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ON BRST QUANTIZATION OF CHIRAL BOSONIC PARTICLE

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ABSTRACT:

A chiral bosonic particle based on a linear constraint, in analogy to a recently proposed similar description for chiral boson, is quantized and its propagator obtained.

Key-words: Self-dual particle; BFV quantization; Chiral boson.

1. INTRODUCTION:

The quantization of chiral boson which are relevant for the formulation of Heterotic string[1] has been much discussed[2,3,4] recently following the the actions proposed by Siegel[2] and Floreanini and Jackiw[3]. In order to gain some more insight on the problem of quantizing self-dual field it has led to the study of chiral bosonic particle[5] based on a constraint quadratic in momenta, inspired from the dimension zero field formulation of chiral boson described in ref. 3. In a recent study[6], motivated by an analogous situation in Yang-Mills theory, it was shown that a self-consistent description and quantization of chiral boson field could be achieved by adding to the Lagrangian of ordinary scalar field an auxilary field to take care of the (linear) constraint. The canonical Hamiltonian formulation using the Dirac's method[7] can be built in a self-consistent manner. In the resulting theory the field itself satisfies the self-duality condition and there is no violation of (micro-) causality principle, contrary to what we find in the description of chiral boson as given in ref.3.

We propose here to describe, in analogy to the description in ref.6 of the self-dual field, a chiral bosonic particle based on a constraint which is linear in momenta and quantize the theory using BRST formulation[8] and obtain also the propagator using functional integral following the BFV[9] formulation. Our action is different and perhaps more faithful compared to the one proposed in ref.5

in that the linear constraint is imposed at the Lagrangian level in the second order formulation which leads to a self-consistent Hamiltonian theory with a constraint again linear in momenta. *We do not agree with ref.5 in that such a term violates reparametrization invariance.* It will become clear from the discussion to follow. Finally we comment on the differences here compared to the theory of self-dual field where a Wess-Zumino like term must be added to compensate the undesired mode.

2. CHIRAL BOSONIC PARTICLE:

In order to make the discussion parallel to that of chiral boson as given in ref.6 and follow the difference we take the following reparametrization invariant action for a left moving massless relativistic particle

$$S= \int [\frac{1}{2e} \dot{x}_\mu \dot{x}^\mu + \alpha_\mu (\eta^{\mu\nu}-\epsilon^{\mu\nu})\dot{x}_\mu]\, d\tau \tag{1}$$

It is convenient to rewrite the Lagrangian as

$$L = \frac{1}{2e} [(\dot{x}^0)^2-(\dot{x}^1)^2] + \alpha(\dot{x}^0-\dot{x}^1) \tag{2}$$

where τ is an invariant monotonic parameter such that $\dot{x}^0=dx^0(\tau)/d\tau>0$ and an overdot indicates derivative with respect to τ. Under the one dimensional diffeomorphisms, $\tau\to\tau'=\tau-\delta\tau$, L transforms like a density ,$\delta L=d(L\delta\tau)/d\tau$, and so do $e(\tau),\dot{x}_\mu$ while x^μ,p_μ transform like scalars,e.g.,

$\delta x = \dot{x}\delta\tau$ etc. and there is no problem with the linear term in (1) as regards the reparametrization invariance.

Following the Dirac method[7] for handling degenerate action we find $p_\alpha \approx 0, p_e \approx 0$ as the primary constraints while the canonical Hamiltonian is given by

$$H_c = e\,[\tfrac{1}{2}(p_0 - p_1) - \alpha](p_0 + p_1) \tag{3}$$

Requiring then the persistency in τ of these constraints leads to the secondary constraint $\phi \equiv (p_0 + p_1) \approx 0$ and the Hamiltonian vanishes as expected due to the reparametrization invariance. No further constraints are generated and the constraints are first class. We may ignore the first two of them by choosing the gauge-fixing conditions $e \approx 0, \alpha \approx 0$ and defining Dirac brackets with respect to them. The Dirac brackets of x, p coincide with the standard Poisson backets and $e = \alpha = 0$ are now strong relations. Hence we arrive at the following (reduced) Lagrangian in the first order formulation

$$L_0 = \dot{x}^\mu p_\mu - \lambda(\tau)(p_0 + p_1) \tag{4}$$

where the first term is by itself a density with respect to reparametrizations. It follows that the Lagrange multiplier λ must transform as a density too and the action (4) is then reparametrization invariant. All the dynamics in the theory is given by the constraint ϕ which generates gauge transformations. We find easily that the following local gauge transformation leaves the first order action (4) invariant

$$\delta x^0=\varepsilon(\tau),\quad \delta x^1=\varepsilon(\tau),\quad \delta p_0=\delta p_1=0,\quad \delta\lambda=\dot{\varepsilon}(\tau) \tag{5}$$

where $\varepsilon(\tau)$ transforms like a scalar.

3. BRST QUANTIZATION:

Corresponding to the local symmetry (5) we define by the well known procedure the following fermionic BRST symmetry s, $s^2=0$, over an extended phase space which carries in addition to x,p two fermionic ghost variabless η and $\bar{\eta}$ along with a gauge invariant bosonic variable b for gauge-fixing

$$sx^0=\eta,\quad sx^1=\eta,\quad sp_0=sp_1=0,\quad s\eta=0,\quad s\lambda=\dot{\eta},\quad s\bar{\eta}=b,\quad sb=0 \tag{6}$$

The BRST invariant action is easily found to be

$$L= L_0+ s\,[\bar{\eta}(\dot{\lambda} +b/2)]= L_0 + b\dot{\lambda} +(1/2)b^2 + \dot{\bar{\eta}}\,\dot{\eta} \tag{7}$$

We may identify the field b with p_λ, the momentum canonical to λ with $sp_\lambda=0$. The canonical quantization may be performed now over the extended phase space with variables $(x^\mu,p_\mu,\lambda,p_\lambda,\eta,\bar{\eta},P,\bar{P})$ where P,and $\bar{P}$ are canonical momenta conjugate to $\bar{\eta}$ and η respectively and satisfy $\{P,\bar{\eta}\}=\{\bar{P},\eta\}= -1$. The Hamiltonian is found to be

$$H=P\bar{P} - \frac{1}{2}\, p_\lambda^2 + \lambda(p_0+p_1) \tag{8}$$

and the graded brackets must be used. In addition to the BRST symmetry generated by the conserved nilpotent charge $\Omega=[\mathcal{P}p_\lambda+\eta(p_0+p_1)]$ we also have an anti-BRST symmetry in our case generated by the nilpotent conserved charge $\bar{\Omega}=[-\bar{\mathcal{P}}p_\lambda+\bar{\eta}(p_0+p_1)]$, $\{\Omega,\bar{\Omega}\}=0$ and the physical states are required to be annihilated by both of these operators.

4. PROPAGATOR:

The quantized propagator in the BFV[9] formulation is constructed as follows. We construct an effective action

$$S_{eff}=\int_0^\tau d\tau\ [\dot{x}^\mu p_\mu+\dot{\lambda}p_\lambda+\dot{\eta}\bar{\mathcal{P}}+\dot{\bar{\eta}}\mathcal{P}-\{\Omega,\Psi\}] \tag{9}$$

where Ψ is an arbitrary gauge-fixing fermionic function. We found above $\Omega=\eta\phi+\mathcal{P}p_\lambda$ and will make a convenient choice for $\Psi=-\lambda\bar{\mathcal{P}}$ which gives rise to proper time gauge $\dot{\lambda}=0$. We find

$$\{\Omega,\Psi\}=\lambda(p_0+p_1)+\mathcal{P}\bar{\mathcal{P}} \tag{10}$$

We consider the BRST invariant boundry conditions (see (8)) $p_\lambda=\eta=\bar{\eta}=0$ for $\tau=0$ and τ and also find from classical eqns. of motion $\dot{x}^0=\{x,H\}=\lambda$ leading to $0<\lambda<\infty$. The quantized theory propagator is then obtained from the following functional integral

$$K(X,Y)=\int_{x^\mu=X^\mu}^{x^\mu=Y^\mu}[d\mu]\ \exp(iS_{eff}) \tag{11}$$

where $x^\mu(o)=X^\mu$, $x^\mu(\tau)=Y^\mu$ are the initial and final values of the particle's position and $[d\mu]$ is the Liouville measure of the extended phase space. We remark[\$] that if we perform the functional integratation in (11) over p_o we find that $\dot{x}^o=\lambda$ is effectively implemented for every trajectory in configuration space and not just for the extremal ones. The functional integration over $P,\bar{P}$ reduces the ghost term in S_{eff} to $\dot{\bar{\eta}}\dot{\eta}$. In view of the boundry conditions, the functional integral over η, $\bar{\eta}$ brings down in the integrand a factor $\det(-\partial_\tau^2)$. This expression may be computed[#] using ζ-function regularization taking care of the same boundry conditions. It results in a multiplicative factor τ in the numerator of K which will be compensated below (see (14)). The integration over p_λ brings in a factor $\delta(\dot{\lambda})$ in the integrand. To integrate over the coordinates we first do a shift transformation from x^μ to ξ^μ

$$x^\mu=X^\mu + \Delta^\mu\tau/\tau + \xi^\mu \quad , \qquad \dot{x}^\mu = \Delta^\mu/\tau + \dot{\xi}^\mu \tag{12}$$

where $\Delta^\mu=(Y-X)^\mu$ so that $\xi_\mu(o)=\xi_\mu(\tau)=0$. The functional integration on ξ now brings down delta functional of $\dot{p}$ obtaining

$$K(\Delta)= \tau \int [d\lambda][dp_o][dp_1]\delta(\dot{\lambda})\delta(\dot{p}_o)\delta(\dot{p}_1) \exp i\int_0^\tau d\tau \, [p_\mu\Delta^\mu/\tau - \lambda\phi] \tag{13}$$

where a numerical normalization factor is suppressed. Only the constant modes survive due to the delta functionals turning the functional integrations into the ordinary ones

$$K(\Delta)= \tau \int d^2p \int_0^\infty d\lambda \; e^{ip.\Delta} \; e^{-i\lambda(p_o+p_1-i\varepsilon)\tau} \tag{14}$$

\$ C. Teitelboim, Phys. Rev.D25, 3159(1982).

\# A. Cohen et.al., Nucl. Phys. B267, 143(1986).

Here we have introduced a damping factor in accordance with that $\lambda > 0$. Thus the factor τ^{-1} coming from the integration over λ compensates the one arising from the ghosts and the propagator (upto a multiplicative constant) is found to be

$$K(x,y) = \int \frac{d^2p}{(2\pi)^2} \; e^{ip.(y-x)} \frac{1}{(p_0+p_1-i\varepsilon)}$$
$$\approx \; \theta(y^0-x^0)\delta(y^1-y^0-x^1+x^0) \qquad (15)$$

5. COMMENTS:

In spite of the apparent similarities with the theory of chiral boson[8] we find that in the case of the chiral particle p_0, p_1 commute and the reparametrization invariance forces the constraint to be first class while in the former case Π_0, Π_1 do not commmute and the constraints are second class. We must add[10] a Wess-Zumino like term to make it into a gauge theory which compensates the undesired mode $(\Pi_0 - \Pi_1)$.

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