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DIQUARK CONTRIBUTIONS TO THE NUCLEON DEEP INELASTIC STRUCTURE FUNCTIONS

by

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ABSTRACT

The contributions of diquarks to the nucleon structure functions are discussed in the framework of the parton model and in the most general case of both vector and scalar diquarks inside unpolarized and polarized nucleons. The vector diquark anomalous magnetic moment and the scalar-vector and vector-scalar diquark transitions are also taken into account. The properties of the diquarks and of their form factors, required in order for the resulting scaling violations to be compatible with the observed ones, are discussed.

Key-words: Deep inelastic scattering; Diquarks.

Introduction

The role of spin 0 and spin 1 constituents in deep inelastic scattering has been studied since the advent of the parton model [1,2]; such spin 0 and spin 1 constituents can arise in a natural way in the pure quark model as bound states of two quarks, the scalar and pseudovector diquarks [3].

More recently the contribution of diquarks to deep inelastic nucleon structure functions has been analysed and compared with the existing data, leading to a nucleon picture allowing for the presence of almost pointlike scalar diquarks and heavier, more extended, vector diquarks [4,5]. Such an analysis has been carried out in the case of lepton and ν deep inelastic scattering on unpolarized nucleons, with vector diquarks lacking anomalous magnetic moment.

We consider here the diquark contributions to the electromagnetic unpolarized and polarized nucleon structure functions in the most general case of scalar and vector diquarks, allowing for a vector diquark anomalous magnetic moment and for scalar-vector and vector-scalar diquark transitions.

In Sect.1 we derive the explicit expressions for the diquark contributions to the unpolarized nucleon structure functions F_1 and F_2, and the polarized ones, g_1 and g_2.

In Sect.2 we look at the scaling violations caused by the introduction of diquarks as constituents, and discuss the Q^2 dependence of the diquark form factors in order for such scaling violations to be compatible with the experimental information. We then give some conclusions.

1 - Diquark contributions to F_1, F_2, g_1 and g_2

Let us recall [6] that the usual electromagnetic tensor $W_{\alpha\beta}(N)$, describing the inclusive interactions of a virtual photon of four-momentum q with a nucleon of four-momentum P and covariant spin vector S, can be written in terms of four structure functions as

$$W_{\alpha\beta}(N) = W^{(S)}_{\alpha\beta}(N) + iW^{(A)}_{\alpha\beta}(N;S) \tag{1.1}$$

with a symmetric (under $\alpha \leftrightarrow \beta$ exchange) part

$$W^{(S)}_{\alpha\beta}(N) = -2m_N\left(\frac{q_\alpha q_\beta}{Q^2} + g_{\alpha\beta}\right)W_1(\nu,Q^2) + \\ + \frac{2}{m_N}\left(P_\alpha + \frac{q_\alpha}{2x}\right)\left(P_\beta + \frac{q_\beta}{2x}\right)W_2(\nu,Q^2) \tag{1.2}$$

and an antisymmetric one

$$W^{(A)}_{\alpha\beta}(N;S) = 2\epsilon_{\alpha\beta\mu\nu}q^\mu\left[m_N^2 S^\nu G_1(\nu,Q^2) + (P\cdot qS^\nu - q\cdot SP^\nu)\,G_2(\nu,Q^2)\right] \tag{1.3}$$

where $q^2 = -Q^2$, $P^2 = m_N^2$, $x = Q^2/(2P\cdot q)$, $P\cdot q = m_N\nu$.

In the parton model the virtual photon interaction with the nucleon is replaced by the sum of the virtual photon interactions with all constituents, supposed to be free. If we neglect the Fermi motion of the constituents inside the nucleon we have [6]

$$W_{\alpha\beta}(N) = \frac{1}{2m_N\nu x}\sum_{j,s} n_j(x,s;S)W_{\alpha\beta}(j,j') \tag{1.4}$$

where $n_j(x,s;S)$ is the number density of partons of type j, covariant spin s and four-momentum $k = xP$ inside a nucleon of four-momentum P and spin S. $W_{\alpha\beta}(j,j') = W^{(S)}_{\alpha\beta}(j,j') + iW^{(A)}_{\alpha\beta}(j,j';s)$ is the electromagnetic tensor which describes the virtual photon exclusive interaction with the parton $j\,(\gamma^* j \to j')$.

From Eqs.(1.1-4) we have, in the parton model

$$W^{(S)}_{\alpha\beta}(N) = \frac{1}{2}\sum_S W_{\alpha\beta}(N) \\ = \frac{1}{4m_N\nu x}\sum_{j,s;S} n_j(x,S;s)W^{(S)}_{\alpha\beta}(j,j') \tag{1.5}$$

$$W^{(A)}_{\alpha\beta}(N,S) = \frac{1}{2m_N\nu x}\sum_{j,s} n_j(x,S;s)W^{(A)}_{\alpha\beta}(j,j';s) \tag{1.6}$$

Let us now compute explicitly $W_{\alpha\beta}(j,j')$ in the case of scalar diquarks ($j = j' = S$), vector diquarks ($j = j' = V$) and of the scalar-vector ($j = S, j' = V$) and vector-scalar ($j = V, j' = S$) diquark transitions.

In the first case we have:

$$W_{\alpha\beta}(S,S) = \left[-ie_S(2k+q)_\alpha D_S(Q^2)\right]^* \left[-ie_S(2k+q)_\beta D_S(Q^2)\right]$$
$$= 4x^2 e_S^2 \left(P_\alpha + \frac{q_\alpha}{2x}\right)\left(P_\beta + \frac{q_\beta}{2x}\right) D_S^2(Q^2) \tag{1.7}$$

where $D_S(Q^2)$ is the (real) scalar diquark electromagnetic form factor, revealing its composite nature, and e_S is the electric charge (in units of the proton charge).

In the case of vector diquarks we start from the most general coupling of a virtual photon with a spin 1 massive particle

$$V^\alpha = ie_V \left\{(2k+q)^\alpha g^{\mu\nu} D_1(Q^2) + \right.$$
$$- \left[(k+q)^\nu g^{\mu\alpha} + k^\mu g^{\nu\alpha}\right] D_2(Q^2) + \tag{1.8}$$
$$\left. + k^\mu (k+q)^\nu (2k+q)^\alpha D_3(Q^2)\right\} \epsilon_{1\nu}(\lambda_1)\epsilon^*_{2\mu}(\lambda_2)$$
$$\equiv ie_V T^{\alpha\mu}\left[\epsilon_2^*(\lambda_2)\right]_\mu \tag{1.9}$$

where $\epsilon_1(\lambda_1)$ and $\epsilon_2(\lambda_2)$ are the polarization vectors of the initial and final diquarks, with helicities λ_1 and λ_2, respectively. The three form factors $D_{1,2,3}(Q^2)$ will be discussed in the next Section. Then we have

$$W_{\alpha\beta}(V,V) = \sum_{\lambda_2} (V_\alpha)^* V_\beta =$$
$$= e_V^2 T^*_{\alpha\mu} T_{\beta\mu'} \left[-g^{\mu\mu'} + \frac{(k+q)^\mu (k+q)^{\mu'}}{x^2 m_N^2}\right] \tag{1.10}$$

From Eqs.(1.8-10) we can derive the symmetric and antisymmetric parts of $W_{\alpha\beta}(V,V)$, which can be cast in the forms:

$$\sum_s W^{(S)}_{\alpha\beta}(V,V) = 4e_V^2 \left\{ -m_N \nu x \left(1 + \frac{\nu}{2m_N x}\right) D_2^2 \left(\frac{q_\alpha q_\beta}{Q^2} + g_{\alpha\beta}\right) + \right.$$
$$+ x^2 \left[\left[\left(1 + \frac{\nu}{m_N x}\right) D_1 - \frac{\nu}{m_N x} D_2 + 2m_N \nu x \left(1 + \frac{\nu}{2m_N x}\right) D_3\right]^2 + \right.$$
$$\left.\left. + 2\left[D_1^2 + \frac{\nu}{2m_N x} D_2^2\right]\right] \left(P_\alpha + \frac{q_\alpha}{2x}\right)\left(P_\beta + \frac{q_\beta}{2x}\right)\right\} \tag{1.11}$$

and

$$W^{(A)}_{\alpha\beta}(V,V;s) = \frac{1}{2m_N} e_V^2 \times$$
$$\left\{ m_N \left(2+\frac{\nu}{m_N x}\right)\left[-\nu D_2^2 + m_N x\left(2+\frac{\nu}{m_N x}\right)(D_1 D_2 + x m_N \nu D_2 D_3)\right] \times \right.$$
$$\epsilon_{\alpha\beta\mu\nu} q^\mu s^\nu +$$
$$+ s\cdot q\left[\left(1+\frac{\nu}{m_N x}\right) D_2^2 - \left(2+\frac{\nu}{m_N x}\right)(D_1 D_2 + x m_N \nu D_2 D_3)\right] \times$$
$$\left. \epsilon_{\alpha\beta\mu\nu} q^\mu P^\nu \right\} \tag{1.12}$$

In writing Eq.(1.12) we have used the relationship between the polarization vector ϵ and the spin four-vector s of a spin 1 particle of mass m and four-momentum k

$$Im(\epsilon^*_\alpha \epsilon_\beta) = \frac{1}{2m}\epsilon_{\alpha\beta\mu\nu} k^\mu s^\nu \tag{1.13}$$

For scalar-vector and vector-scalar transitions we have

$$\begin{aligned} W_{\alpha\beta}(S,V) &= \sum_{\lambda_2} e_S^2 \left[\epsilon_{\alpha\mu\nu\rho} q^\mu k^\nu \epsilon_2^*(\lambda_2)^\rho\right]^* \times \\ &\left[\epsilon_{\beta\mu'\nu'\rho'} q^{\mu'} k^{\nu'} \epsilon_2^*(\lambda_2)^{\rho'}\right] D_T^2 \\ &= 2 m_N \nu x^3 e_S^2 D_T^2 \left\{ -m_N^2 \left(1+\frac{\nu}{2m_N x}\right)\left(\frac{q_\alpha q_\beta}{Q^2} + g_{\alpha\beta}\right) + \right. \\ &\left. + \left(P_\alpha + \frac{q_\alpha}{2x}\right)\left(P_\beta + \frac{q_\beta}{2x}\right)\right\} \end{aligned} \tag{1.14}$$

and, we find

$$\sum_s W_{\alpha\beta}(V,S) = W_{\alpha\beta}(S,V) \tag{1.15}$$

where $D_T(Q^2)$ is the transition form factor.

From Eqs.(1.5) and (1.6) we can write the total diquark contribution to

$W_{\alpha\beta}(N)$ as

$$\left[W^{(S)}_{\alpha\beta}(N)\right]_{Diquarks} = \frac{1}{2m_N\nu x}\times \left\{S(x)\left[W^{(S)}_{\alpha\beta}(S,S)+W^{(S)}_{\alpha\beta}(S,V)\right]+ + \frac{1}{3}V(x)\sum_{s}\left[W^{(S)}_{\alpha\beta}(V,V)+W^{(S)}_{\alpha\beta}(V,S)\right]\right\} \tag{1.16}$$

$$\left[W^{(A)}_{\alpha\beta}(N;S)\right]_{Diquarks} = \frac{1}{2m_N\nu x}\Delta V(x;S)W^{(A)}_{\alpha\beta}(V,V;s=S) \tag{1.17}$$

where $S(x)(V(x))$ is the density number of scalar (vector) diquarks with momentum $k = xP$ and $\Delta V(x;S)$ is the difference between the number of vector diquarks with spin parallel to the proton spin S and those with spin antiparallel to S. In case there are different kinds of scalar and/or vector diquarks inside a nucleon a sum over all of them has to be included in Eqs.(1.16,17).

In deriving Eqs.(1.16,17) we have used the $SU(6)$ and parity relationships

$$V_1^{\frac{1}{2}}(x) + V_{-1}^{\frac{1}{2}}(x) = 2V_0^{\frac{1}{2}}(x) = \frac{2}{3}V(x) \tag{1.18a}$$

$$V_\lambda^\Lambda(x) = V_{-\lambda}^{-\Lambda}(x) \tag{1.18b}$$

where $V_\lambda^\Lambda(x)$ is the number density of vector diquarks with spin projection λ inside a nucleon with spin projection Λ. Eqs.(1.18a) are true, in general, for any collinear decomposition of a spin 1/2 particle into two constituents, one of which has spin 1.

By insertion of Eqs.(1.7,11,12,14,15) into Eqs.(1.16,17) and comparison with Eqs.(1.2) and (1.3) we get the explicit expressions for the diquark contributions to the nucleon deep inelastic electromagnetic structure functions. Although some of these results are well known [1,2,5,7] we present all of them here for a comprehensive treatment of the subject. With obvious notations, we have:

$$F_1^{(S)} = 0$$

$$F_2^{(S)} = e_S^2 S(x) x D_S^2$$

$$
\begin{aligned}
F_1^{(V)} =& \frac{1}{3}e_V^2 V(x)\left(1+\frac{\nu}{2m_N x}\right)D_2^2\\
F_2^{(V)} =& \frac{1}{3}e_V^2 V(x)x\left\{\left[\left(1+\frac{\nu}{m_N x}\right)D_1+\right.\right.\\
&\left.\left.-\frac{\nu}{m_N x}D_2+2m_N\nu x\left(1+\frac{\nu}{2m_N x}\right)D_3\right]^2+2\left[D_1^2+\frac{\nu}{2m_N x}D_2^2\right]\right\}\\
F_1^{(S-V)} =& \frac{1}{2}e_S^2 S(x)x^2 m_N^2\left(1+\frac{\nu}{2m_N x}\right)D_T^2\\
F_2^{(S-V)} =& \frac{1}{2}e_S^2 S(x)x^2 m_N\nu D_T^2\\
F_1^{(V-S)} =& \frac{1}{6}e_S^2 V(x)x^2 m_N^2\left(1+\frac{\nu}{2m_N x}\right)D_T^2\\
F_2^{(V-S)} =& \frac{1}{6}e_S^2 V(x)x^2 m_N\nu D_T^2\\
g_1^{(V)} =& \frac{1}{4}e_V^2\Delta V\left[\left(2+\frac{\nu}{m_N x}\right)(D_1D_2+xm_N\nu D_2D_3)-\frac{\nu}{2m_N x}D_2^2\right]\\
g_2^{(V)} =& \frac{1}{4}e_V^2\Delta V\frac{\nu}{2xm_N}\left[\left(2+\frac{\nu}{m_N x}\right)\times\right.\\
&\left.(D_1D_2+xm_N\nu D_2D_3)-\left(1+\frac{\nu}{m_N x}\right)D_2^2\right]
\end{aligned}
\tag{1.19}
$$

where we have given the results for the scaling structure functions $F_1 = m_N W_1$, $F_2 = \nu W_2, g_1 = m_N^2 \nu G_1$ and $g_2 = m_N \nu^2 G_2$. Obviously, only the vector diquarks contribute to the polarized structure functions g_1 and g_2.

2 - Diquark form factors and anomalous magnetic moment

Let us consider first the results (1.19) in the limit of pointlike diquarks. In such a case the form factors are given by

$$
\begin{gathered}
D_S(0) = 1\\
D_1(0) = 1 \quad D_2(0) = 1+\kappa \quad D_3(0) = 0\\
D_T(0) = 0
\end{gathered}
\tag{2.1}
$$

where κ is the vector diquark anomalous magnetic moment.

Whereas scalar pointlike diquarks do not introduce any Q^2 dependence in the structure functions F_1, F_2, g_1 and g_2, vector pointlike diquarks lead to strong scaling violations:

$$\begin{aligned}
F_1^{(V)} &= \frac{1}{3}e_V^2 V(x)\left(1+\frac{Q^2}{4m_N^2x^2}\right)(1+\kappa)^2 \\
F_2^{(V)} &= \frac{1}{3}e_V^2 V(x)x\left[3+\frac{Q^2}{2m_N^2x^2}(1+\kappa^2)+\left(\frac{Q^2}{2m_N^2x^2}\right)^2\kappa^2\right] \\
g_1^{(V)} &= \frac{1}{4}e_V^2 \Delta V(x,S)(1+\kappa)\left[2+\frac{Q^2}{4m_N^2x^2}(1-\kappa)\right] \\
g_2^{(V)} &= \frac{1}{4}e_V^2 \Delta V(x,S)\frac{Q^2}{4m_N^2x^2}(1+\kappa)\left[1-\kappa-\frac{Q^2}{2m_N^2x^2}\kappa\right]
\end{aligned} \tag{2.2}$$

which would be incompatible with experiment. Notice also that the scaling violations in $F_2^{(V)}$ and $g_2^{(V)}$ are of order Q^4, unless the anomalous magnetic moment of the vector diquark, κ, is zero, in which case all scaling violating terms are proportional to Q^2.

Of course, diquarks, bound states of two quarks, are not pointlike objects and any realistic comparison with experimental data should take into account their form factors $D_S, D_{1,2,3}$ and D_T. Dimensional counting rules would give for these form factors the following (or faster) large Q^2 decrease

$$\begin{aligned}
D_{S,1,2} &\sim \frac{1}{Q^2} \\
D_3 &\sim \frac{1}{Q^4} \\
D_T &\sim \frac{1}{Q^3}
\end{aligned} \tag{2.3}$$

It is then clear from Eqs.(1.19) and (2.3) that the scaling violations due to terms proportional to Q^2 or Q^4 will be mitigated by the Q^2 dependence of the diquark form factors. The question is now: is the final balance compatible with experimental information [8]?

Rather than attempting a detailed analysis of deep inelastic scattering data in the framework of the parton model with diquarks, which, in some particular cases, has already been done [4,5,7], we prefer to draw here some more general conclusions. We know that, apart from the QCD logarithmic ones, power like scaling violations of order $1/Q^2$ are allowed in F_1 and F_2 by the experimental data [8]. We also know the large Q^2 behaviour of the ratio

$$R = \frac{F_2}{2xF_1}\left(1 + \frac{2m_N x}{\nu}\right) - 1 \sim \frac{1}{Q^2} \tag{2.4}$$

correctly predicted in the quark parton model due to the Callan-Gross relationship $F_2^{(q)} = 2xF_1^{(q)}$. We then demand that these two conditions

i) scaling violations proportional to $1/Q^2$ or smaller (we do not deal here with the QCD ones)

ii) $R \sim 1/Q^2$

still hold true when introducing diquarks as constituents. We extend point i) to the polarized structure functions g_1 and g_2 as well.

Let us rewrite for convenience the large ν, Q^2 limits of some of Eqs.(1.19) (dropping some non leading terms)

$$\begin{aligned}
F_1^{(V)} &\simeq \frac{\nu}{6m_N x} D_2^2 e_V^2 V(x) \\
F_2^{(V)} &\simeq \frac{\nu}{3m_N}\left[\left(1 + \frac{\nu}{m_N x}\right)(D_1 - D_2)^2 + D_1^2 + m_N \nu^3 x D_3^2 + \right. \\
&\quad \left. + 4m_N \nu x \left(1 + \frac{\nu}{2m_N x}\right) D_3(D_1 - D_2) + 2m_N \nu x D_1 D_3\right] e_V^2 V(x) \\
F_2^{(S-V)} &\simeq 2xF_1^{(S-V)} \simeq \frac{\nu m_N}{2} x^2 D_T^2 e_S^2 S(x) \\
F_2^{(V-S)} &\simeq 2xF_1^{(V-S)} \simeq \frac{\nu m_N}{6} x^2 D_T^2 e_V^2 V(x) \\
g_1^{(V)} &\simeq \frac{\nu}{8m_N x}\left[2D_1 D_2 + 2m_N \nu x D_2 D_3 - D_2^2\right] e_V^2 \Delta V(x) \\
g_2^{(V)} &\simeq \frac{\nu}{8m_N x}\left[\left(1 + \frac{\nu}{m_N x}\right) D_2(D_1 - D_2) + D_1 D_2 + \nu^2 D_2 D_3\right] e_V^2 \Delta V(x)
\end{aligned} \tag{2.5}$$

One can see from Eqs.(1.19), (2.5) and the usual expression for $F_1^{(q)}$ and $F_2^{(q)}$ that it is possible to satisfy the above demands i) and ii) if

$$\begin{aligned}
D_1 = D_2 &\sim Q^{-2} \\
D_3 &\sim Q^{-6}
\end{aligned} \tag{2.6}$$

For D_S and D_T we do not get any further restrictions than those already given in Eqs.(2.3). In principle one might satisfy the conditions i) and ii) also by choosing different form factors D_1 and D_2 which both decrease at least like Q^{-4}, but this would be too far from the reasonable expectations given in Eq.(2.3).

Notice that the condition $D_1 = D_2$ implies that the vector diquarks have no anomalous magnetic moment (see Eq.(2.1)). Eqs.(2.6) also imply

$$F_2^{(V)} \simeq 2xF_1^{(V)} \tag{2.7}$$

so that all vector diquark contributions to F_1 and F_2 satisfy, at leading order, the Callan-Gross relationship. Our results (2.6) justify the often used assumption $D_1 = D_2 \sim Q^{-2}, D_3 = 0$ [5,7].

It is often stated that the Callan-Gross relation $F_2 = 2xF_1$ is a "proof" that partons have spin 1/2. Eq.(2.7) clearly indicates that this is not so.

Conclusions

We have computed the full diquark contributions to the electromagnetic nucleon structure functions. The resulting scaling violations have been discussed in terms of the diquark form factor behaviours at large Q^2 and of the vector diquark anomalous magnetic moment.

Even without attempting a detailed analysis of deep inelastic scattering data, some definite conclusions can be obtained by demanding that the scaling violations be compatible with the general trend of the observed ones.

It turns out that vector diquarks, if present as constituents inside nucleons, should have a zero anomalous magnetic moment and one of their form factors, $D_3(Q^2)$, should be much smaller than expected on the basis of simple dimensional arguments. Failing these conditions, the scaling violations would be too big and incompatible with experiments.

The final emerging picture, $D_1 = D_2 \sim Q^{-2}, D_3 \simeq 0$ is the same as that already assumed, for simplicity reasons, in some diquark analysis of deep inelastic scattering data [5,7].

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Note added in Proofs. Bearing in mind other applications of the diquark model, it is plausible that diquark form factors $D_1 \sim D_2 \sim Q^{-4}$, suggested by a perturbative QCD analysis [9], are to be preferred to $D_1 = D_2 \sim Q^{-2}$. We are grateful to Dr. Kroll for pointing this out to us.

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