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AN EFFECTIVE LAGRANGIAN DESCRIPTION OF SUPERNOVA-CORE BOUNCE

by

H. RODRIGUES, V. d'AVILA, S.J.B. DUARTE and T. KODAMA

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

ABSTRACT

The global dynamical aspects of a supernova event is studied in terms of an effective Lagrangean formulation. The equation of motion derived from this Lagrangian is solved numerically for diferent supernova core masses. An equation of state for cold matter is introduced by means of an adiabatic index parametrization which is a smooth function of the matter density. The energy transfer from the inner to the outer core is estimated in the context of the hydrodynamic bounce mechanism. It is found that only a very restricted mass distribution to pre-supernova core configuration generate a strong enough shock wave leading to a prompt bounce ejection.

Key-words: Effective Lagrangian; Supernova Bounce; Shock wave.

I. INTRODUCTION

The study of supernova processes has attracted a great deal of interests in the field of high energy astrophysics for many years [1,4] ; in special, very recently the neutrino and optical observation of **SN1987A** have rosen the opportunity to test some aspects of current supernova theories. The study of such a violent event is expected to give an insight not only to the property of nuclear matter at extreme conditions of density and temperature, but also to the phases transitions of the hadronic matter.

A supernova is an event related to the sudden and catastrophic death of a massive star ($8-10 < M/M_{\odot} < 60-100$), at the end of its evolution, which may lead to the formation of a neutron stars or a black hole. However, in spite of many theoretical investigations, we still face several controversial points of view concerning the basic mechanism determining the event of supernova explosion. This is mainly due to the very complicated nature of the hydrodynamical calculation including shock wave formation and several unknown fundamental factors. such as neutronization processes, nuclear matter equation of state, neutrino opacity, thermonuclear explosive fuel, etc. In fact, the quantitative results of those calculations are strongly dependent on these factors [5,11]. If the phenomena really is such a critical one, it seems unlike to justify the rather regular occurrence of supernova events in the Universe. In our opinion the explosion mechanism should have a more general origin which can be understood in terms of the pre-supernova configuration and the global properties of stellar dynamics, without depending on the very details of a particular model.

It is well known that the gravitational collapse of the pre-supernova core is triggered by endothermic processes such as e-capture and/or photo-disintegration of nuclei, leading to an almost free fall motion of the core. In the ultimate instants of this collapse, the stellar matter density attains a value even higher than that of the nuclear matter equilibrium. However, the essential and basic question is how the violent implosion is suddenly inverted into equally or even more violent expansion and subsequent ejection of matter. Several possible mechanisms have been proposed, and we may classify them as follows:

1. Hydrodynamical bounce with shock wave formation;
2. Nuclear fuel detonation with shock wave propagation;
3. Energy and Momentum transfer from the core to the envelope by neutrinos;
4. Stellar rotation and magnetohydrodynamical effects.

Here, we are interested in investigating the kinematical aspects of the explosive mechanism and its relation to the general feature of the stellar matter equation of state. Therefore, we avoid introducing any mechanisms which depend explicitly on microscopic processes, and we will concentrate ourselves to the first type of mechanism, i.e.,hydrodynamic bounce.

Usually, the hydrodynamical equations of motion for supernovae have been solved by transforming the corresponding partial differential equation into a finite difference equation. This procedure requires many variables to be treated in order to keep a good mathematical approximation for solving numerically the hydrodynamics of the system with shock wave generation. Although sophisticated computer codes can solve satisfactorily the problem, many physical aspects are masked by the inumerous variables unnecessary for the specific study of, for example, shock wave generation. In order to investigate general aspects of shock wave formation, it is worthwhile to develop a more physical approximation procedure specific to it.

In this work we establish a physical mapping of the hydrodynamic equation into a few-variable effective Lagrangian system in order to avoid an approximation merely based on mathematical arguments. In this sense, our main interest is not to discuss the local properties of hydrodynamic evolution of the system, but to describe the more global nature of core explosion, namely the separation of its mass into a remnant neutron star and the exploding outer shell, as well as the amount of energy transfer from the inner to the outer core.

In the following section we construct the effective Lagrangian for our model and derive the equation of motion of the system. In Section III, we introduce a simple parametrization to describe the equation of state of the stellar matter, and we show some examples of application. In the last section, we discuss our results.

II. THE MODEL

As illustrated in **Fig.1**, we divide the pre-supernova core into n- shells, characterized by their radii $\vec{R} = [R_1, R_2, \ldots, R_n]$ and mean densities $\{\rho_1, \rho_2, \cdots, \rho_n\}$. Their masses,

$$m_i = \frac{4\pi}{3}\rho_i(R_i^3 - R_{i-1}^3), \quad i = 1, .., n \tag{1}$$

are kept constant in time. In this equation, $R_0 = 0$ by convention.

Neglecting possible energy losses, we may write the Lagrangian of the system,

$$\begin{aligned} L &= L(\{R_i\}, \{\dot{R}_i\}) \\ &= K - V_G - E_{int}, \end{aligned} \tag{2}$$

where K represents the kinetic energy associated with the hydrodynamic motion of matter, V_G the total gravitational energy and E_{int} the internal energy. The quantities K and E_{int} are the sum of the contribution from each shell, namely, $K = \sum_{i=1}^{n} K_i$ and $E_{int} = \sum_{i=1}^{n} E_i$. The kinetic energy of the i-th shell is calculated as

$$K_i = \frac{1}{2} \int_V d^3\vec{r}\ \rho_i v_i^2(\vec{r}), \tag{3}$$

where $\vec{v}_i(\vec{r})$ is the velocity field inside the i-th shell. The velocity field can be determined from the continuity equation. For a spherically symmetric homogeneus shell, we get

$$v(r) = \frac{(\dot{R}_i R_i^2 - \dot{R}_{i-1} R_{i-1}^2) r^3 + (\dot{R}_{i-1} R_{i-1}^2 R_i^3 - \dot{R}_i R_i^2 R_{i-1}^3)}{r^2 (R_i^3 - R_{i-1}^3)}, \quad R_{i-1} \le r \le R_i, \tag{4}$$

which satisfies the boundary conditions $v(r = R_{i-1}) = \dot{R}_{i-1}$, and $v(r = R_i) = \dot{R}_i$. Again for i=0, the convention $R_0 = 0$ and $\dot{R}_0 = 0$ is assumed.

With this velocity field, the kinetic energy of the i-th shell is calculated as

$$K_i = \frac{3}{10} (\, \dot{R}_{i-1} \quad \dot{R}_i \,) \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} \dot{R}_{i-1} \\ \dot{R}_i \end{pmatrix}, \tag{5}$$

where

$$T_{11} = \frac{5\xi^3 + 6\xi^2 + 3\xi + 1}{(1 + \xi + \xi^2)^3} m_i; \tag{6-a}$$

$$T_{12} = T_{21} = \frac{3}{2} \frac{\xi^2(\xi^2 + 3\xi + 1)}{(1 + \xi + \xi^2)^3} m_i; \tag{6-b}$$

$$T_{22} = \frac{\xi^3(\xi^3 + 3\xi^2 + 6\xi + 5)}{(1 + \xi + \xi^2)^3} m_i; \tag{6-c}$$

with $\xi \equiv R_i / R_{i-1}$.

The gravitational potential energy is found to be

$$V_G = -G \sum_{i=1}^{n} [\frac{3}{5} m_i f(\xi) + \frac{3}{2} M_i g(\xi)] m_i / R_{i-1}, \tag{7}$$

where G is the gravitational constant, and

$$f(\xi) = \frac{1}{2}(2\xi^3 + 4\xi^2 + 6\xi + 3)/(1 + \xi + \xi^2),$$

$$g(\xi) = (1 + \xi)/(1 + \xi + \xi^2)^2,$$

with

$$M_i = \begin{cases} 0, & \text{for } i = 1; \\ \sum_{j=1}^{j=i} m_j & \text{for } i \geq 2\ . \end{cases}$$

The first term in Eq.(7) represents the gravitational self-energy of the i-th shell, and the last term comes from the interaction among the shells.

The internal energy of each shell is expressed as

$$E_i = m_i \epsilon_i / \rho_i, \tag{8}$$

where ϵ_i is the volumetric energy density of the i-th shell given by the equation of state as a function of the density ρ.

The Euler-Lagrange equation leads to the following equation of motion,

$$\hat{T}\frac{d^2\vec{R}}{dt^2} = \hat{Q}\frac{d\vec{R}}{dt} + \vec{F}, \tag{9}$$

where $\hat{T}$ and $\hat{Q}$ are $n \times n$ matrices given by

$$\hat{T} = \begin{pmatrix} T_{11}^{(1)} & T_{12}^{(1)} & 0 & 0 & \cdots & 0 \\ T_{21}^{(1)} & T_{22}^{(1)} + T_{11}^{(2)} & T_{12}^{(2)} & 0 & \cdots & \vdots \\ 0 & T_{21}^{(2)} & T_{22}^{(2)} + T_{11}^{(3)} & T_{12}^{(3)} & \cdots & \vdots \\ 0 & 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & & & 0 \\ \vdots & \vdots & 0 & T_{21}^{(n-1)} & T_{22}^{(n-1)} + T_{11}^{(n)} & T_{12}^{(n)} \\ 0 & \cdots & 0 & 0 & T_{21}^{(n)} & T_{22}^{(n)} \end{pmatrix}, \tag{10}$$

and

$$\hat{Q} = -\frac{d}{dt}\hat{T}, \tag{11}$$

where $T_{lm}^{(i)}$ is the matrix element of i-th shell given by Eq.(7). The force term $\vec{F}$ is given as

$$\vec{F} =$$

$$= \begin{pmatrix} \frac{20\pi}{3}(P_1 - P_2)R_1^2 - \frac{G}{R_1^2}[M_1^2 f_2^{(1)} + M_2^2 f_1^{(2)} + \frac{5}{2}(m_1 M_1 g_2^{(1)} + m_2 M_2 g_1^{(2)})] \\ \frac{20\pi}{3}(P_2 - P_3)R_2^2 - \frac{G}{R_2^2}[M_2^2 f_2^{(2)} + M_3^2 f_1^{(3)} + \frac{5}{2}(m_2 M_2 g_2^{(2)} + m_3 M_3 g_1^{(3)})] \\ \vdots \\ \frac{20\pi}{3}(P_i - P_{i+1})R_i^2 - \frac{G}{R_i^2}[M_i^2 f_2^{(i)} + M_{i+1}^2 f_1^{(i+1)} + \frac{5}{2}(m_i M_i g_2^{(i)} + m_{i+1} M_{i+1} g_1^{(i+1)})] \\ \vdots \\ \frac{20\pi}{3} P_n R_1^2 - \frac{G}{R_n^2}[M_n^2 f_2^{(n)} + \frac{5}{2} m_n M_n g_2^{(n)}] \end{pmatrix}, \quad (12)$$

where P_n represents the pressure of the gas in each shell, and

$$f_1^{(i)} = \frac{3}{2}\frac{(\xi^2 + 3\xi + 1)}{(1 + \xi + \xi^2)^3};$$

$$f_2^{(i)} = \frac{\xi^3(\xi^3 + 3\xi^2 + 6\xi + 5)}{(1 + \xi + \xi^2)^3};$$

$$g_1^{(i)} = \frac{2\xi + 1}{(1 + \xi + \xi^2)^2};$$

$$g_2^{(i)} = \frac{\xi^3(\xi + 2)}{(1 + \xi + \xi^2)^2}.$$

At this stage, the partial differential equation of hydrodynamic motion is converted into a set of second order ordinary differential equations. This mapping is a physical one in the sense that the total energy of the system is strictly conserved holding the effective meaning of the variables associated to each shell. Note that our equations of motion reduce again to the hydrodynamic equation in the limit of $n \to \infty$. Due to this physical nature of our Lagrangian, we expect that the global behavior of stellar core dynamics can well be described without introducing many subshells in the calculation. In particular, in this paper, we are interested only in the study of the kinematical aspect of shock formation which separates the core into the remnant neutron star and the exploding outer shell. Therefore, we may take the simplest case, $n = 2$, without spoiling the basic physics of the process.

The equations of motion are then numerically integrated for a specified initial configuration.

III. BOUNCE SHOCK GENERATION

The gravitational stability of the iron core prior to the onset of the collapse is basically supported by the degenerate electron pressure whose adiabatic index γ is close to 4/3. In this electronic phase we safely neglect contributions from eventual free nuclei and nucleons. For higher densities, due to the e-capture and/or photo-disintegration reactions, which reduce the pressure, the core becomes gravitationally unstable, and the collapse sets in. The increase of density accompanying the collapse further neutronizes the core matter, and the effective adiabatic index becomes smaller than the critical value 4/3, leading to an almost free-fall collapse. On the other hand, when the density reaches a value close to that of the equilibrium nuclear matter, the equation of state suddenly stiffens and the adiabatic index γ rises to over 4/3. At this stage, the collapse is halted due to the high incompressibility of the nuclear matter and the bounce will take place.

In order to simulate the above properties of the stellar matter in a simple way, we parametrize the relative concentration of electrons in the stellar matter as a function of the density. This function should simulate the deleptonization as well as the neutronization of the stellar matter, which are the basic physical ingredients of the supernovae implosion phase. At the same time our equation of state should represent the increase of adiabatic index near the nuclear matter density. By keeping this in mind, we write the relative concentration of electron gas as

$$F(\zeta) = \frac{1}{1 + e^{(\zeta - \zeta_c)/\zeta_d}}, \tag{13}$$

where,

$$\zeta = log_{10}\rho;$$

$$\zeta_c = \frac{1}{2}(\zeta_n + \bar{\zeta});$$

$$\zeta_d = \frac{1}{2}(\zeta_n - \bar{\zeta});$$

with $\bar{\zeta}$ and ζ_n as free parameters. We then write the pressure as the sum of the electron pressure P_e and the neutron gas pressure P_n, given by

$$P_e = K_1\{x(2x^2 - 3)(x^2 + 1)^{1/2} + 3ln[x + (x^2 + 1)^{1/2}]\}, \tag{14}$$

$$P_n = K_2\rho^{5/3}, \tag{15}$$

where $K_1 = 6.002 \times 10^{22}(erg/cm^3)$; $K_2 = 5.454 \times 10^9(erg \cdot cm^2/g^{5/3})$;

$$x = \frac{(3\pi^2)^{1/3}}{m_e c}\hbar(FY_0\rho/m_u); \tag{16}$$

and m_e is the electron mass, Y_0 is the initial electron-nucleon ratio, ρ is the barionic density and m_u is the atomic mass unit.

With the above equation of state, the effective adiabatic index $\gamma(\equiv \partial lnP/\partial ln\rho)$ can be calculated as,

$$\gamma = \frac{1}{3}\frac{1}{P}\{[\frac{1}{\zeta_d}(F-1)+1][8K_1x^5(x^2-1)^{-1/2}]+5P_n\}, \tag{17}$$

where $P = P_e + P_n$.

Figure 2 shows the behavior of adiabatic index versus logarithm of the density for the three cases shown in Table 1. As expected, the adiabatic index is a smooth function of the density which connects the behavior of the adiabatic index of the relativistic degenerate electron gas ($\gamma = 4/3$) to the one of the nonrelativistic degenerate neutron gas ($\gamma = 5/3$).

Table 1

equation of state	$\zeta_d(g/cm^3)$	$\zeta_e(g/cm^3)$	γ_{min}
A	11.0	9.0	1.06
B	11.0	8.5	1.12
C	10.5	9.0	0.92

In order to analyse the dynamical evolution of the stellar core according to our equation of motion, we first prepare the initial condition corresponding to a pre-supernova core configuration. Let M be the total mass of the core. As mentioned before, we divide the core in two parts, with $m_1 = \alpha M$ and $m_2 = (1-\alpha)M$, and obtain the equilibrium configuration of these two shells using the equation of state of pure degenerate electron gas [12] corresponding to the iron core ($Y_0 \cong 0.46$). We assign this configuration to the pre-supernova core. Then, we switch on our equation of state given by Eqs.(14-17), which triggers the collapse since the adiabatic index is lower than 4/3 at the equilibrium density of the pre-supernova core for this equation of state.

In **Fig.3**, we show an example of solution of Eq.(9) for $M = 1.34M_\odot$ and $\alpha = 0.940$. with the equation of state (A). In this example, the outer core was ejected by a strong bounce shock leaving an oscillating high density inner core. At the instant of bounce, a large amount of energy was transferred from the inner core to the outer one, as is shown in **Fig.4**.

The other interesting example is seen in **Fig.5**, where the energy transfer was not enough to eject the outer core, and the system gets into a nonlinearly coupled oscillation. In this example, all parameters are identical to the first one, except for the value of α which was changed to 0.767.

It is interesting to note that, even for such non-explosive configurations, if the two-shell system continues to oscillate, it will fall into an exploding condition after a finite number of oscillations. This situation becomes more clear if we represent the dynamics of the system by a trajectory in the two-dimensional configuration space. **Fig.6-a** represents the potential surface (sum of the internal and gravitational energies) as a function of coordinates R_1 and R_2 (see also **Fig.6-b**). This surface is characterized by the narrow and deep valley which has the minimum at the $(R_{1,n}, R_{2,n})$ corresponding to the equilibrium neutron star configuration of the total system. This valley extends to $R_2 \to \infty$ keeping R_1 almost constant. There are two steep walls, one at $R_1 \sim 0$ and the other at $R_1 \sim R_2$. The former corresponds to the high density core and the latter to the high density outer shell.

In this figure, we also plotted the trajectory of the system corresponding to the solution given in **Fig.3**. The system which started from the pre-supernova configuration enters into the displayed region from the right-hand side (point A). The trajectory is pulled into the valley by the gravitational force and then it hits almost perpendicularly the steep wall at $R_1 \sim 0$ (core bounce, point B), being reflected into the direction of $R_1 \sim R_2$ wall. The second reflection at the point C in the $R_1 \sim R_2$ wall orientates the trajectory just in the direction of the outgoing door.

Fig.7-a,b shows the potential surface corresponding to the configuration of the example shown in **Fig.5**. In this case, it is seen that the potential surface has a narrow and shallow valley with a very deep pocket at the corner of the two walls. The system finds the outgoing way only after a somewhat long forth-and-back motion inside the valley, sometime going around the edge of the pocket. The initial part of the trajectory is shown in **Fig.7-a**. It may be possible that such 'non-first-chance' explosion be also a physical mechanism for supernova phenomena[13, 14]. However, our present model is not adequate for such cases, since no dissipative processes are taken into account here.

Anyway, it is important to clarify under what conditions a pre-supernova implosion causes a bounce shock strong enough to lead to the explosion of the outer core at the first chance.

V. DISCUSSION AND FINAL REMARKS

It is worthwhile to emphasize that the quantitative values of energy transfer calculated in the present model does not necessarily correspond to the final amount of energy or ejected mass of a supernova. Dissipative processes, as well as the the neutrino transport phenomena during the shock propagation may strongly influence the scenario of after-shock dynamics. On the other hand, in our Lagrangian formulation the violent energy transfer from the inner to outer core through a formation of a shock wave is naturally incorporated to the kinematical feature of the two colliding shells. Therefore, we expect that our model may serve to estimate the upper limit for the amount of energy transferred via bounce shock formation.

In **Fig.8** we show the dependence of energy transfer as a function of α for pre-supernova cores of mass $M = 1.18, 1.24$ and $1.34 M_{\odot}$ calculated using the equation of state A. It is interesting to observe that in each case, the strongest shock is formed at $\alpha \cong 0.9$, namely only 10 % of the core mass is exploded. The amount of energy transferred is of the order of $3-4 \times 10^{51}$ ergs which corresponds to the characteristic energy estimated for a type II supernova event [15].

The existence of such a maximum of energy transfer as a function of mass separation α can be understood qualitatively from the form of the potential surface (**Figs. 6,7**). For smaller values of α, the valley of the potential as a doorway route of the supernova explosion becomes higher and furnishes less kinetic energy to the outgoing outer core. On the other hand, if the value of α approaches unity, although the vallley becomes deep and wide, the smallness of the outer shall mass does not permit to carry a large kinetic energy from the inner core at the instant of maximun compression of the outer shell.

Similar results are obtained using other equations of state (see **Figs.9,10**), although the details vary from one equation of state to another. The largest energy transfer is obtained when a bounce shock is generated at the position which divides the pre-supernova core into two parts, the inner part carrying approximately 90 % of the core mass. If the bounce shock takes place on other locations, the energy transfer decreases rapidly and the explosion will be unsuccessful, at least for the first chance. In other words, the success of a supernova explosion at the first chance will depend crucially on the mass configuration at the instant of the generation of the bounce shock.

The result of the present calculation suggests that, in order to estimate the global quantities of a supernova explosion, it suffices to know what is the proportion of the mass which is contained inside the radius where the bounce shock is generated.

In the present calculation with only 2 shells of constant mass, we searched for the mass separation ratio which maximizes the energy transfer due to the bounce shock genaration. Of course, a hydrodynamic calculation naturally leads to a mass separation of the explosion. However, in order to estimate the amout of the ejected mass and energy output we expect that the full hydrodynamic calculation might not be necessary. For a given pre-supernova configuration, a relatively accurate estimate for these quantities may be predicted allowing for the mass transfers between shells and making use of a few more shells. A study on this line is now in progress.

Figure Captions

1. Homogeneus shells for supernova core.
2. Adiabatic index as a function of matter density for different cases shown in Table 1.
3. Shell radii as functions of time for the total core mass equal to $1.34M_\odot$ and $\alpha = 0.940$. The equation of state A is used.
4. Total energy of each shell in units of 10^{51} ergs as functions of time for the case shown in **Fig.3**. Solid curve is for the innner core and the dashed one is for the outer core. Note a sudden change in these energies at the instant of the outer core bounce, showing a large energy transfer from the inner to the outer shell.
5. Same as **Fig.3**, with $\alpha = 0.767$.
6. **a)** Contour map of potential energy in $log_{10}R_1 - log_{10}R_2$ plane for the case of **Fig.3**. Radii are in centimeters. The numbers on the equipotential curves represent the energy in units of 10^{51} ergs. The corresponding trajectory is indicated in this figure (curve A-B-C), with arrows indicating the increasing time direction. **b)** Potential surface view.
7. **a)** Contour map of potential energy in $log_{10}R_1 - log_{10}R_2$ plane for the case of **Fig.5**. Radii are in centimeters. The numbers on the equipotential curves represent the energy in units of 10^{51} ergs. The corresponding trajectory is indicated in this figure. Note the oscilatory behavior of the trajectory (see text). **b)** Potential surface view.
8. Energy transfer in units of 10^{51} ergs from the inner core to the outer core calculated with equation of state A as a function of the mass ratio α for different total core masses. Numbers attached to each curve represents the total mass in units of solar mass.
9. Same as **Fig.8** for equation of state B.
10. Same as **Fig.8** for equation of state C.

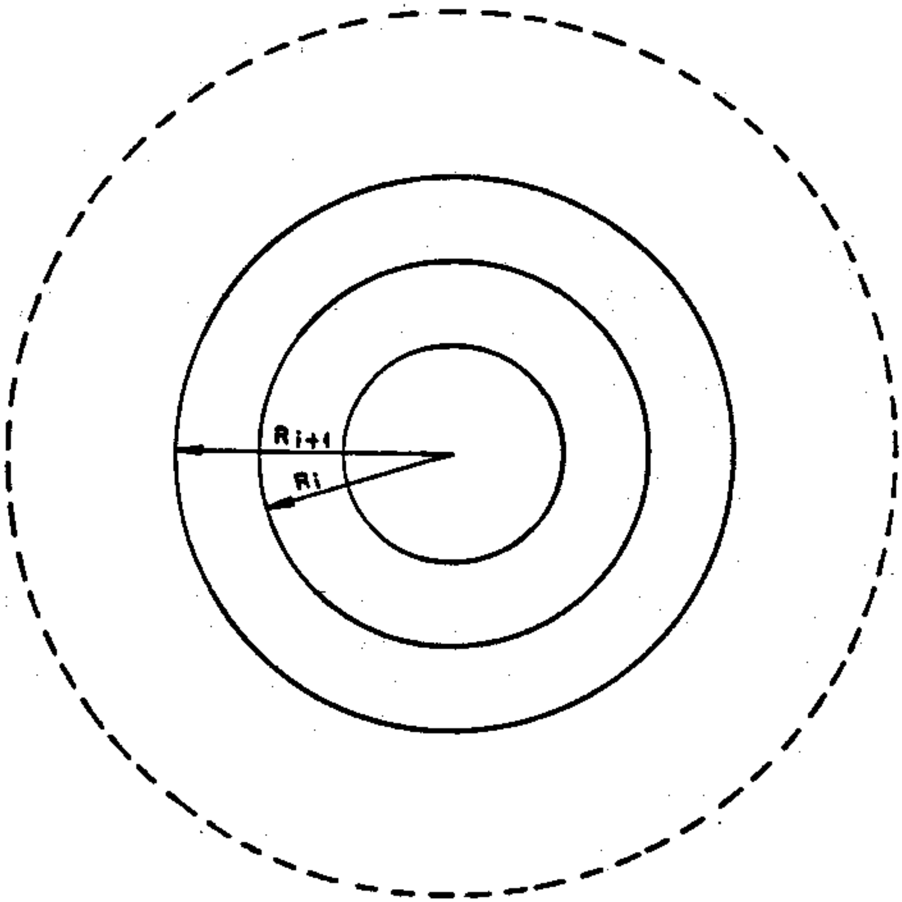

FIG. 1

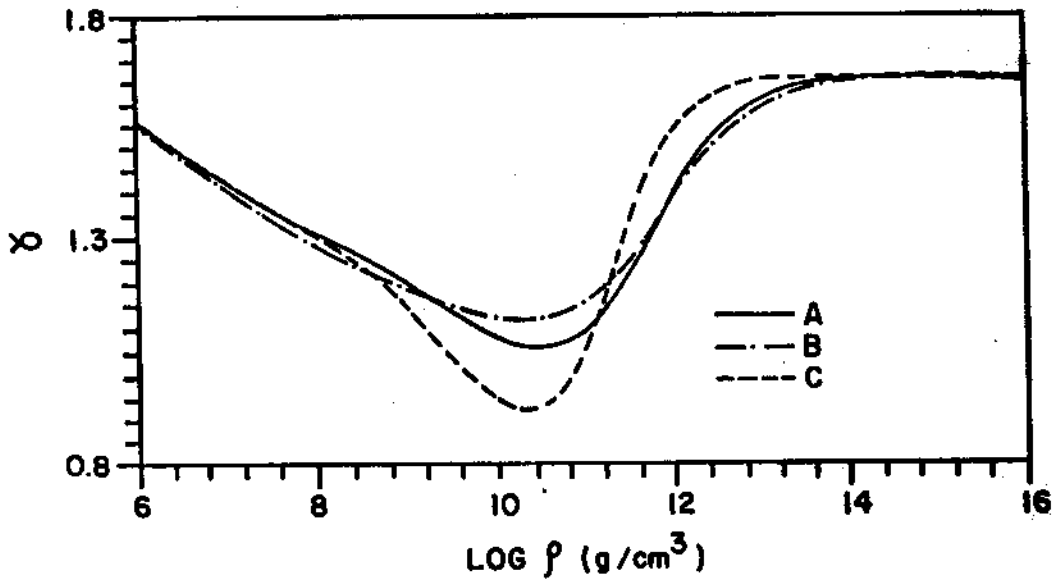

FIG. 2

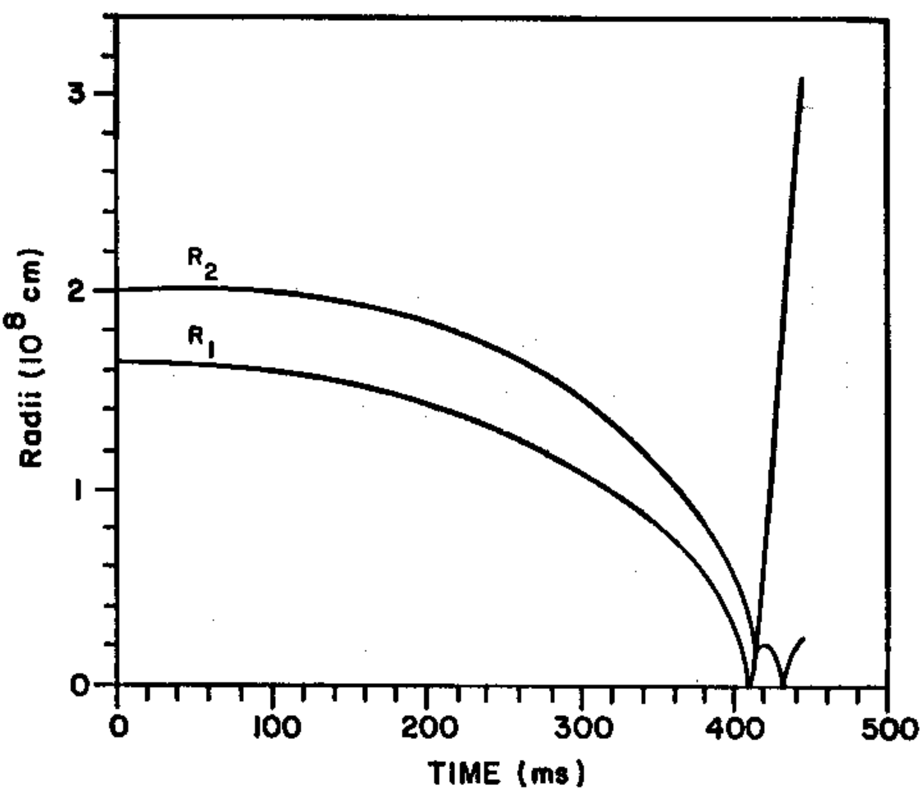

FIG. 3

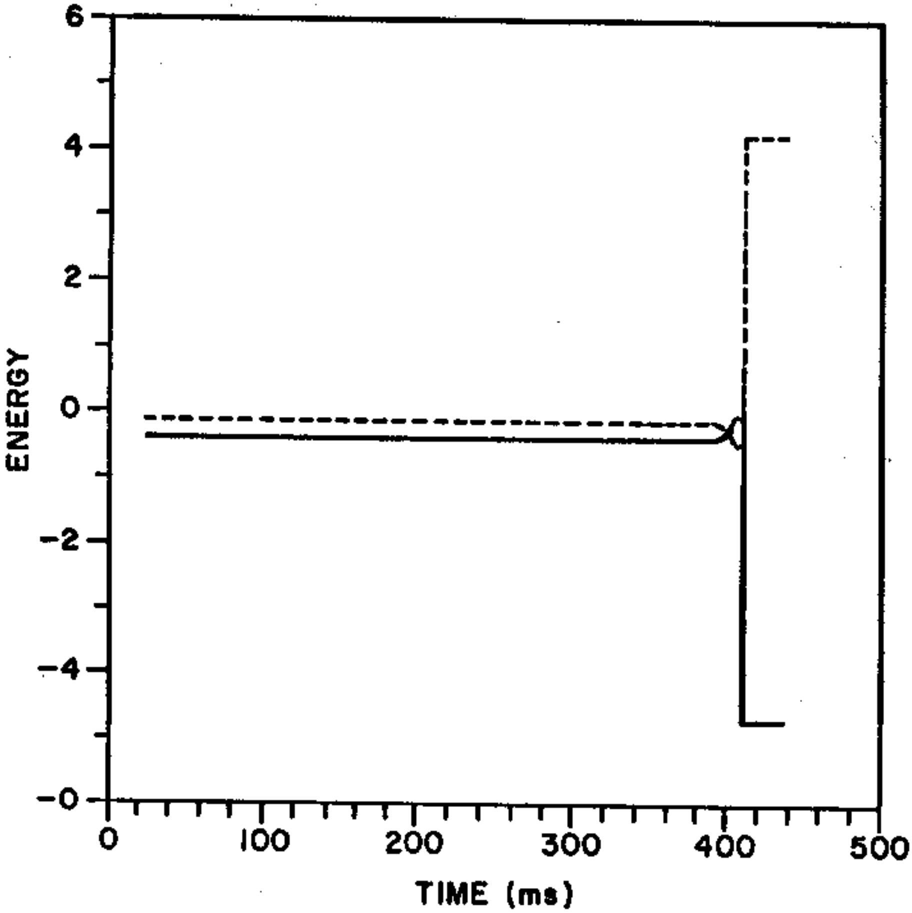

FIG. 4

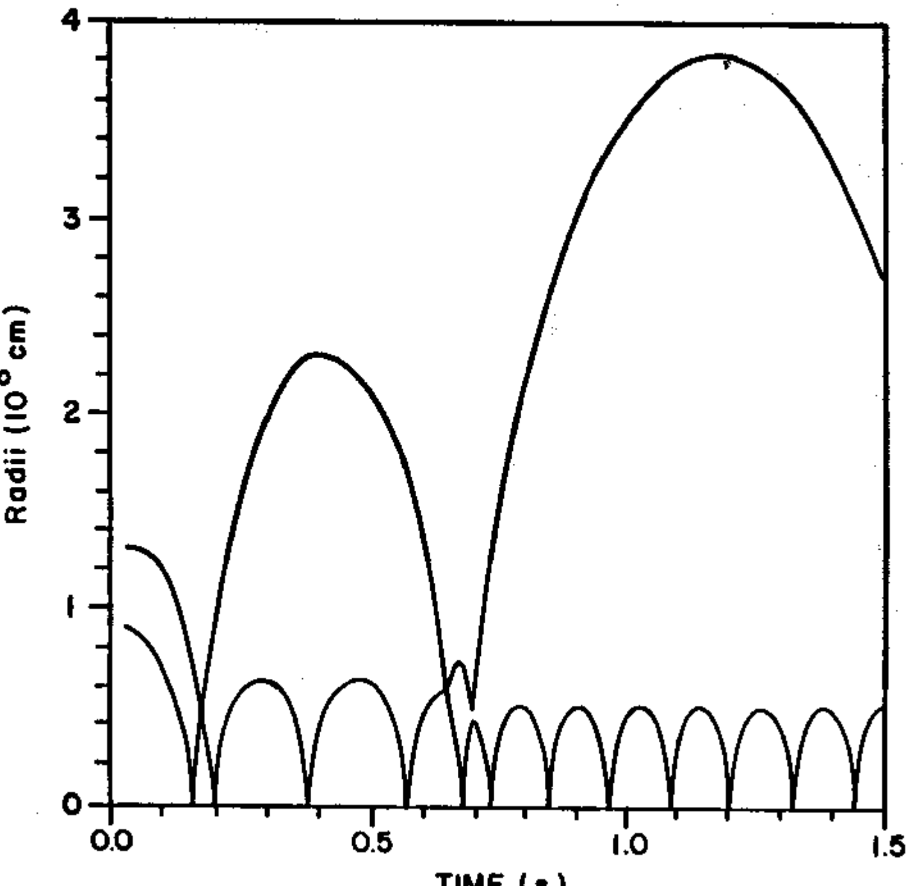

FIG. 5

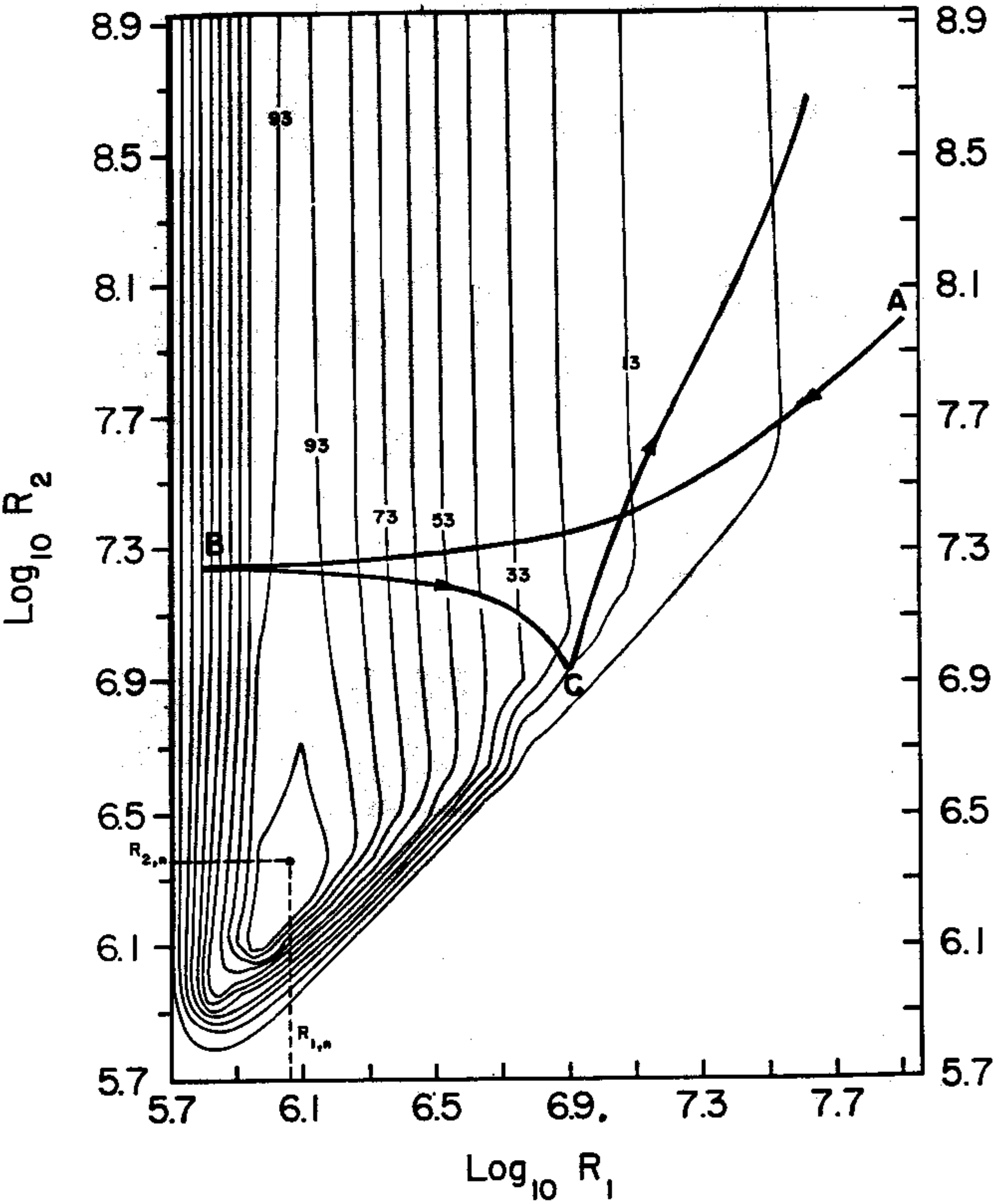
8.9
8.5
8.1
7.7
7.3
6.9
6.5
6.1
5.7
Log10 R2
Log10 R1
5.7
6.1
6.5
6.9
7.3
7.7
93
93
73
53
33
13
A
B
C
R2,a
R1,a

FIG. 6-a

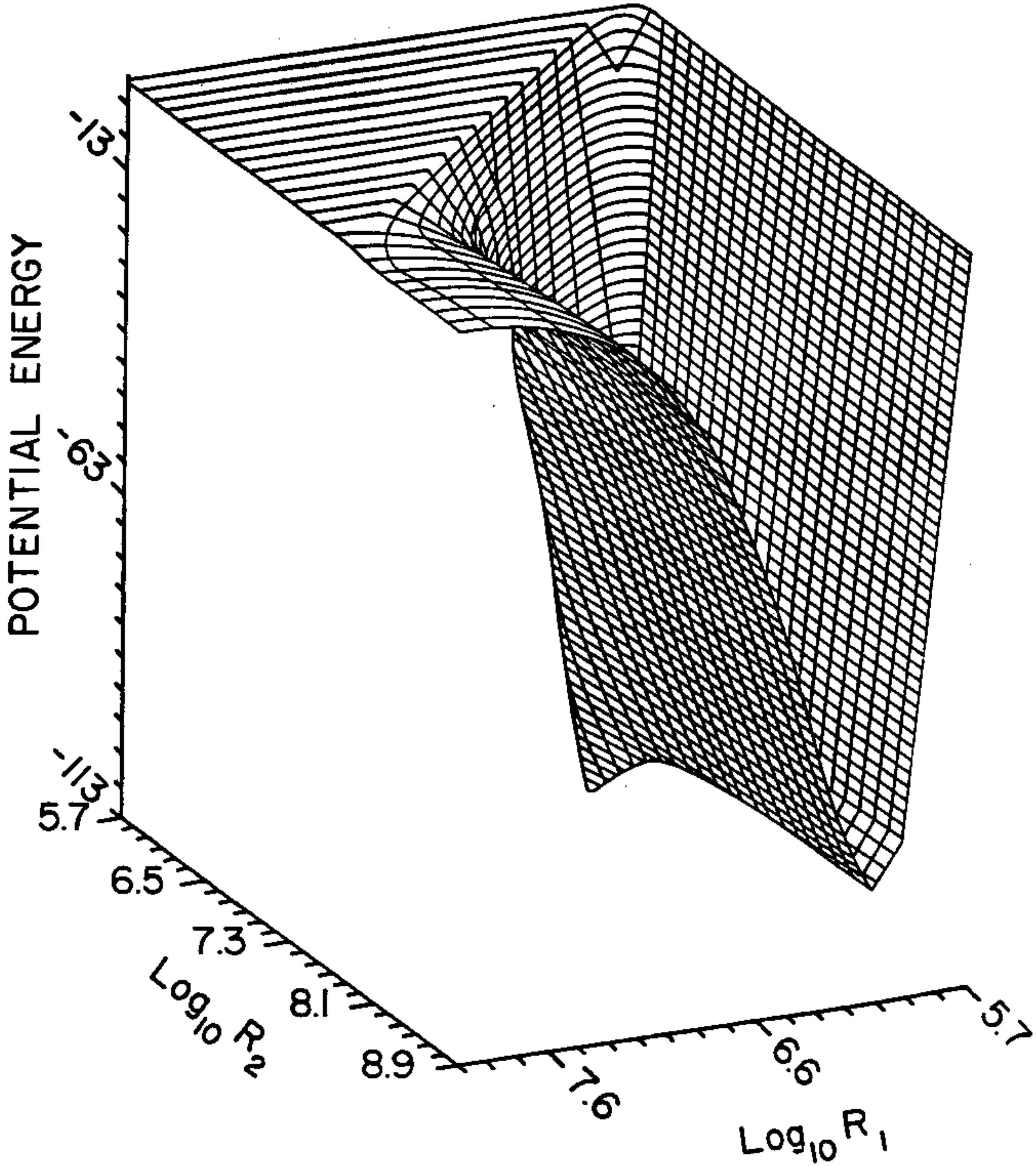
POTENTIAL ENERGY
-13
-63
-113
5.7
6.5
7.3
8.1
8.9
$Log_{10} R_2$
7.6
6.6
5.7
$Log_{10} R_1$

FIG. 6-b

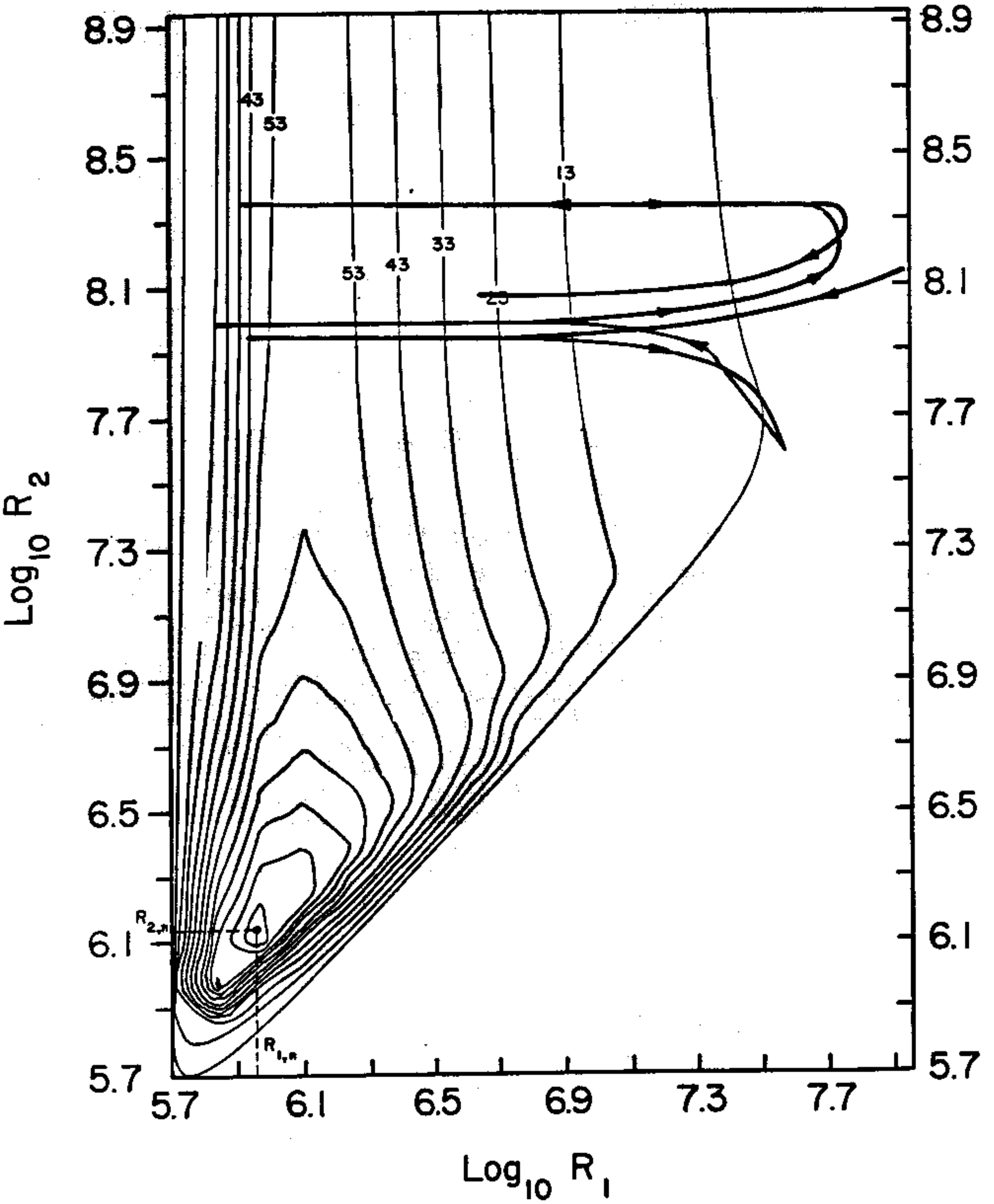
8.9
8.5
8.1
7.7
7.3
6.9
6.5
6.1
5.7
5.7
6.1
6.5
6.9
7.3
7.7
Log10 R2
Log10 R1
43
53
13
33
53
43
23
R2,R
R1,R

FIG. 7-a

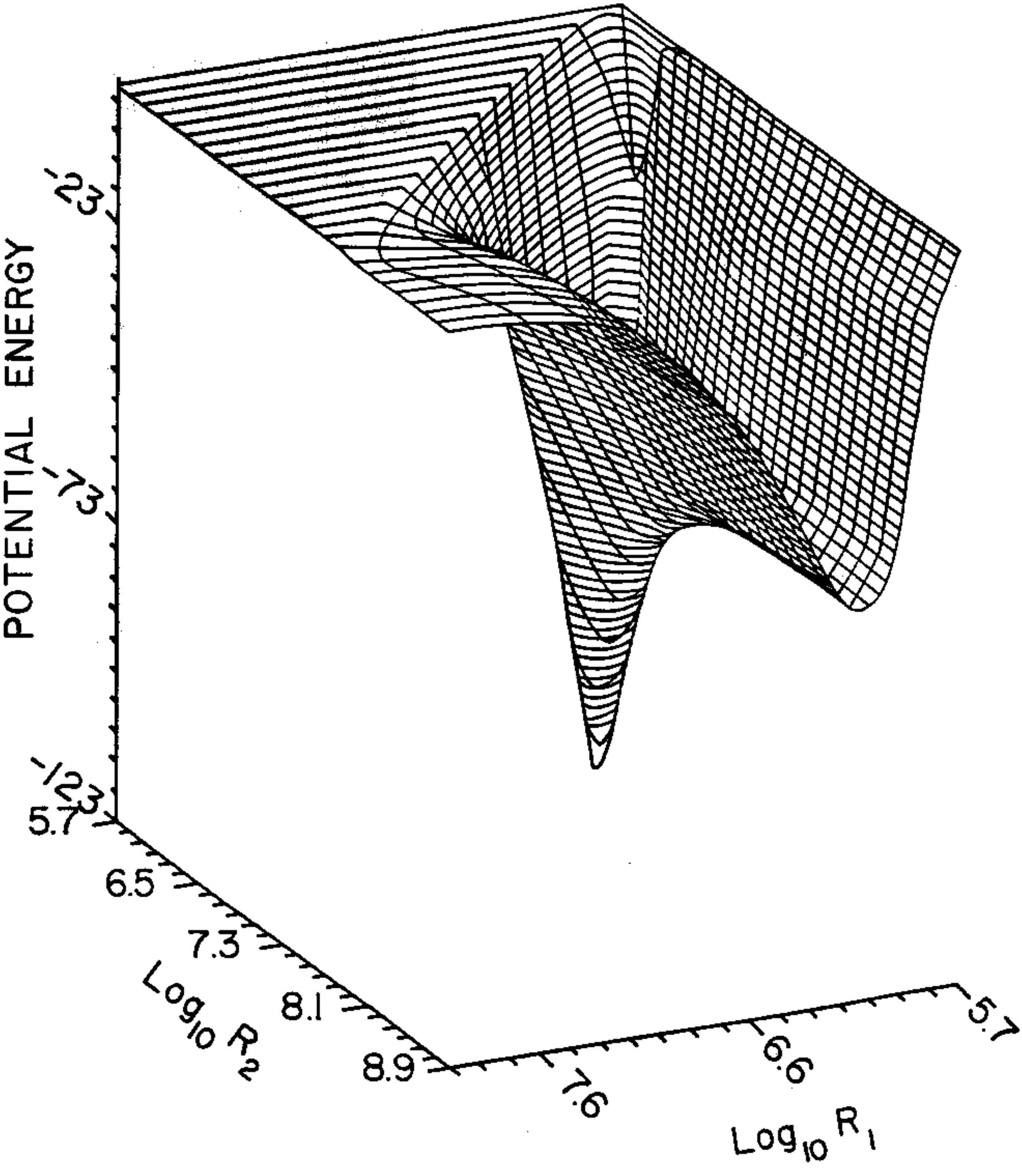

FIG. 7-b

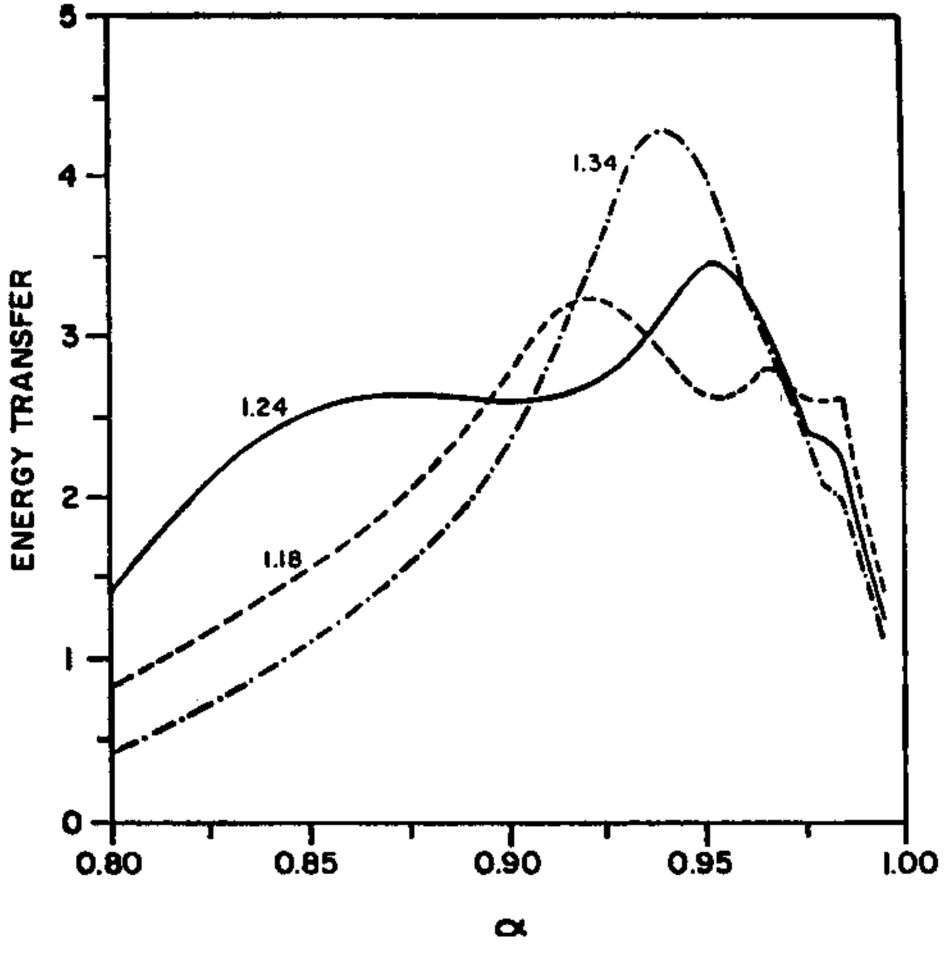

FIG. 8

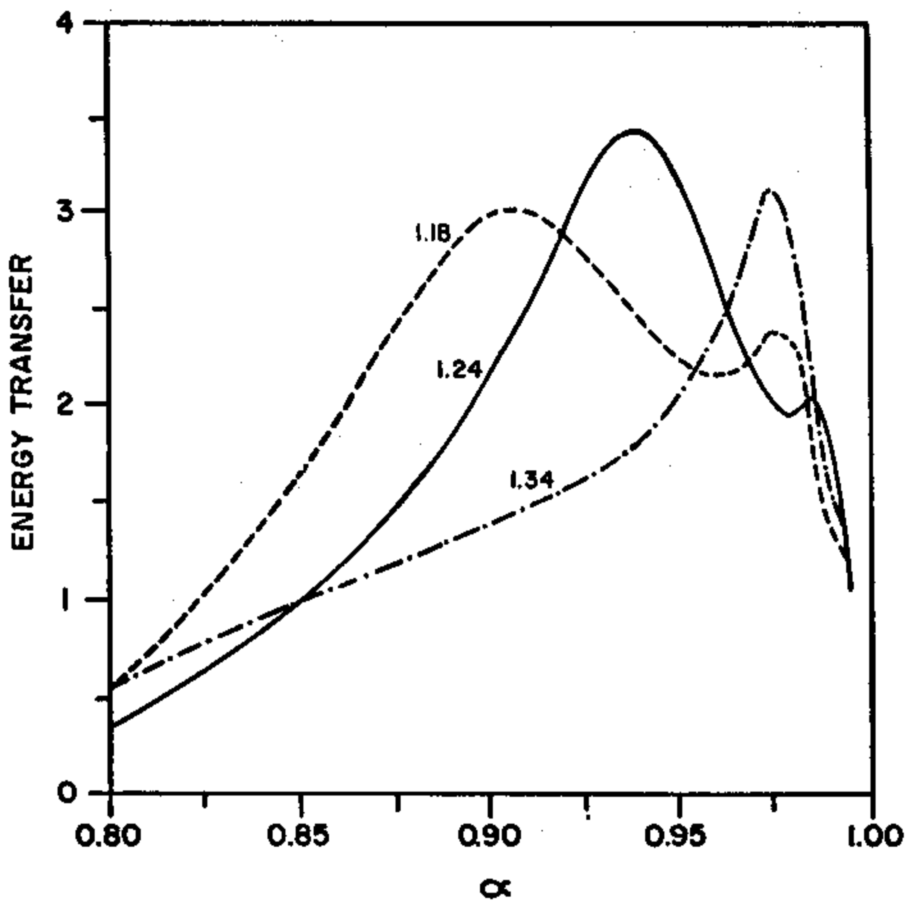

FIG. 9

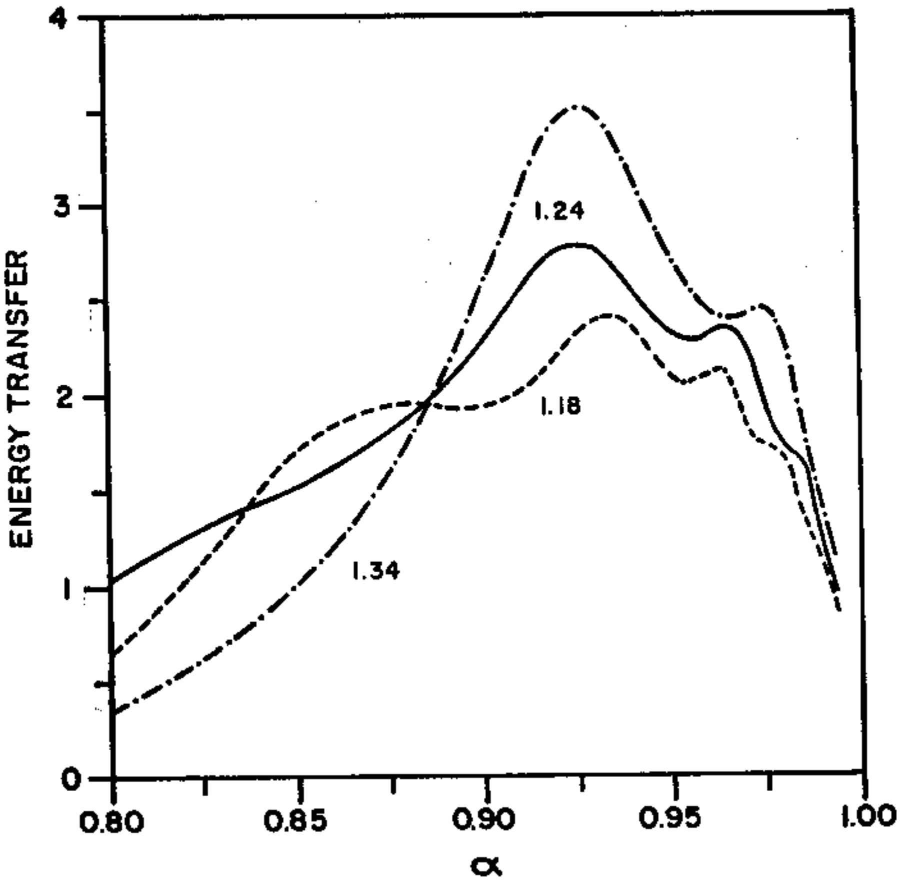

FIG. 10

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