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ON WORLD-SHEET GAUGE FIELDS IN (1,0)- AND (1,1)-SUPERSYMMETRIC GAUGE THEORIES

by

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Abstract

In this letter, one-loop corrections are perfomed to integrate out matter fields in (1,0)- and (1,1)-supersymmetric gauge models. The results obtained are put in correspondence with central charge calculations.

Key-words: Supersymmetry; Conformal theories; Bidimensional models.

The introduction of world-sheet gauge fields has been exploited as a classificatory scheme for the various string models [1]. Moreover, if one starts off models that exhibit (p,q)-supersymmetry the gauge fields dynamics may be used to reveal remarkable properties of the corresponding superstring models.

It is our main purpose in this letter to contemplate (1,0)- and (1,1)-supersymmetric gauge models in order to understand how the matter-gauge field couplings may affect the dynamics of the starting classical theory. More specifically, we shall integrate out matter fields in order to discuss the effective action governing the dynamics of the gauge fields. Issues like the Quillen's counterterm associated to the anomaly in the holomorfic factorization [2] and Schwinger-like mass generation are discussed. These results may be analysed on the basis of a geometrical formulation, but we rather adopt here a field-theoretical analysis. The eventual relevance of the results we shall obtain is pointed out in the concluding remarks.

We shall adopt the following conventions in our discussion: the holomorfic part of the (1,0)-superspace is parametrized by the coordinates $(z,\overline{z};\theta)$, whereas the (0,1) sector is descroibed by $(z,\overline{z};\overline{\theta})$. As for the superconformal (1,1)-model, the coordinates are taken by $(z,\overline{z};\theta,\overline{\theta})$. Here

$$z = x^1 + ix^2 \qquad \text{and} \qquad \overline{z} = x^1 - ix^2 \tag{1}$$

and the space-time metric is taken Euclidian. The Grassmann variables are left-handed

Weyl spinors. The supersymmetry covariant derivatives are defined by

$$D_\theta = \partial_\theta - i\theta\partial_z \qquad \text{and} \qquad D_{\overline{\theta}} = \partial_{\overline{\theta}} - i\overline{\theta}\partial_{\overline{z}} \tag{2}$$

For the (1,0) model, we describe the fundamental matter superfields by means of their compononent-field expressions as below:

$$\Phi_i(z,\overline{z},\theta) = \varphi_i(z,\overline{z}) + \theta\lambda_i(z,\overline{z}), \qquad D_{\overline{\theta}}\Phi_i = 0, \tag{3a}$$

$$\overline{\Phi}_i(z,\overline{z},\overline{\theta}) = \overline{\varphi}_i(z,\overline{z}) - \overline{\theta}\overline{\lambda}_i(z,\overline{z}), \qquad D_\theta\overline{\Phi}_i = 0, \qquad \text{i=1,2} \tag{3b}$$

The assignment of charges to the superfields is such that

$$\Phi_i' = e^{iq\Lambda}\,\Phi_i \tag{4}$$

with $q = +q$ if $i = 1$ and $q = -q$ if $i = 2$, φ_i and $\overline{\varphi}_i$ are scalars and λ_i, $\overline{\lambda}_i$ are right-handed spinors.

Now, for the gauging of the Abelian symmetry (4), gauge superconections have to be introduced. The gauge-covariantization of D_θ and $D_{\overline{\theta}}$ requires superfields

$$\Gamma_\theta = \gamma + \theta V_z \qquad \text{and} \qquad \Gamma_{\overline{\theta}} = -\overline{\gamma} - \overline{\theta}V_{\overline{z}} \tag{5a}$$

with

$$\delta\Gamma_\theta = \frac{1}{g}\,D_\theta\Lambda \qquad \text{and} \qquad \delta\Gamma_{\overline{\theta}} = \frac{1}{g}\,D_{\overline{\theta}}\overline{\Lambda} \tag{5b}$$

where $\overline{\gamma}$, γ are spinors fields, V_z and $V_{\overline{z}}$ are components of the gauge fields, Λ and $\overline{\Lambda}$ are gauge superparameters and g the coupling constant. They build up the gauge-covariant derivatives:

$$\nabla_\theta = D_\theta - igq\Gamma_\theta \qquad \text{and} \qquad \nabla_{\overline{\theta}} = D_{\overline{\theta}} - igq\Gamma_{\overline{\theta}} \tag{6}$$

To discuss the remaining gauge superfields, we should perhaps notice that the supersymmetry algebra suggests that Γ_z appears as an independent connection. Then, by using the conventional constraint [3], we arrive at the gauge-covariant derivatives ∇_z and $\nabla_{\overline{z}}$ as follows:

$$\Gamma_z = iD_\theta\Gamma_\theta \qquad \text{with} \qquad \nabla_z = \partial_z - igq\Gamma_z \tag{7a}$$

and

$$\Gamma_{\overline{z}} = iV_{\overline{z}} + \theta(\overline{\eta} + \partial_{\overline{z}}\gamma) \qquad \text{with} \qquad \nabla_{\overline{z}} = \partial_{\overline{z}} - igq\Gamma_{\overline{z}} \tag{7b}$$

where $\overline{\eta}$ (η) is plays the role of the photino field. From the supersymmetry algebra with suitable constraints on the torsion, the holomorfic field-strength superfield can be found:

$$[\nabla_\theta, \nabla_{\overline{z}}] = igqW \tag{8}$$

with

$$W = \overline{\eta} + \theta F_{z\overline{z}} \tag{9}$$

and

$$F_{z\overline{z}} = \partial_z V_{\overline{z}} - \partial_{\overline{z}} V_z \tag{10}$$

which is the usual Abelian field-strength. Analogously, we can calculate the anti-holomorfic part, which is necessary as a reality constraint of the (super)action.

This superaction results to be

$$S = \frac{i}{4} \sum_{a \neq b = 1,2} \int dz\, d\bar{z} \left[d\theta\, (\nabla_{\bar{z}} \Phi_a)(\nabla_\theta \Phi_b) + d\bar{\theta}\, (\nabla_z \overline{\Phi}_a)(\nabla_{\bar{\theta}} \overline{\phi}_b) \right] \tag{11}$$

whose component-field expression takes over the form:

$$\begin{aligned} S \;=\; & \frac{1}{2} \int dz\, d\bar{z}\, \{ (\partial_{\bar{z}}\, \varphi_1)(\partial_z \varphi_2) - i\lambda_2 \partial_{\bar{z}} \lambda_1 + \frac{1}{2} [\, gq(2iV_{\bar{z}} \lambda_1 \lambda_2 + \overline{\eta} \varphi_1 \lambda_2 + \\ + \;& \overline{\eta} \varphi_2 \lambda_1 - (\partial_{\bar{z}} \varphi_1) V_z \varphi_2 + (\partial_{\bar{z}} \varphi_2) V_z \varphi_1 + V_{\bar{z}} \varphi_1 (\partial_z \varphi_2) - V_{\bar{z}} \varphi_2 (\partial_z \varphi_1) + \\ - \;& 2gq V_{\bar{z}} \varphi_1 V_z \varphi_2 \,] + \text{h. c.} \} \end{aligned} \tag{12}$$

where use was been made of (8), (9) and (10). To work out the 1-loop corrections to the effective action, we wrote below the propagators for the component fields flowing inside the loops. They are:

$\varphi_1(\overline{\varphi}_1)$ ——▶—— $\varphi_2(\overline{\varphi}_2)$ $\longrightarrow$ $\langle \varphi_1(Z_a) \varphi_2(Z_b) \rangle = \langle \overline{\varphi}_1(Z_a) \overline{\varphi}_2(Z_b) \rangle = \frac{i4}{k^2}\, \delta(Z_a - Z_b)$ (13a)

$\lambda_1(\overline{\lambda_1})$ - - - -▶- - - - $\lambda_2(\overline{\lambda_2})$ $\longrightarrow$

$$\langle \lambda_1(Z_a) \lambda_2(Z_b) \rangle = \frac{4}{k_{\bar{z}}}\, \delta(Z_a - Z_b) \tag{13b}$$

$$\langle \overline{\lambda}_1(Z_a) \overline{\lambda}_2(Z_b) \rangle = \frac{4}{k_z}\, \delta(Z_a - Z_b) \tag{13c}$$

The relevant vertices for the loop calculation are depicted in Fig. 1

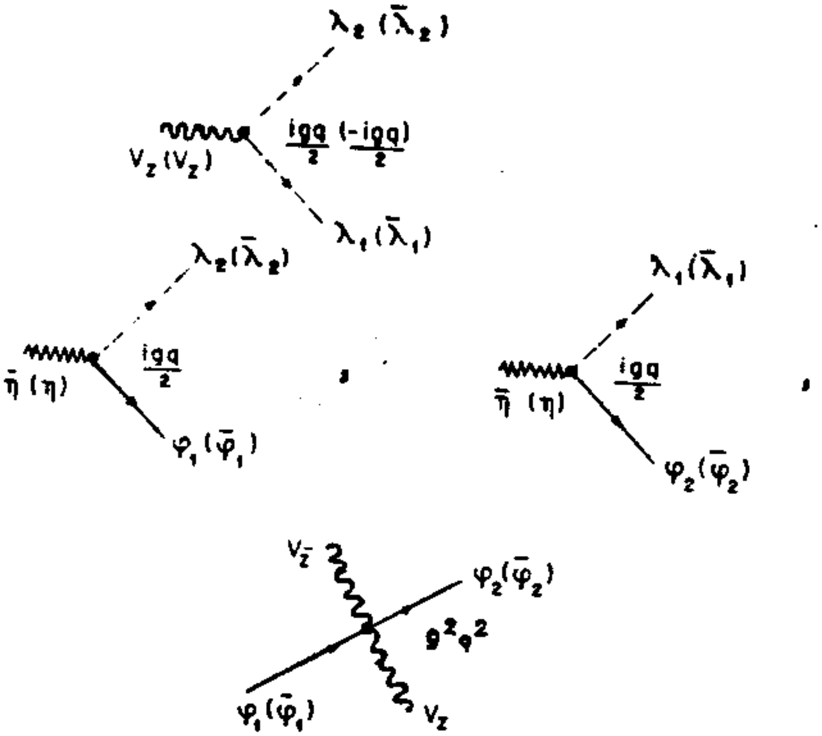

Figure 1: Matter-gauge vertices relevant for the one-loop graphs

With the help of the matter-gauge interaction terms above, one can fix the 1-loop corrections to the gauge field self-energy diagram, whose complete answer is found to be:

$$S_{eff} = \frac{q^2 g^2}{\pi} \int dz\, d\bar{z} \left\{ \frac{1}{4} V_{\bar{z}} V_z - 2V_z \frac{\partial_{\bar{z}} \partial_{\bar{z}}}{\Box} V_z - \eta \frac{\partial_z}{\Box} \eta + \text{h. c.} \right\} + \text{cubic terms} \qquad (14)$$

The finite local term appearing in (14) is crucial for the survival of the gauge simmetry at the quatum level; it however breaks the holomorfic factozation property and, therefore, it is interpreted as the anomaly for the latter [1].

On the other hand, by analysing the conformal charge of the holomorfic part using

(12), we read off the following stress tensor:

$$
\begin{aligned}
T^{zz}(z) &= -\frac{1}{2}\{((\partial^z\varphi_1)(\partial^z\varphi_2) - i\overline{\lambda}_2\partial^z\overline{\lambda}_1 + \frac{1}{2}[\, gq\,(V^z\varphi_1\,(\partial^z\varphi_2) + \\
&- \; V^z\varphi_2(\partial^z\varphi_1) + (\varphi_i \rightarrow \overline{\varphi}_i)\,\} \qquad (15)
\end{aligned}
$$

whose OPE turns out to be

$$
\langle\, T(z)T(w)\,\rangle \;=\; \frac{1}{2}\,\frac{1/2}{(z-w)^4} \qquad (16a)
$$

or

$$
c \;=\; 1/2 \qquad (16b)
$$

This is the central charge of (0,1)-model. We point out that this results plays an important role in the classification of the string models [1,4].

For the time being, we have only calculated and identified some quantum-mechanical properties of the (1,0) or (0,1) model. We shall repeat the same analysis to the superconformal (1,1) model in order to stress on the connection between the 1-loop corrections and the central charge of this model. To do this, we present below our relevante matter superfields:

$$
\Phi(z,\overline{z};\theta,\overline{\theta}) \;=\; \varphi(z,\overline{z}) + \theta\lambda(z,\overline{z}) - \overline{\theta}\overline{\eta}(z,\overline{z}) + \theta\overline{\theta}h(z,\overline{z}) \qquad (17a)
$$

$$
\Phi(z,\overline{z};\theta,\overline{\theta}) \;=\; \overline{\varphi}(z,\overline{z}) + \theta\eta(z,\overline{z}) - \overline{\theta\lambda}(z,\overline{z}) + \theta\overline{\theta}(z,\overline{z}) \qquad (17b)
$$

where we use these component-fields with the definitions

$$
\varphi = \varphi_1 + i\varphi_2 \qquad , \quad h = h_1 + ih_2,
$$

$$\lambda = \lambda_1 + i\lambda_2 \qquad , \quad \eta = \lambda_1 - i\lambda_2 \tag{18}$$

with φ_i, h_i real fields, λ_i complex field and the index i refers to the Noether charge of the fields.

The gauge background superfields read as below:

$$\Gamma_\theta(z,\overline{z};\theta,\overline{\theta}) = \gamma + \theta V_z - \overline{\theta} f - \theta\overline{\theta}\rho \tag{19a}$$

$$\Gamma_{\overline{\theta}}(z,\overline{z};\theta,\overline{\theta}) = -\gamma + \theta V_{\overline{z}} + \theta \overline{f} - \theta\overline{\theta}\overline{\rho} \tag{19b}$$

with the conventional constraint:

$$D_\theta \Gamma_\theta = -i\Gamma_z \qquad \text{and} \qquad D_{\overline{\theta}}\Gamma_{\overline{\theta}} = -i\Gamma_{\overline{z}} \tag{20}$$

Here he note, contrary to the (1,0) or (0,1) model that the superfields Γ_z and $\Gamma_{\overline{z}}$ are not independent. The superaction for this model is

$$S = \frac{1}{2}\int dz\, d\overline{z}\, d\theta\, d\overline{\theta} (\nabla_{\overline{\theta}}\overline{\Phi})(\nabla_\theta \Phi) \tag{21}$$

where the covariant derivatives are defined as

$$\nabla_\theta = D_\theta - igq\Gamma_\theta \qquad \text{and} \qquad \nabla_{\overline{\theta}} = D_{\overline{\theta}} - igq\Gamma_{\overline{\theta}} \tag{22}$$

Here, q has the same definition as given in (4). For component-field calculations, the relevant commutators in the supersymmetry algebra are:

$$[\nabla_\theta, \nabla_{\overline{z}}] = -igq\overline{\Omega} \qquad \text{and} \qquad [\nabla_{\overline{\theta}}, \nabla_z] = -igq\Omega \tag{23}$$

Then, (21) can be written as:

$$
\begin{aligned}
S &= \frac{1}{2}\int dz d\bar{z}\,\{\,(\partial_{\bar{z}}\overline{\varphi})(\partial_z\varphi) - i\lambda\partial_{\bar{z}}\eta - i\overline{\lambda}\partial_z\overline{\eta} + \overline{h}h + \\
&+ gq\,[\,iV_{\bar{z}}\eta\lambda - iV_z\overline{\eta}\overline{\lambda} - \overline{\sigma}\varphi\eta + \overline{\sigma\varphi}\lambda + \sigma\varphi\overline{\lambda} - \sigma\overline{\varphi\eta} + i\gamma\overline{\lambda}h + \\
&- i\overline{h}\overline{\gamma}\lambda + f\overline{\varphi}h + \overline{h}f\varphi + i\overline{\gamma}\eta h - i\overline{h}\gamma\overline{\eta} + \frac{1}{2}\{\,-(\partial_{\bar{z}}\overline{\varphi})V_z\varphi + \\
&- (\partial_z\overline{\varphi})V_{\bar{z}}\varphi + V_{\bar{z}}\overline{\varphi}(\partial_z\varphi) + V_z\overline{\varphi}(\partial_{\bar{z}}\varphi)\,\}\,] + \\
&+ g^2q^2\,[\,-V_{\bar{z}}\overline{\varphi}V_z\varphi + \gamma\overline{\lambda}\overline{\gamma}\lambda + \overline{\gamma}\eta\gamma\overline{\eta} + \gamma\overline{\gamma\varphi}h + \overline{h}\gamma\overline{\gamma}\varphi + \\
&+ f\overline{\varphi}f\varphi - f\overline{\varphi\gamma}\lambda - \overline{\gamma}\eta f\varphi - f\overline{\varphi}\gamma\overline{\eta} - \gamma\overline{\lambda}f\varphi\,]\,\}
\end{aligned}
\qquad (24)
$$

Where the photino now is given by $\sigma = -i\rho - \partial_z\overline{\gamma}$ (and $\overline{\sigma} = -i\overline{\rho} + \partial_{\bar{z}}\gamma$). Now, the relevant propagators are analogous to ones given in (13). Then using the peturbation scheme the only one-loop contribuition to the effective action are represent in the fig. 2

Figure 2: One-loop correction involving matter fields in the loops

The final answer for the 1-loop Feynmann graphs drawn in fig. 2 is given by

$$\begin{aligned} S_{eff} &= \frac{q^2g^2}{\pi}\int dz d\bar{z}\,\{\,\frac{1}{2}V_{\bar{z}}V_z - 2V_z\frac{\partial_{\bar{z}}\partial_{\bar{z}}}{\Box}V_z - 2V_{\bar{z}}\frac{\partial_z\partial_z}{\Box}V_{\bar{z}} + \\ &+ i\sigma\frac{\partial_{\bar{z}}}{\Box}\sigma + i\overline{\sigma}\frac{\partial_z}{\Box}\sigma + i8ff\gamma\frac{\partial_z}{\Box}\gamma + i8ff\overline{\gamma}\frac{\partial_z}{\Box}\overline{\gamma}\,\} \end{aligned} \tag{25}$$

Notice, now, that the first term in (25) is a dinamically-generated mass term, as it happens in Schwinger model [5]. Finally, we calculate the central charge using the OPE to the stress tensor associated to (24):

$$\begin{aligned} T_{zz}(z) &= -\frac{1}{2}\{\,(\partial_z\overline{\varphi})(\partial_z\varphi) - i\lambda\partial_z\eta - \frac{1}{2}[\,gq(\,V_z\overline{\varphi}(\partial_z\varphi) + \\ &- (\partial_z\overline{\varphi})V_z\varphi\,)\,]\,\} \end{aligned} \tag{26}$$

whose tensorial algebra is given by

$$\langle T(z)T(w)\rangle = 0 \tag{27a}$$

or

$$c = 0 \tag{27b}$$

As long as conformal symmetry is concerned, this results represents a classical-like theory.

To conclude this letter, we would like to point out a few comments on the calculations we have carried out. According to the discussions of refs [1], gauge field dynamics may play a significant role in the classification scheme of string models and possibly in the issue of supersymmetry breaking in superstrings. Therefore it behooves us a better understanding of the quantum-mechanical properties of models that involve 2-dimensional gauge fields coupled to matter fields through supersymmetry. We have specified Abelian gauge theories with (1,0) (or (0,1)) and (1,1) supersymmetries. The results we find here show that in those supersymmetric models there appears an anomaly in the holomorfic factorization of the gauge field piece of the effective action. The results presented in eq. (14) is nothing but the Quillen's counterterm necessary to enforce gauge invariance of the effective action at the cost of factorization. On the other hand, we calculate the central charge of the (1,0) (or (0,1)) model and verify, by eq. (16), that $c \neq 0$. The central charge signals the presence of a factorization anomaly (Quillen's counterterm when $c = 1/2$ to (1,0) (or (0,1) model)) or a dynamical mass generation [6] (Schwinger-model-like when $c = 0$ to (1,1) model) in the framework of Euclidean 2-dimensional supersymmetry. This is relevant for exact calculation of partition functions on Riemann surfaces. It remains to the analysed the generalization of the values of c to other (p,q) gauge models and the constraints to the pertubation terms. This would be very important for classifying superstring models.

The component-field results reported here for (1,0) model are supported by a superspace calculation carried out in ref. [3,6]. It would be further interesting to analyse the consequences of our conclusions in a model that possesses local supersymmetry, since this would be the case actually interesting for string calculations. A breaking of factorization also in the gravitational sector would signal a remarkable observation in the process of solving partition functions for theories on Riemann surfaces.

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