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HADRON PRODUCTION IN A FIREBALL RADIATION MODEL FOR ELECTRON-POSITRON COLLISIONS

by

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Summary

A model of fireballs radiated in e^+e^- collisions and radiating in turn other fireballs (i.e. hadrons) is shown to describe well the large bulk of particle production (multiplicities, inclusive distributions, fractions as functions of momenta). One adjustable parameter is used for each kind of hadron emitted (minus an overall constraint). When considered as a process in the mean, the scheme is shown to be equivalent to a sequential decay where the only assumption made is that at each step a particle is emitted with an energy proportional to that of the decaying parent and the proportionality factor turns out to be, numerically, nothing but the running coupling constant of QCD. The consequences of this scheme are investigated.

Key-words: Hadron production by electron-positron collisions; Fireball radiation model; Total and inclusive cross sections; Multiplicities.

1. Introduction.

The problem of multiple hadronic production has been approached by means of several kinds [1-5] of models since the early days of the discovery of strongly interacting particles. Perhaps the most ambitious program has been the one developed by Hagedorn [6] assuming that in a high energy collision the production of hadrons occurs through the formation of highly excited matter, a fireball that starts emitting fireballs that keep emitting more fireballs and so on until all the energy has been released. It was not immediate to realize that these fireballs (also called clusters[7]) are nothing but the hadrons themselves and, in fact, an even more stringent local hadron-parton duality has been advocated in recent times [8].

While actually closer to Heisenberg's works (our model is not a statistical one as we assume a radiation law), in spirit our approach is very similar to Hagedorn's in that we arrive at a sort of hadron-fireball duality. We begin by developing a model for $e^+e^- to$ hadrons which starts from the moment when a system of a highly excited quark-antiquark pair has been created through a highly virtual photon and the resulting fireball begins its decay process. This is to say that here we will not be concerned with jets and the large p_T physics for which the appropriate language is presumed be that of perturbative QCD. Similarly in this paper we are not going to make any dynamical consideration concerning the production of the $q\bar{q}$ pair, of the resulting angular distribution around the beam axis and of the p_T distribution around the jet axis.

These topics will be the subject of a forthcoming publication[9]. In the present paper we are going to concentrate on the bulk of e^+e^- yields: inclusive distributions, multiplicities, etc. It is quite extraordinary that over 95% of these data turn out to be in excellent agreement with the predictions from the scheme outlined above: a fireball is created that keeps decaying into fireballs which in turn decay into fireballs etc. under the following simplifying assumptions: i) a two step process occurs (as we will see, higher iterations would be needed if we were to account for the finer details) and ii) the energy distribution is assumed to be due to a simple radiation mechanism at each step. A grand total of three adjustable parameters is used: one for each kind of hadrons considered (pions, kaons and nucleons if we ignore smaller contributions) minus an overall constraint due to energy conservation.

It is quite amusing that if one assumes the iteration procedure to continue indefinitely (fireballs creating fireballs ...) and if one takes a slightly modified radiation energy distribution ($\propto [E\ell n^{1/2}E]^{-1}$ instead of E^{-1}) one arrives for the multiplicity of particles produced to the same functional energy dependence that one encounters in QCD, namely $\exp[c\ell n\sqrt{s}/\Lambda]^{1/2}$ where c is a constant. We do not believe that any special significance should be attached to this particular point but it certainly opens remarkable possibilities.

The model is discussed in detail in Sec. 2 whereas the comparison with the data for the multiplicities of the various kinds of hadrons is given in Sec. 3. Here the three adjustable parameters are fixed and all that follows is parameter-free.

Sec. 4 is devoted to compare the model with the inclusive yields for producing the various hadrons. It should be stressed that one of the reasons for the agreement we find with the data lies also in our careful taking into account threshold effects. The problem of the fraction of particles of various momenta and energies is considered in Sec. 5. Up to energy thresholds,our model predicts a constant energy fraction for the various kinds of hadrons and the momenta fractions are in reasonably good agreement with the data.

In the second part of the paper we explore the consequances of taking a different viewpoint whereby the production occurs in a statistical way:a mean number of fireballs is produced and so on. First, we show that nothing much changes as far as averaged multiplicities are concerned. Next, we show the effective equivalence of this scheme in the mean to that of adopting an apparently completely different approach, namely that particle production occurs through sequential emissions. At each step of this sequential decay the only assumption we make is the most economical one: the energy of the particle (fireball) emitted is simply proportional to that of the parent fireball. No energy distribution law is postulated here. In Sec. 6 the equivalence of these schemes is discussed and it is seen to imply as a numerical consequence that the proportionality factor of the sequential decay scheme is,perhaps not unexpectedly, nothing but the running coupling constant of strong interactions. Some conclusions are given in Sec. 7.

It is quite obvious that the present attempt lacks the sophistication of all the QCD armory for which we refer the interested reader to the specialized literature [10][11]. Also, several distributions [12][13] are known that fit multiplicities very well. It is, however, to be said that our approach provides a quite satisfactory account of the large bulk of e^+e^- yields with oversimplified means and, most of all, that it allows one to understand these yields in a simple and physically intuitive language while showing appealing connections among different production mechanisms.

A final comment is in order: for the model we are using here, the case of $e^+e^- \to$ hadrons is the ideal reaction to look at. An extension to the case of hadronic initiated multiple production will be the subject of future work but this is a much more complex problem. While, in fact, it is most natural to assume that elementary particles (such as e^+e^-) when colliding in their c.m., coalesce into a unique fireball that then decays into two back to back fireballs which keep decaying, the same kind of argument does not apply so directly to the collisions of hadrons. The latter, in fact, being composite systems, one must first disentangle the formation and decay of fireballs resulting from the collision of two elementary constituents (quarks) from the big mess of what all the other constituents do. As nearly fifty years of models of strong interactions have taught us, this is not a simple task and this is also the primary reason why non-perturbative QCD has been so far so untractable.

2. Description of the model

Our starting assumption is that a collision e^+e^- occurs via the exchange of a virtual photon of mass $\sqrt{s}$ which manifests itself as a fireball of hot hadronic matter which decays radiating fireballs of lower energy. The energy distribution of these fireballs is simply assumed to be the usual radiation law $1/E$ so that the number of fireballs of energies between E and $E + dE$ is (the suffix "c" being for "cluster")

$$dn_c = g_c \frac{dE}{E} \tag{2.1}$$

where we assume g_c to be a constant (to be determined). This is exactly Heisenberg's starting point [2].

The total number of clusters that can be emitted is then

$$L(s) \equiv n_c(\sqrt{s}) = g_c \int_{m_\pi}^{\sqrt{s}} \frac{dE}{E} = g_c ln\left(\frac{\sqrt{s}}{m_\pi}\right) \tag{2.2}$$

where we are implicitly assuming already (through the lower limit of integration) that fireballs are nothing but hadrons.

For the time being, we will assume $L(s)$ to be an integer but this will soon become irrelevant.

The number of fireballs with energy between E and $\sqrt{s}$ will be

$$n_c(E,\sqrt{s}) = g_c \int_E^{\sqrt{s}} \frac{dE}{E} = g_c \ell n \left(\frac{\sqrt{s}}{E}\right) \tag{2.3}$$

Thus, the energy of the $\ell - th$ cluster is, from the preceeding equation

$$E_\ell = \sqrt{s} e^{-n_c(E_\ell)/g_c} = \sqrt{s} e^{-\ell/g_c} \tag{2.4}$$

Let us now demand that the total energy carried away by the $n_c(\sqrt{s})$ clusters be exactly $\sqrt{s}$ i.e. that energy is conserved throughout the first chain of decays. From (2.4) we must have

$$\sqrt{s} = \sum_{\ell=1}^{L} E_\ell = \sqrt{s}\, \frac{1 - e^{-L/g_c}}{1 - e^{-1/g_c}} e^{-1/g_c} \tag{2.5}$$

which, assuming L to be large enough, implies

$$e^{1/g_c} = 2 \;\Rightarrow\; g_c = 1.44 \tag{2.6}$$

Notice that, not surprisingly, from (2.4,2.6) the energy of the first cluster is nothing but $\sqrt{s}/2$. This is in keeping with the physical picture that the first step of the process is represented by two back to back partons each of which carries an energy $\sqrt{s}/2$. In this sense, the fireball-hadron duality is nothing but the hadron-parton duality already mentioned[8].

If the above process is repeated, we will have a second generation of cluster formation and then possibly a third one and so on.

In general, after $k-$steps, the total amount of clusters which will have been formed by an initial energy E will be given by

$$n_c^{(\kappa)}(E) = \int_{m_\pi}^{E} f_c(E_1)dE_1 \int_{m_\pi}^{E_1} f_c(E_2)dE_2 \cdots \int_{m_\pi}^{E_{\kappa-1}} f_c(E_\kappa)dE_\kappa \tag{2.7}$$

where we have written $f_c(E)$ for our basic radiation distribution

$$f_c(E) = g_c\, \frac{1}{E} \tag{2.8}$$

Let us defer for a moment the consideration of what might happen if the above process is repeated indefinitely and let us now suppose that the second generation of fireballs are already the final hadrons. Assuming the same energy distribution law, the number of hadrons of type h whose energy is between E and $E + dE$ emitted by the $\ell - th$ cluster will be

$$dn_h^{(\ell)}(E) = g_h \frac{dE}{E} \theta(E_\ell - E) \tag{2.9}$$

where g_h is a constant (to be adjusted) that discriminates among the various kinds of hadrons $(\pi, K, N, \ldots)$.

In the following we will consider π, K and N for a total of 3 adjustable parameters.

As we will see in what follows, the data will be perfectly reproduced by stopping at just this point of the decay chain of fireballs and we can therefore consider at this point what gives us the constraint of energy conservation.

For this, we write the total mean number of all kinds of hadrons emitted by the $\ell-$cluster with energy E_i. From (2.9) we find

$$N^{(\ell)}(E_i) = G\ \ell n \left(\frac{E_\ell}{E_i}\right) \theta(E_\ell - E_i) \tag{2.10}$$

where we have defined

$$G = \sum_{h=1}^{H} g_h \tag{2.11}$$

if H is the total number of kinds of hadrons we are going to consider.

Summing over the energies of all hadrons produced, assuming the total mean number of the latter to be $>> 1$ and demanding that the result coincides with the energy E_ℓ of the parent fireball, we get, in analogy with (2.6)

$$\sum_{h=1}^{H} g_h = g_c = 1.44. \tag{2.12}$$

Remark that in the sum (2.12) both charged and neutral hadrons are to be taken into account.

We end this section by returning to the general formula (2.7). First of all, it is quite obvious (and will be discussed in Sec. 3) that at the second iteration the total multiplicity will be of a ℓn^2 type if the form (2.8) is used; this, as it is well known, is a classical form assumed for hadron multiplicities.

One may, however, ask oneself what other more general forms would be allowed should one iterate an indefinite number of times the scheme of fireball decay and, also, should one allow for slight modifications of the radiation energy distribution law (2.8).

To look into this, remember first that eq. (2.7) can be recast in the form

$$n_c^{(\kappa)}(E) = \frac{1}{\kappa!}\left[\int_{m_\pi}^{E} f_c(E_1)\, dE_1\right]^{\kappa} \tag{2.13}$$

Next, let us assume that we modify as little as possible the radiation law (2.8), i.e. we take

$$f_c(E) = \frac{g_c\,(\ell n(E/m_\pi))^{\beta}}{E} \tag{2.14}$$

so that (for $\beta > -1$)

$$n_c^{(\kappa)}(E) = \frac{1}{k!} g_c^{\kappa}\left[\frac{(\ell n\frac{E}{m_\pi})^{\beta+1}}{\beta+1}\right]^{\kappa}. \tag{2.15}$$

The decay constant g_c will, of course, depend on the specific choice of β.

If we now allow for the iteration procedure to go to infinity, the sum of all particles produced will give a multiplicity

$$N \underset{E\to\infty}{\simeq} \exp\left[g_c \frac{(\ell n\frac{E}{m_\pi})^{\beta+1}}{\beta+1}\right] \tag{2.16}$$

where we have ignored all thresholds since we have taken $E \to \infty$.

Eq. (2.16) is most interesting indeed. If $\beta = 0$ (pure radiation law) one finds that the infinitely recursive fireball scheme gives a power growth for the multiplicity. If, on the other hand we take $\beta = -\frac{1}{2}$ (implying a somewhat harder energy distribution) we recover the growth suggested by QCD[8] i.e.

$$N \simeq \exp\left(C\ell n^{\frac{1}{2}}\left(\frac{E}{\Lambda}\right)\right). \tag{2.17}$$

where we have assumed $\Lambda_{QCD} \simeq m_\pi$ as suggested by deep inelastic data.

It is, of course, quite possible that this result is totally accidental and we are not going to attach too much significance to it; nevertheless it is quite intriguing that the QCD energy dependence for the mean multiplicities should be obtained in the infinitely recursive scheme of fireball decay with the pure radiative decay law hardened by a $(\ell n E)^{-\frac{1}{2}}$ factor.

3. Fit to the multiplicities.

If we go back to eq. (2.9), the total number of hadrons of type h emitted by the $\ell - th$ cluster will be

$$\begin{aligned} n_h^{(\ell)}(E_\ell) &= g_h \int_{m_h}^{E_\ell} \frac{dE}{E} = g_h \ \ell n \left(\frac{E_\ell}{m_h}\right) \theta(E_\ell - m_h) = \\ &= g_h \ \ell n \left(\frac{\sqrt{s} e^{-\ell/g_c}}{m_h}\right) \theta(\sqrt{s} e^{-\ell/g_c} - m_h) = \\ &= g_h \ \ell n \left(\frac{\sqrt{s} 2^{-n_c}}{m_h}\right) \theta(\sqrt{s} 2^{-n_c} - m_h) \end{aligned} \tag{3.1}$$

where the last line follows from eq. (2.6).

The total number of particles of type h emitted by all the fireballs obtains from (3.1) by summing it over ℓ from 1 to $L(s)(\equiv n_c(s))$.
We find

$$\begin{aligned} N_h(s) &= \sum_{\ell=1}^{L(s)} n_h^{(\ell)}(E_\ell) = g_h \sum_{\ell=1}^{L(s)} \left(\ell n \frac{\sqrt{s}}{m_h} - \frac{\ell}{g_c}\right) \theta(E_\ell - m_h) \\ &= \frac{1}{2} g_h g_c \ \ell n \left(\frac{\sqrt{s}}{m_h}\right) \left[\ell n \left(\frac{\sqrt{s}}{m_h}\right) - \frac{1}{g_c}\right] \theta(\sqrt{s} e^{-\ell_<} - m_h) \end{aligned} \tag{3.2}$$

where use has been made of eq. (2.3) and where $\ell_<$ is the smallest cluster whose mass is larger than that of the hadron under consideration.

Notice that from this point on we can completely forget the initial requirement that $L(s)$ should be an integer. All we have to do is use eq. (3.2) in its last form.

Notice also that eq. (3.2) is exactly in the form of the $0(\ell n^2 \sqrt{s})$ dependence which is usually taken to fit the data. Eq. (3.2) represents only the multiplicity of hadrons of type h. The total multiplicity will be given by

$$N(s) = \sum_{h=1}^{H} N_h(s) \tag{3.3}$$

if we allow for a total of H types of hadrons of all possible kinds (charged and neutral).

The comparison with the charged multiplicity data[14] for $\pi^\pm$, $K^\pm$, p extrapolated up to $\sqrt{s} = 100$ GeV is shown in Fig. 1 where we have taken

$$\begin{cases} g_{\pi^\pm} \simeq 0.6 \\ g_{K^\pm} \simeq 0.18 \\ g_p \simeq 0.11 \end{cases} \tag{3.4}$$

From now on there are no more adjustable parameters.

Notice that a blind counting of the contribution of neutral particles (1/3 of all pions are π^0's, i.e. $g_{\pi^0} \simeq 0.3$, 1/2 of kaons are K^0, $\bar{K}^0$, i.e. $g_{K^0} = 0.18$ and 1/2 of the nucleons are neutrons, i.e. $g_n = 0.11$) would lead to

$$\sum_{h=1}^{H} g_h \simeq 1.48 \tag{3.5}$$

i.e. pions, kaons, and nucleons alone seem to saturate the energy conservation constrain (2.12). Some care has, however, to be paid to the fact that a fraction of about 10% of the charged pions comes from K decay (mostly)[15], and a fraction of $\sim 25\%$ of the protons comes from Λ decay[14]; one then arrives at an estimate of $\sum_h g_h = 1.33$ counting pions, kaons and nucleons so that some room is left to produce all the remaining particles.

The total charged multiplicity, obtained summing the pionic, kaonic and nucleonic contributions i.e. using eqs. (3.2, 3.4) is compared with the data in Fig. 2 and extrapolated to the TeV region. In this figure, the SPS Collider data[16] have been added with the simple minded recipe[17] of halving the $p\bar{p}$ energy in order to compare with e^+e^-.

Fig. 3 shows how also the few data on Λ production [18] are well reproduced by the model. A value $g_\Lambda \simeq 0.055$ is used in Fig. 3.

Notice, once again, how important thresholds effects are to get good agreement with the data at not too high energies.

4. Inclusive distributions and scale breaking

The well known energy momentum conservation of inclusive reaction[19] tells us that the differential number of particles produced with energy between E and $E + dE$ is proportional to the inclusive yield $\frac{1}{\sigma}\, d\sigma/dx$ where $x = 2E/\sqrt{s}$.

From eq. (2.9) summing over the contribution of all clusters whose energy exceeds E (eq.(2.3)), we find

$$dN_h(E) = g_c g_h \, \ell n \left(\frac{\sqrt{s}}{E} \right) \, \frac{dE}{E} \tag{4.1}$$

Notice that, according to eq. (4.1), the inclusive distributions $\frac{1}{\sigma}\, d\sigma/dE$ to produce the various hadrons would be all proportional. Some mass effects are introduced in the quantity for which the data are given[20]

$$\frac{1}{\beta\sigma}\frac{d\sigma}{dx}.$$

The comparison for the various hadrons is shown in Fig. 4-6. The agreement, as one can see is excellent for $K^{\pm}$ and p yields and is still very good for $\pi^{\pm}$ for which the tail of the distribution only (few percent of the yields) is not well reproduced.

Several comments can be made. First of all, no fragmentation function has been used here. Second, the deviation of our curves from the data can simply be understood as an energy scale breaking effect. As it can be seen, in fact, the data show an excellent scaling behaviour in the beam energy $\sqrt{s}$ up to values of x, typically, around ~ 0.3. After this value, this kind of scaling appears to break down and the lowest energy data are seen to accompany better our curve.

The reason for this can easily be understood as a consequence of scale breaking. To see this, we begin by evaluating up to what x^h_{max} our curve for the inclusive cross section can be trusted for each hadron. For this, we write the mean energy of the hadron of type h emitted by the $\ell - th$ cluster as

$$\langle E_h^{(\ell)} \rangle = \frac{\int \frac{E}{\sigma} \frac{d\sigma}{dx}\Big|_h dx}{\int \frac{1}{\sigma} \frac{d\sigma}{dx}\Big|_h dx} = \frac{g_h \int_{m_h}^{E_\ell} dE \theta(E_\ell - E)}{g_h \int_{m_h}^{E_\ell} \frac{dE}{E} \theta(E_\ell - E)} \tag{4.2}$$

where E_ℓ is the energy of the $\ell - th$ cluster (2.4). Performing the integrations we get

$$\langle E_h^{(\ell)} \rangle = \frac{E_\ell - m_h}{\ell n \left(\frac{E_\ell}{m_h} \right)} \tag{4.3}$$

Therefore, given that the most energetic hadron (in the mean) must come from the most energetic cluster, using eq. (2.4) with $\ell = 1$, we get

$$\langle E_h^{(\ell)} \rangle = \frac{\frac{\sqrt{s}}{2} - m_h}{\ell n(\sqrt{s}/(2m_h))} \tag{4.4}$$

i.e., from the definition $x = 2E/\sqrt{s}$, we find

$$\langle x^h_{\rm max}\rangle = \frac{1 - x^h_{\rm min}}{\ell n\left(\frac{1}{x_{\rm min}}\right)} \quad ; \quad \left(x^h_{\rm min} = \frac{2m_h}{\sqrt{s}}\right). \tag{4.5}$$

Eq. (4.5) gives, for each kind of hadron h, the maximum mean value of x^h in our model i.e. the maximum value for which we can trust our results. For larger x-values we would need to take fluctuations into account. It is quite recomforting that these fluctuations affect our conclusions only to a few percent.

Notice that the value of $x^h_{\rm max}$ recedes to lower and lower values (logarithmically) as the energy increases and this effect is less and less marked the heavier the particle. Thus, the deviations of our curves from the data occur around $x_{\rm max} \simeq 0.33$ for $\sqrt{s} = 34$GeV and $x_{\rm max} \simeq 0.43$ for $\sqrt{s} = 14$GeV for the proton but already around $x_{\rm max} \simeq 0.21$ for $\sqrt{s} = 34$GeV and $x_{\rm max} = 0.25$ for $\sqrt{s} = 14$GeV for the pion.

The situation is summarized in Table I whereas in Figures 4-6 we have shown with arrows the various values of $x^h_{\rm max}$ for the different kinds of hadron.

TABLE 1

$\sqrt{s}$(GeV)	5.2	14	22	34	55
$x^\pi_{\rm max}$	0.32	0.25	0.23	0.21	0.19
$x^k_{\rm max}$	0.49	0.35	0.31	0.28	0.25
$x^p_{\rm max}$	0.63	0.43	0.37	0.33	0.29

The last comment one can make is that any hardening of the simple minded $1/E$ distribution energy would improve the agreement. This would be the case should one use the distribution $(E\ell n^{\frac{1}{2}}E)^{-1}$ that in an infinitely recursive scheme has been shown to give us the QCD multiplicity law (2.17).

As already stressed, however, the point is that only few percent of the data are not well reproduced in Fig.4 and this is entirely acceptable given the simplicity of our approach.

5. Fraction of particles

The fraction of the various hadrons with energies between E and $E + dE$ i.e.

$$f_h(E) = \frac{dN_h(E)}{\sum_h dN_h(E)} \tag{5.1}$$

is obtained directly by the use of eq.(4.1) i.e. it is a pure constant (up to threshold effects)

$$f_h(E) = \frac{g_h}{\sum g_h}. \tag{5.2}$$

The data, however, are usully given as a function of the momentum rather than of the energy. Performing the appropriate transformation we find

$$f_h(p) = \frac{g_h \frac{p}{p^2+m_h^2}\left[\ell n\left(\sqrt{s}/(p^2+m_h^2)^{\frac{1}{2}}\right)\right]}{\sum_h g_h \frac{p}{p^2+m_h^2}\left[\ell n\left(\sqrt{s}/(p^2+m_h^2)^{\frac{1}{2}}\right)\right]} \tag{5.3}$$

The agreement of (5.3) with the data[18] is good for pions,kaons and protons (Fig.8) up to the various values of p_{max}.We note that our p_{max} follow the variation of the point at which,for the various hadrons,our model deviates from the data at each c.m. energy.Of course,to the extent that we are discussing ratios,the deviations from the data occur at the earliest point i.e. for the pion.

It should anyway be recalled that our model is not expected to hold at large x values. Furthermore,we have not introduced any fragmentation function which could have been chosen in such a way as to improve the agreement with the large x data.

6. Properties in the mean and sequential decay.

6.1 Independent cluster radiation.

In this second part of the paper, we begin, first of all, by considering what the previous model might imply if taken in the mean. More specifically, suppose that the decay process is seen as the emission of a mean number of clusters $n_c(\sqrt{s})$. Under the same previous radiations law (2.1) for the energy distribution, we find

$$n_c(\sqrt{s}) = k_c \ell n \left(\frac{\sqrt{s}}{m}\right) \tag{6.1}$$

where k_c is a constant (which may or may not coincide with the one, g_c, we introduced in Sec.2)and m is the mass of the lightest hadron,i.e. the pion.

The mean energy cluster will be

$$\langle E_c \rangle = \sqrt{s}/n_c(\sqrt{s}). \tag{6.2}$$

Each cluster will emit a mean number of particles (again we take the radiation law) given by

$$n_p(E_c) = k_p \ell n \left(\frac{\sqrt{s}}{m\, n_c(\sqrt{s})}\right) \tag{6.3}$$

with k_p some proportionality factor (possibly not the same as g_h introduced earlier).

The main assumption is now that the production and the decay of fireballs are two totally independent phenomena. In this case, the total multiplicity will be

$$n(\sqrt{s}) = k\ \ell n \left(\frac{\sqrt{s}}{m}\right) \left[\ell n \left(\frac{\sqrt{s}}{m}\right) - \ell n(k_c \ell n(\sqrt{s}/m))\right] \tag{6.4}$$

where

$$k = k_c k_p \tag{6.5}$$

Notice how extremely close is the law one arrives at in this way of reasoning with our previous law for the total multiplicity (3.2) which we obtained by keeping the energy correlation constraint i.e. demanding that the total energy carried by all the clusters be just $\sqrt{s}$ (in the present case this is automatically guaranteed by eq.(2.6) which is why k_c may or may not coincide with g_c). The closeness of (6.4) to (3.2) tells us that the total charged multiplicity must be well reproduced by (6.4). This is indeed the case and the result is undistinguishable from Fig.2 if we take

$$k \simeq k_c \simeq 1 \tag{6.6}$$

6.2 Sequential decay.

Next, we consider the following simple picture: the decay occurs by emitting a hadron at the time under the assumption that the hadron's energy is just proportional to that of the parent fireball, i.e.

$$\begin{cases} E_1 = \alpha\sqrt{s} \\ E_2 = \alpha(\sqrt{s} - E_1) = \alpha\sqrt{s}(1-\alpha) \\ \dots \\ E_n = \alpha\sqrt{s}(1-\alpha)^{n-1} \\ \dots \end{cases} \tag{6.7}$$

The chain goes on until the last fireball has not enough energy to still emit another hadron, i.e. $E_n \simeq m$.

One could discuss this decay chain at the light of the QCD angle ordering[8] but this is quite irrelevant for our purposes.

Notice that, contrary to what we have done so far, no specific energy distribution such as energy radiation is taken here.

If we iterate eq.(6.7) up to exhausting all available energy, we find for the total number of particles

$$N = \frac{\ell n(\alpha\sqrt{s}/m)}{\ell n\,(1/(1-\alpha))} + 1. \tag{6.8}$$

That the average charged particle multiplicity as a function of the c.m. energy is well reproduced by (6.8) needs not being further considered after we have already said that this is the case for (6.4) given the equivalence of these two laws.

So far we have said nothing about α. Certainly it must be $\alpha < 1$ not to violate energy conservation and the most natural thing is that α should be just the running coupling constant of QCD. As we will see, this conjecture follows under the requirement that the present scheme gives the same multiplicity law (6.4) that we have derived previously.

If $\alpha \ll 1$, eq.(6.8) gives

$$N = \alpha\ \ell n(\alpha\sqrt{s}/m) + 1 \tag{6.9}$$

What is most remarkable about (6.9) is that it follows without making any special requirement about the energy distribution.

Assuming now eqs.(6.9) and (6.4) to match, we find (up to an extra term 1 in (6.9))

$$\alpha = \frac{1}{n_c(s)} \simeq \frac{1}{k_c} \frac{1}{\ell n(\sqrt{s}/m)} \tag{6.10}$$

where we have taken $k_c \simeq 1$ to fit the data.

Recalling the form of the running coupling constant α_s

$$\alpha_s = \frac{2\pi}{11 - \frac{2}{3}n_f} \frac{1}{\ell n(\sqrt{s}/\Lambda)} \tag{6.11}$$

we see that the various parameters match if we take the number of flavors $n_f = 6$ and $\Lambda \simeq m_\pi \simeq 0.14$GeV (as suggested by deep inelastic data). Notice, in passing, that had we insisted in keeping $k_c \simeq g_c \simeq 1.44$, $n_f = 3$ would have given a closer numerical agreement.

We are thus led to the following chain of rather intriguing considerations:

i) the sequential decay chain fits very well the total multiplicity in e^+e^- without any specific choice of energy distribution, provided the running coupling constant α_s is used for α;

ii)from the equivalence between the above scheme and that of cluster production in which a radiation-law form was used, the latter receives an indirect support by the above point i);

iii) the mean number of cluster produced $n_c(s)$ is related to α_s by

$$n_c(s) \simeq \frac{1}{\alpha_s}; \tag{6.12}$$

iv) the mean energy of the cluster produced (6.2) can now be rewritten as

$$\langle E_c \rangle = \alpha_s \sqrt{s} \tag{6.13}$$

or it coincides with the energy released at the first step of the sequential decay chain i.e. with the hadron's energy

$$E_1 = \alpha_s \sqrt{s}; \tag{6.14}$$

v) the above finding is perfectly in keeping with the duality scheme whereby fireballs are hadrons[6].

7. Conclusions.

We have discussed a series of related arguments that can all be viewed in a general scheme of fireballs formation and decay to account for the production of hadrons in e^+e^-.

In the first part (Secs.2-5) we have discussed a model that describes well the large bulk of e^+e^- yields and does so in an extremely simple and physically intuitive way.

Although a specific radiation law of energy distribution is assumed here like in Heisenberg (ref.2) and the model is not a statistical one, in its spirit the model is reminiscent of the original work of Hagedorn on fireballs (where fireballs turned out to be nothing but hadrons) but some of our results are rather in support of an extended fireball - parton - hadron duality closer perhaps to some more recent approach[8]. The novelty here, which really dates back to the early days of multiple production (see ref.2 for a complete discussion) was to assume a radiation-type energy distribution for the hadrons emitted.

In the second part of the paper (Sec. 6), we view our approach in the mean and we develop a sequential decay scheme which displays intriguing features leading to rather interesting considerations .

Throughout the whole paper we have not used a QCD language (we have not even put color in the game) but, amusingly enough, we have found some analogies and points of contact on which we have elaborated rather at length. The model is not intended to compete in data fitting and in numerical analyses with the highly sophisticated models (Lund, Webber, QCD cluster etc) which are available on the market. Our intention is not an accurate reproduction of the data in the fine details for which extremely powerful techniques have been developed[11,12]. Our aim was to provide a physical insight by which the large bulk of data could be accounted for in a simple way. As we have shown, our model does so with practically one adjustable parameter for each kind of hadron considered (we have limited ourselves to three: pions, kaons and nucleons). For the more dynamical aspects of $e^+e^- \to$ hadrons, such as angular distributions, p_T distribution around the jet axis and so on we refer the reader to a forthcoming paper[9] where some of the results developed here will be used in a hadronization scheme which has been elaborated recently [21].

An extension of the present model to hadronic initiated reactions will be the subject of future work.

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Figure Captions.

Fig. 1:Pion,kaon and proton multiplicities as functions of $W = \sqrt{s}$.

Fig. 2:Total multiplicity as function of $W = \sqrt{s}$,a)in electron-positron collisions,b)up to the Tevatron energies.

Fig. 3:Lambda and kaon multiplicities as functions of $W = \sqrt{s}$.

Fig. 4:Pion differential multiplicity,i.e. inclusive distribution, as a function of Feynman x ($W = \sqrt{s}$).

Fig. 5:Same as Fig. 4 for protons.

Fig. 6:Same as Fig. 4 for kaons.

Fig. 7:Same as Fig. 4 for lambdas.

Fig. 8:Charged hadron fractions as functions of momenta.

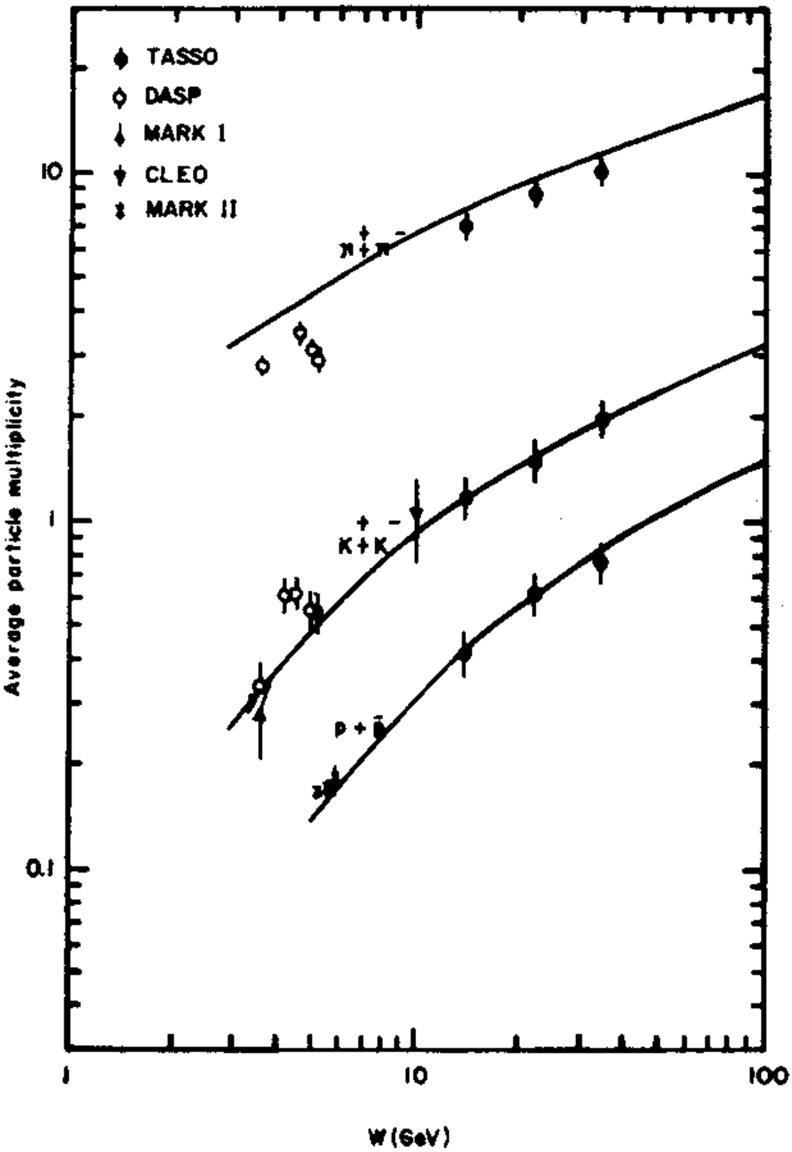
TASSO
DASP
MARK I
CLEO
MARK II
Average particle multiplicity
W (GeV)
10
1
0.1
1
10
100
p + p̄

Fig.1

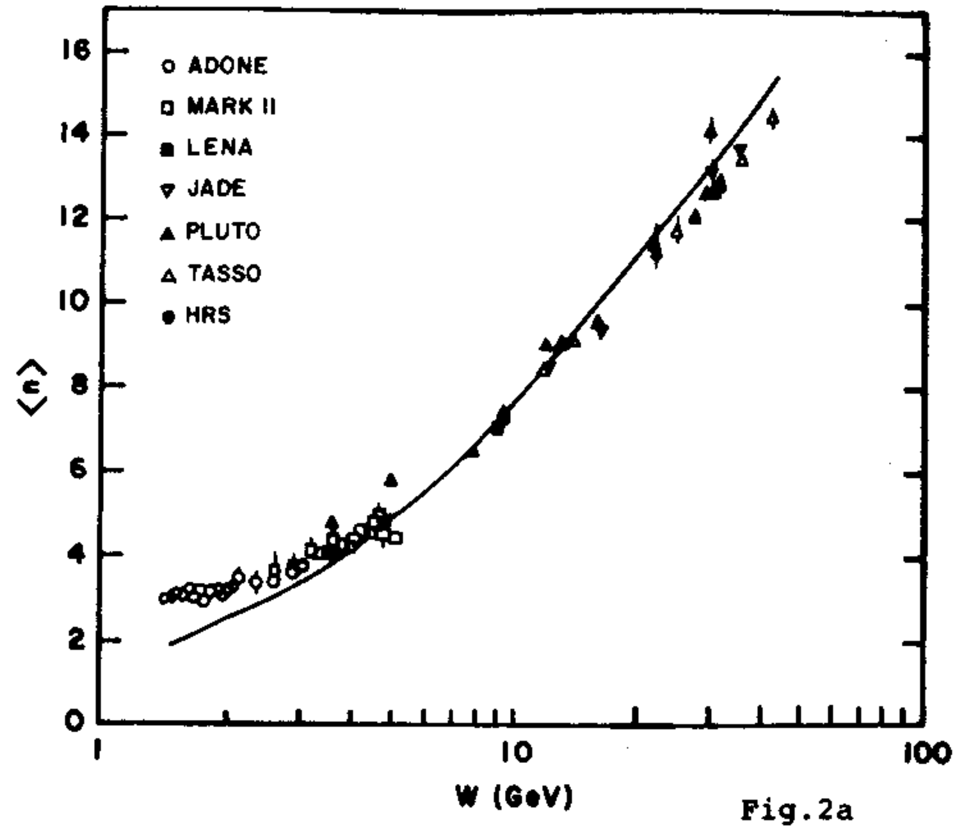
o ADONE
MARK II
LENA
JADE
PLUTO
TASSO
HRS
⟨n⟩
W (GeV)

Fig.2a

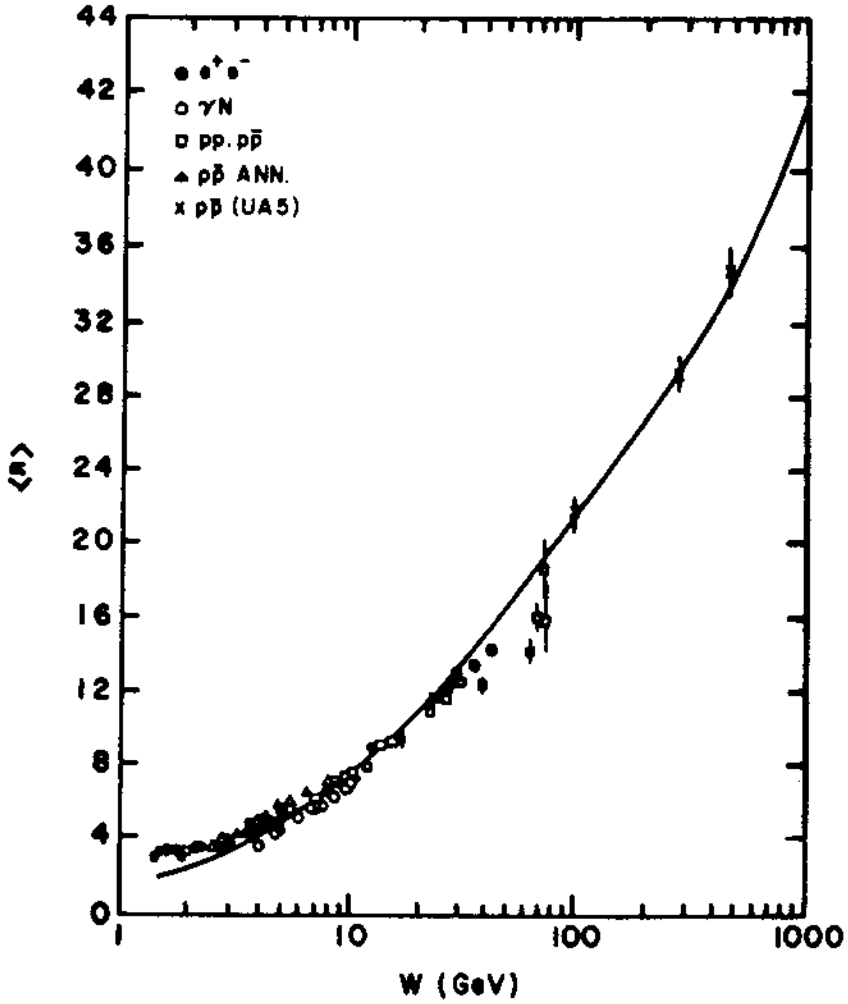
e^+e^-
γN
pp, $p\bar{p}$
$p\bar{p}$ ANN.
x $p\bar{p}$ (UA5)
⟨n⟩
W (GeV)

Fig.2b

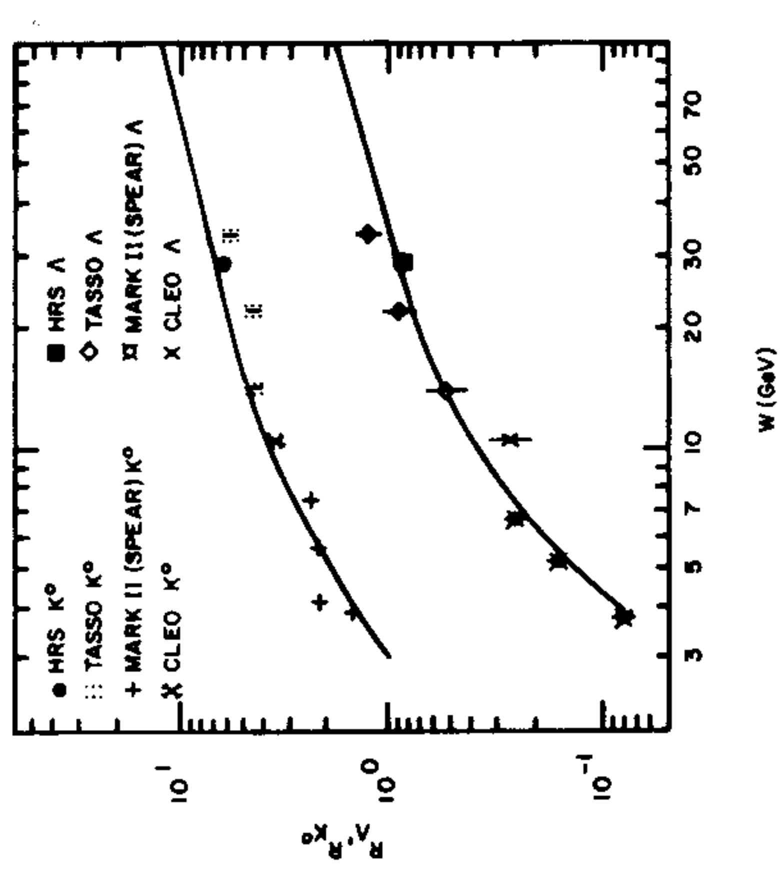
HRS K°
TASSO K°
+ MARK II (SPEAR) K°
CLEO K°
HRS Λ
TASSO Λ
MARK II (SPEAR) Λ
X CLEO Λ
W (GeV)
R_Λ, R_K°

Fig.3

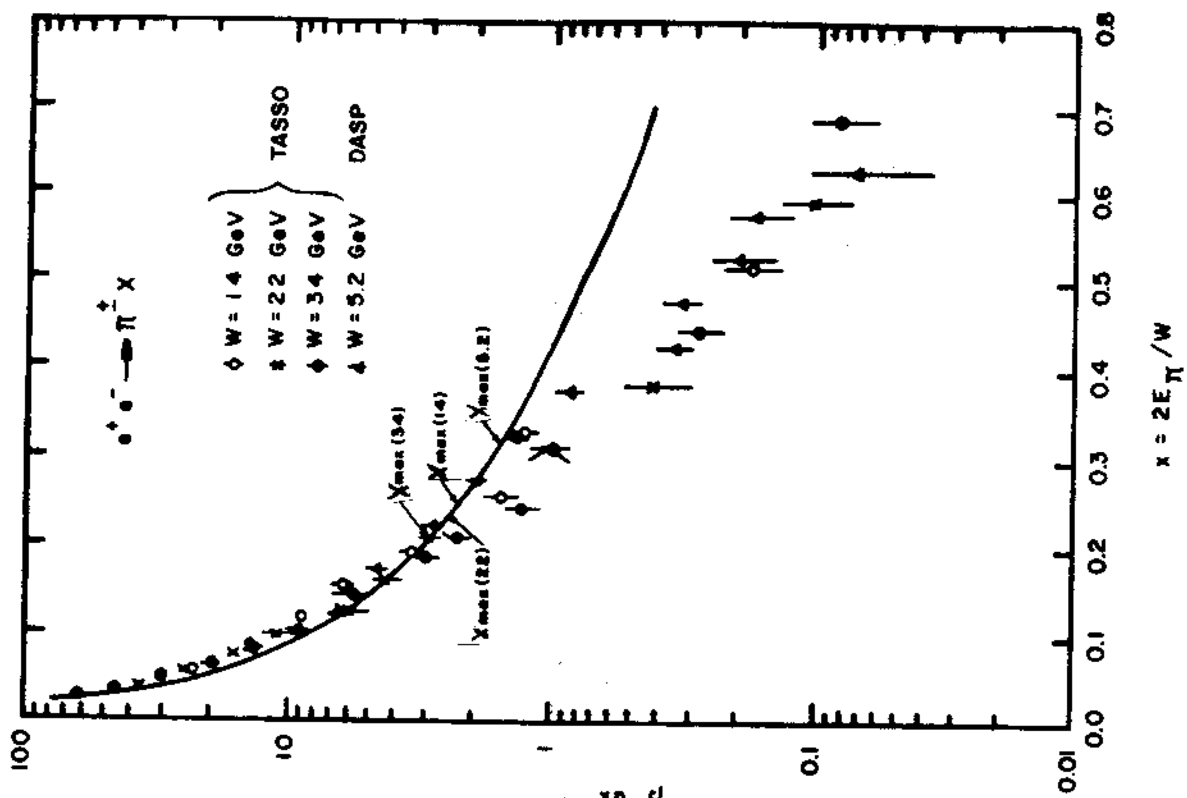
W = 14 GeV
W = 22 GeV
W = 34 GeV
W = 5.2 GeV
TASSO
DASP
x = 2E_π /W

Fig.4

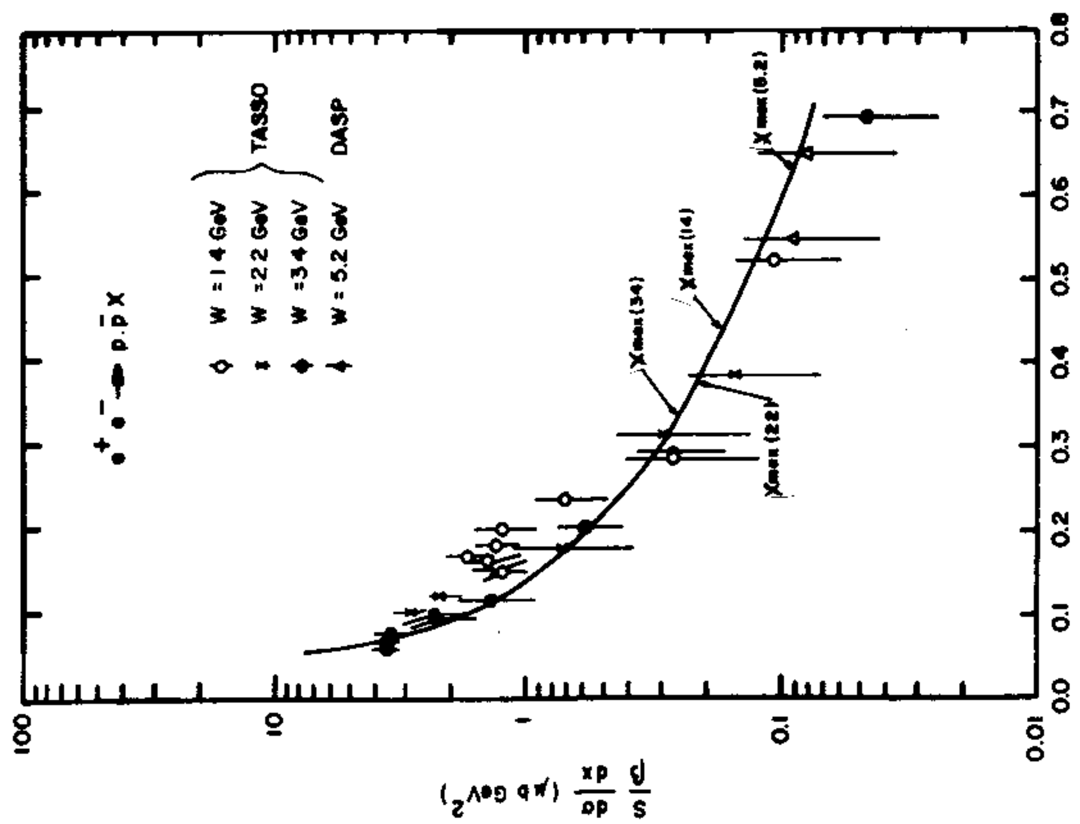
$e^+e^- \to p,\bar{p}\,X$
W = 14 GeV
W = 22 GeV
W = 34 GeV
W = 5.2 GeV
TASSO
DASP
$X_{max}(34)$
$X_{max}(14)$
$X_{max}(22)$
$X_{max}(5.2)$
$\frac{s}{\beta}\frac{d\sigma}{dx}$ (μb GeV^2)
$x = 2E_{p,\bar{p}}/W$
100
10
1
0.1
0.01
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Fig.5

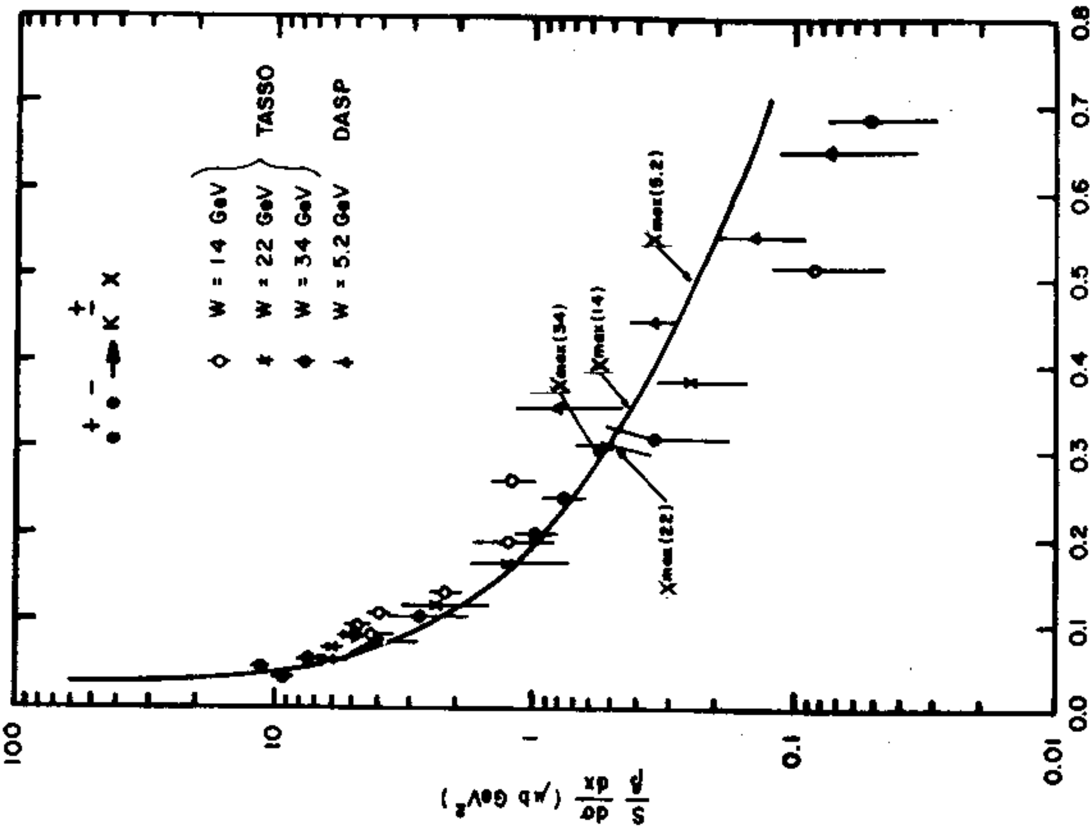
$e^+e^- \to K^{\pm}X$
W = 14 GeV
W = 22 GeV
W = 34 GeV
W = 5.2 GeV
TASSO
DASP
$X_{max}(34)$
$X_{max}(14)$
$X_{max}(22)$
$X_{max}(5.2)$
$\frac{s}{\beta}\frac{d\sigma}{dx}$ (μb GeV^2)
$x = 2E_K/W$
100
10
1
0.1
0.01
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Fig.6

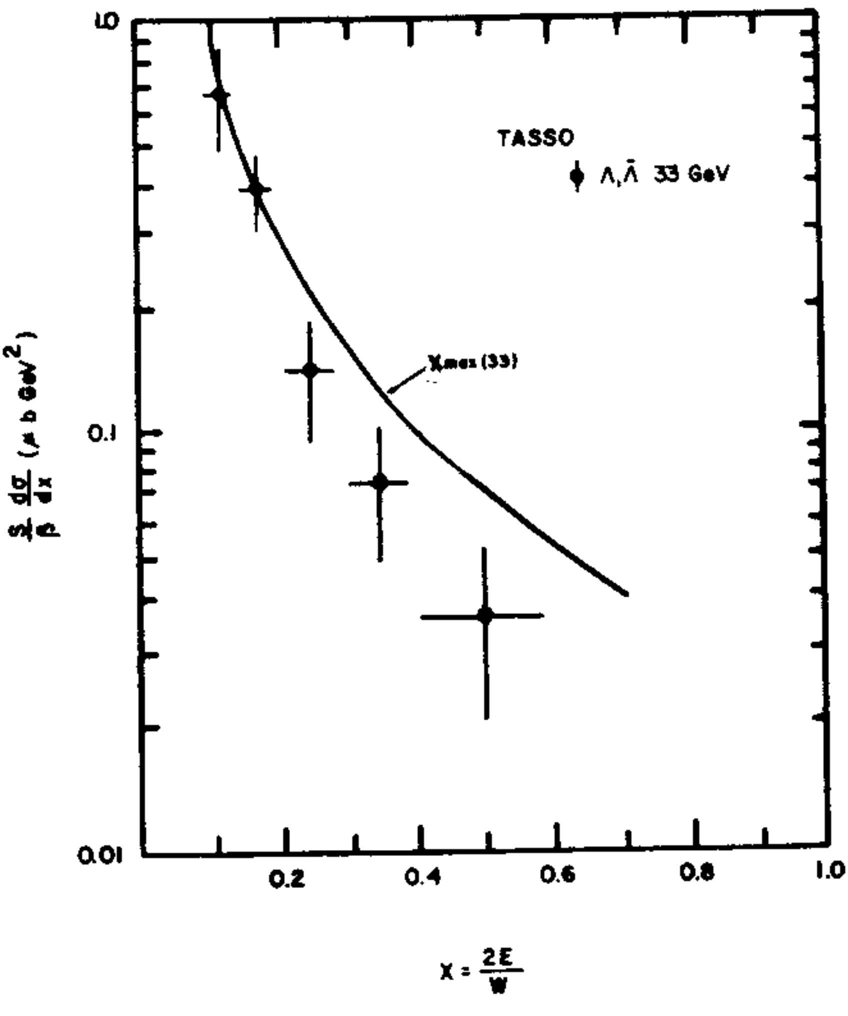
TASSO
Λ,Λ̄ 33 GeV
Xmax (33)
1.0
0.1
0.01
0.2
0.4
0.6
0.8
1.0
s/β dσ/dx (μb GeV²)
x = 2E/W

Fig.7

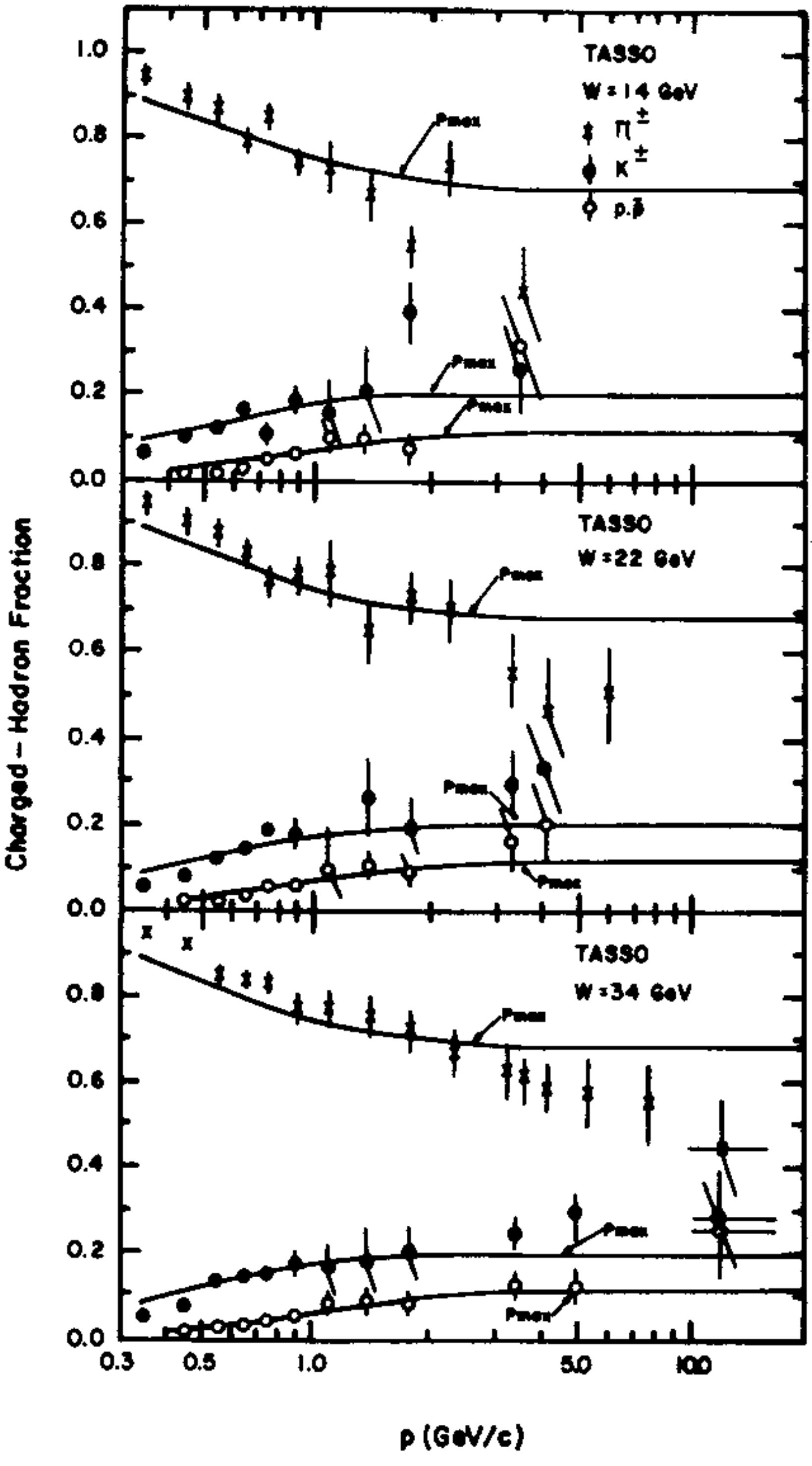
TASSO
W = 14 GeV
π±
K±
p, p̄
Pmax
TASSO
W = 22 GeV
TASSO
W = 34 GeV
Charged - Hadron Fraction
p (GeV/c)
0.0
0.2
0.4
0.6
0.8
1.0
0.3
0.5
1.0
5.0
10.0

Fig.8

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