \left[\frac{\Gamma(\alpha-\frac{D}{2})p^\mu p^\nu - \frac{1}{2}\Gamma\left(\alpha-1-\frac{D}{2}\right)\eta^{\mu\nu}(c+p^2)}{\Gamma(\alpha)}\right]. \tag{44}$$

We quote in the sequel the results for the Feynman parametrical integrals performed after the integration over the loop momenta :

$$(I_1)_{\alpha\beta} = \begin{cases} i\frac{\omega^2}{8\pi M}\Big\{p_\alpha p_\beta\left[-\frac{3(p^2+M^2)}{4(p^2)^2}M + \frac{3(p^2)^2+2M^2p^2+3M^4}{8(p^2)^2}W\right] + \\ +\eta_{\alpha\beta}\left[\frac{M^3}{4p^2} - \frac{(p^2-M^2)^2}{8p^2}W\right]\Big\} + i\frac{\omega^2}{32\pi}\eta_{\alpha\beta}\ ,\ p^2>0 \\ i\frac{\omega^2}{8\pi M}\Big\{p_\alpha p_\beta\left[-\frac{3(p^2+M^2)}{4(p^2)^2}M + \frac{3(p^2)^2+2M^2p^2+3M^4}{8(p^2)^2}V\right] + \\ +\eta_{\alpha\beta}\left[\frac{M^3}{4p^2} + \frac{(p^2-M^2)^2}{8p^2}V\right]\Big\} + i\frac{\omega^2}{32\pi}\eta_{\alpha\beta}\ ,\ p^2<0 \end{cases} \tag{45}$$

$$(I_2)_{\alpha\beta} = \begin{cases} i\frac{\omega^2 M}{8\pi}\eta_{\alpha\beta}W\ ,\ p^2>0 \\ i\frac{\omega^2}{8\pi M}\eta_{\alpha\beta}\left[\frac{3(p^2)^2-2M^2p^2+3M^4}{4p^2}V\right]\ ,\ p^2<0 \end{cases} \tag{46}$$

$$(I_3)_{\alpha\beta} = \begin{cases} i\frac{\omega^2 M}{8\pi}\eta_{\alpha\beta}W \ , \ p^2>0 \\ i\frac{\omega^2 M}{8\pi}\eta_{\alpha\beta}V \ , \ p^2<0 \end{cases} \tag{47}$$

$$(I_4)_{\alpha\beta} = \begin{cases} i\frac{\omega^2}{8\pi M}\Big\{p_\alpha p_\beta\Big[-\frac{3(p^2+M^2)}{4(p^2)^2}M + i\frac{3\pi}{8}\frac{1}{\sqrt{p^2}} + \frac{3(p^2)^2+2M^2p^2+3M^4}{8(p^2)^2}W\Big] + \\ + \eta_{\alpha\beta}\Big[\frac{M^3}{4p^2} - i\frac{\pi}{8}\sqrt{p^2} - \frac{(p^2-M^2)^2}{8p^2}W\Big]\Big\} + i\frac{\omega^2}{32\pi}\eta_{\alpha\beta} \ , \ p^2>0 \\ i\frac{\omega^2}{8\pi M}\Big\{p_\alpha p_\beta\Big[-\frac{3(p^2+M^2)}{4(p^2)^2}M - \frac{3\pi}{8}\frac{1}{\sqrt{-p^2}} + \frac{3(p^2)^2+2M^2p^2+3M^4}{8(p^2)^2}V\Big] + \\ + \eta_{\alpha\beta}\Big[\frac{M^3}{4p^2} - \frac{\pi}{8}\sqrt{-p^2} + \frac{(p^2-M^2)^2}{8p^2}V\Big]\Big\} + i\frac{\omega^2}{32\pi}\eta_{\alpha\beta} \ , \ p^2<0 \end{cases} \tag{48}$$

where W and V are defined as

$$W \equiv \frac{1}{\sqrt{p^2}}\Big[\ln(|p^2-M^2|) - 2\ln(|\sqrt{p^2}-M|) - i\pi\theta(p^2-M^2)\Big] \ \text{ if } \ p^2>0 \tag{49}$$

and

$$V \equiv \frac{1}{\sqrt{-p^2}}\left[\frac{\pi}{2} - \arctan\left(\frac{p^2+M^2}{2M\sqrt{-p^2}}\right)\right] \ \text{ if } \ p^2<0 \ . \tag{50}$$

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