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# EFFECTIVE FIRE-TUBE AND GEOMETRIC SCALING IN PROTON-PROTON AND PROTON-ANTIPROTON COLLISIONS

by

R.A.M.S. NAZARETH[+], N. PRADO[+] and T. KODAMA[1]

[1]Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

[+]Instituto de Física
Universidade Federal do Rio de Janeiro
21945 - Rio de Janeiro, RJ - Brasil

The simple phenomenological model for proton-proton process from Ref.1 is developed. The model combines a fireball formation process via string fragmentation mechanism for the space-time development of a non-perturbative fire-tube of excited vacuum, with a subsequent final hadron production through fireball decays. The effective string tension is proportional to the fire-tube transversal area determined by the overlap area of the two colliding protons. The overlap area is a function of the impact parameter and the proton-proton total cross-section. An excellent agreement with experimental data is obtained for a wide energy range covering from $\sqrt{s} \sim 20\,GeV$ to $1\,TeV$.

Key-words: String fragmentation; p-p collision; rapidity distributions.

## 1. Introduction

In *Ref.*1, we proposed a simple phenomenological model for multiple pion production mechanism in proton - proton collision process. There, we suppose that two colliding protons transform themselves into coloured objects due to exchange of their sea quarks(anti-quarks). The two receding coloured objects generate a nonperturbative chromoeletrical flux confined to a tube-like volume between them whose transversal area corresponds to the proton-proton overlap area, for each impact parameter. This fire-tube is regarded as a single effective one-dimensional classical string with a tension coefficient given by,

$$k = \epsilon_o \, A(b) \tag{1.1}$$

where $\epsilon_o$ is the volumetrical energy density of the chromoeletric tube, and $A(b)$ represents the transversal area of the tube. We may express this transversal area as,

$$A(b) = f(b) \, \sigma_{tot}(\sqrt{s}) \tag{1.2}$$

where $f(b)$ is a universal function of impact parameter determined by the geometric property of proton structure[1], and $\sigma_{tot}(\sqrt{s})$ is the total nucleon cross-section.

The space-time development of this effective string is discribed by the mechanism analogous to that of *Lund model*[2], except for the observable hadron production process. In our model, instead of direct production of hadrons, the string fragmentation leads to a formation of a set of intermediate statistical massive objects (*fireballs*) which subsequently decay into observable hadrons. (This two-step mechanism was also suggested for $e - \bar{e}$ jet phenomena[3]).

A preliminary *Monte Carlo* calculation of the model described above[1] has shown it to be promising in reproducing the rapidity and transversal momentum distributions and pion multiplicity for $\sqrt{s} = 19 GeV$. In this paper we present results of calculations based on analytic formulae for the string fragmentation process[4], covering a wide energy range from $\sqrt{s} \approx 19\, GeV$ *to* $1\, TeV$

## 2. Fireball Mass and Rapidity Distributions

As previously mentioned, we obtain analytical expressions for fireball distributions by a effective string fragmentation mechanism. The string constant, in our model, is proportional to the energy density of the firetube and to the two protons overlap interaction area. For simplicity, we consider that the endpoint particles are massless. This is a good approximation when the total incident energy is much higher than masses of endpoint particles.Classical trajectories of the two endpoint particles are represented by two interwined zig-zag lines which form periodically inclined rectangles in the $x - t$ plane (see *fig*.1). The inclination and area of these rectangles are related directly to the velocity and invariant mass of this system[2]. In addition to the above described motion of a classical string, another basic ingredient of the model is the stochastic breaking of the string at any point in between the two endpoint particles. Such a breaking is supposed to occur due to quantum mechanical pair creation of quark and antiquark. Once a breaking takes place, the original string splits into two substrings, each of them having a definite energy and momentum according to the coordinates of the space-time point where the string breaking took place. If string breaking occurs at $n$ different points, $n+1$ substrings are formed. Now, instead of assuming that the above process of string breaking continues indefinitly, we consider here that when any of substrings closes its period of oscillation for the first time, the substring turns into a highly excited object (*fireball*), converting the collective oscillation energy into internal excitation energy (*fig*.2). We, therefore, will not consider the fragmentation of substrings after the first period of oscillation.

Let $w$ be the probability of string breaking per unit time and unit length. Then the probability $P_n$ that the original string breakes up into exactly $n+1$ substrings is given by[4],

$$P_n = \int \dots \int d^2x_1\, d^2x_2 \dots d^2x_n\, w^n\, e^{-wA} \tag{2.1}$$

where $A = A(x_1, x_2, ..., x_n)$ is the area of hatched domain in fig.2. The integration should be done in the rectangle $S_0$ with the condition that all $x$'s are causally disconnected from each other. The probability $P_n$ is a Lorentz scalar and a function of $w, k, and M$, where $M$ is the initial $CM$ energy ($\sqrt{s}$). Eq. 2.1 can be rewritten in the form of a recursion relation as,

$$P_n(M; w, k) = \int d^2x_n\, w\, e^{-wA_1}\, P_{n-1}(M'; w, k) \tag{2.2}$$

This recursion relation can be solved explicitly ($Appendix A$).

Now, with the probability $P_n$ for (n+1) generated strings, it is possible to calculate the mass and rapidity distribution of fireballs as the probability to find, in the final configuration of string fragmentation, a fireball for which the mass and rapidity are specified. For convenience, we treat separately two groups of fireballs, those containing the two endpoint particles of the original string and those coming from the inner parts of the string. We shall refer to the first group as "*endpoint*" fireballs (ep) and the second as "*non−endpoint*" (n.ep) ones. (Fireball distributions are calculated in detail in $Appendix B$). For endpoint fireball mass and rapidity distribution we obtain,

$$\left(\frac{d^2P}{dmdy}\right)_{ep} = \frac{mw}{k^2}\exp\left(-\frac{zm}{M}e^{\mp y}\right)\,\Theta(\mp y + ln(M/m)) \tag{2.3}$$

where the minus and plus signs of $y$ are for the fireball from the right and left respectively and $Z = wM^2/(2k^2)$. For non-endpoint case we have,

$$\left(\frac{d^2P}{d\bar{m}dy}\right)_{n.ep} = 2\bar{m}\Big[E_1(z) - E_1(\bar{m}^2e^{y_{max}-y}) - E_1(\bar{m}^2e^{y_{max}+y}) + E_1(\bar{m}^2)\Big]\,\Theta(y_{max}^2 - y^2) \tag{2.4}$$

where $E_1(x)$ is the exponential integral function, $\bar{m} = m\sqrt{w/(2k^2)}$, $y_{max} = ln(M/m)$ and the Heaviside $\Theta$ function comes from the energy conservation.

## 3. Final Hadron Spectra

If we suppose that the fireballs are really statistical objects, the decay properties of a fireball should be completely specified by its mass. For example, for a thermal model, the hadron (for simplicity, we consider only mesons) spectrum from the decay of a fireball with mass $m$ may be expressed as,

$$E\,(d^3N_\pi/dp^3) = \frac{A}{\pi}\,e^{-E/T(m)} = \frac{A}{\pi}\,e^{-E_t\,\cosh(y-y_{fb})/T(m)} \tag{3.1}$$

where $E$ and $p$ are the energy and momentum of emitted pions and $T = T(m)$ is the temperature of the fireball. For non-endpoint fireballs the normalization constant $A_{n.ep}$ can be determined by the conservation of the total energy,

$$\int E(d^3N_\pi/dp^3)d^3p = 4A_{n.ep}\, m_\pi^2 T K_2(m_\pi/T) \;=\; m \tag{3.2}$$

where $m_\pi$ is the pion mass and $K_n(z)$ is the modified Bessel function of $n$ order. The average multiplicity of produced pions from a fireball $< n(m) >_{n.ep}$ can be calculated as a function of $m$ as,

$$\begin{aligned} < n_\pi(m) >_{n.ep} &= \int d^3p(d^3N/dp^3) = 4A_{n.ep} m_\pi T K_1(m_\pi/T) \\ &= (m/m_\pi)K_1(m_\pi/T)/K_2(m_\pi/T) \end{aligned} \tag{3.3}$$

and

$$< n >_{charged} = \frac{2}{3} < n_\pi >$$

.

The $p_t$ distribution of pions from a fireball is given by,

$$\left(\frac{dN_\pi}{dp_t^2}\right)_{n.ep} = \int \frac{dN}{dy dp_t^2}\, dy \;=\; 2AK_0\left(\frac{E_t}{T}\right) = \frac{1}{2}\,\frac{m}{m_\pi^2}\,\frac{1}{T}\,\frac{K_0\left(\frac{E_t}{T}\right)}{K_2\left(\frac{m_\pi}{T}\right)} \tag{3.4}$$

where $E_t = \sqrt{p^2 + m^2}$ is the transversal energy. The avarage $p_t$ value is then,

$$< p_t^2 >_{n.ep} = \frac{1}{< n_\pi >_{n.ep}} \int p_t^2 \left(\frac{dN_\pi}{dp_t^2}\right) dp_t^2 \;=\; \frac{m_\pi T K_2\left(\frac{m_\pi}{T}\right)}{K_1\left(\frac{m_\pi}{T}\right)} \tag{3.5}$$

For the rapidity ($y$) distribution of pions we have,

$$\left(\frac{dN_\pi}{dy}\right)_{n.ep} = \int \frac{d^2N_\pi}{dydp_t^2}dp_t^2 = \frac{2Am_\pi T}{\pi}\frac{1}{\cosh(y-y_B)}\left[1+\frac{1}{\cosh(y-y_B)}\right]e^{-\frac{T}{m_\pi}\cosh(y-y_B)} \tag{3.6}$$

where $y_B$ is the rapidity of the fireball.

On the other hand the pseudorapidity distribution is calculated as:

$$\left(\frac{dN_\pi}{d\eta}\right)_{n.ep} = 2A\left(\frac{m_\pi}{cosh\eta}\right)^2 \Phi\left(\frac{m_\pi}{T}\cosh y_B, \tanh\eta\tanh y_B\right) \tag{3.7}$$

where $\Phi(a,b)$ is

$$\Phi(a,b) = \int_0^\infty \frac{x^2dx}{\sqrt{1+x^2}}e^{-a(\sqrt{1+x^2}-bx)} \tag{3.8}$$

with $a = (m_\pi/T)\cosh y_B$ and $b = \tanh\eta\ \tanh y_B$.

To fix those quantities uniquely, we have to specify the temperature $T$, or equivalently the mean multiplicity $< n_\pi(m) >$, as a function of fireball mass. It has been suggested that the mean $p_T$ value is related to the fireball mass[5] as,

$$< p_T > = .25m^{1/7} \tag{3.9}$$

which in term of the temperature $T$ also becomes similar to $\simeq m^{1/7}$ according to Eq. 3.5. For large $m$, $T$ is a very slowly varying function in $m$, so that the mean multiplicity becomes almost linear in $m$. Here we parametrize $< n_\pi(m) >$ as

$$< n(m) > = \sqrt{Const.\, m^2 + n_0{}^2} \tag{3.10}$$

where *Const.* and $n_o$ are parameters. From the energy and momentum conservation, $n_o \geq 2$. Once *Const.* and $n_o$ are given, we calculate $T = T(m)$ from Eq. 3.3.

In the case of the endpoint fireballs, we have decay of pions as well as a leading particle decay (proton decay). For these distributions, the normalization conditions are given by:

$$\begin{aligned} \int \left(\frac{d^3N_p}{dp^3}\right) d^3p &= 1 \\ \int \left(\frac{d^3N_\pi}{dp^3}\right) d^3p &= n_\pi \\ \int \left[\left(E_p \frac{d^3N_p}{dp^3}\right) + \left(E_\pi \frac{d^3N_\pi}{dp^3}\right)\right] d^3p &= m \end{aligned} \tag{3.11}$$

and

$$< n_{charged} > = \frac{2}{3} < n_\pi > +1 \tag{3.12}$$

Finally, from all the expressions above we can calculate the final pion spectrum of a string fragmentation as a superposition of fireball and pion spectra. For example, for pions from non-endpoint fireballs, the average total multiplicity of pions $n$ , is given by,

$$n_{\pi_{n.ep}} = \int_{m_{min}}^{m_{max}} dm < n_\pi(m) >_{n.ep} \left(\frac{dP}{dm}\right)_{n.ep} \tag{3.13}$$

where $m_{min}$ is the inferior limit of fireball mass and $m_{max}$ is the maximum fireball mass limited by the kinematics. The fireball mass distribution $dP/dm$ can be obtained by integrating eq.2.4 on $y$ (see Appendix B).

Also, the transverse momentum distribution can be calculated by,

$$\left(\frac{dN_\pi}{dp_t^2}\right)_{n.ep} = \int_{m_{min}}^{m_{max}} dm \left(\frac{dN_\pi}{dp_t^2}\right)_{n.ep} \left(\frac{dP}{dm}\right)_{n.ep} \tag{3.14}$$

and the rapidity and pseudorapidity distributions are given by,

$$\left(\frac{dN_\pi}{dy}\right) = \int_{m_{min}}^{m_{max}} dm \int_{-y_{max}}^{y_{max}} dy_{fb} \left(\frac{dN}{dy_\pi}\right)_{n.ep} \left(\frac{d^2P}{dm dy_{fb}}\right)_{n.ep} \tag{3.15}$$

$$\left(\frac{dN_\pi}{d\eta}\right) = \int_{m_{min}}^{m_{max}} dm \int_{-y_{max}}^{y_{max}} dy_{fb} \left(\frac{dN}{d\eta_\pi}\right)_{n.ep} \left(\frac{d^2P}{dm dy_{fb}}\right)_{n.ep} \tag{3.16}$$

Identical expressions are obtained for the final pion spectrum for the endpoint fireball decay, except that we should calculate the temperature $T_{ep} = T(m - m_p)$.

The nondiffractive distributions are finally obtained as,

$$\frac{dN_\pi}{dy}|_{ndif} = \left(\frac{dN_\pi}{dy}\right)_{n.ep} + \left(\frac{dN_\pi}{dy}\right)_{ep}$$
$$\frac{dN_\pi}{d\eta}|_{ndif} = \left(\frac{dN_\pi}{d\eta}\right)_{n.ep} + \left(\frac{dN_\pi}{d\eta}\right)_{ep} \tag{3.17}$$

and

$$< n >_{ndif} = < n >_{n.ep} + < n >_{ep} \tag{3.18}$$

## 4. Results and Discussion

For a given incident energy, the final fireball mass and rapidity distributions should be integrated over all possible impact prameter values. Our model, however, does not include quantum diffractive mechanism, which is essentially a peripheral phenomena. In fact, in Eq.(1.1), $k$ tends to zero for large impact parameters, resulting in a series of very small-mass fireballs which are incompatible with the assumption of the existence of $m_{min}$. Experimentally this diffractive process has $\simeq$ 20% of the total cross section. Therefore we assume that our model is applicable for impact parameter values up to 90% of its maximum value $b_{max} = \sqrt{\sigma_{tot}/\pi}$, and compare the results to the experimental non-diffractive data.

Our model contains the following adjustable parameters:

$m_{min}$ : lower bound of fireball mass.

$\omega$ : string breaking probability.

$\epsilon_o$ : volumetrical energy density of fire-tube.

$Const.$, $n_o$ : Parameters of fireball decay.

In this calculation we have assumed the lower bound of the $m_{min}$ parameter as 0.5 $GeV$; the final pion distributions are not much sensitive to this threshold mass except for low energy collision. In addition, as seen in *Appendix A*,the string dynamics almost scales with the parameter $\epsilon_o^2/\omega$ at high energies. Thus we choose arbitrarily the string breaking probability value as $\omega = 0.02/fm^2$. Then we are left only with one adjustable free parameter $\epsilon_o$ for the string fragmentation mechanism.

The value of $n_o$ should be of the order of $\simeq 3$ according to the well-studied low energy data. We took $n_o = 2.7$. We adjusted $\epsilon_o$ and *Const.* to reproduce the energy dependence of the pion multiplicity data (Table 1). We obtain $\epsilon_o = 0.63\ GeV/fm^3$ and $Const. = 0.225$. Note that this value of $\epsilon_o$ is of the same order of the proton energy density.

Table 1

| $\sqrt{s}(GeV)$ | $<n>_{inel}$ | $<n>_{nsd}$ | $<n>_{calc}$ |
|---|---|---|---|
| 20 | 7.7 | 8.5 | 6.3 |
| 53 | 11.8 | 12.97 | 11.5 |
| 200 | 18.9 | 20.8 | 19.6 |
| 540 | 27.5 | 29.4 | 27.4 |
| 900 | | | 32.6 |

With these parameters, we calculate the final pion pseudorapidity distributions for various *CM* energies. The results are compared with the available experimental nondiffractive data[7-10] in *fig*.3. The energy dependence of the central value of pseudorapidity distribution $dN_\pi/d\eta|_0$ is also shown in *fig*.4.

We would enphasize the excellent agreement of our results with the experimental ones in a very wide *CM* energy region.

The parameters *Const.* and $n_o$ govern the decay of fireballs, determining uniquely the temperature as a function of fireball mass $m$ (see *fig*.5). In *fig*.6

we also show the pion multiplicity as a function of fireball mass. The value of $Const. = 0.225$ which adjusts the final pion multiplicity seems to give a relatively high temperature value for heavier fireballs, resulting in a too high $< p_t >$ at $\sqrt{s} = 900GeV$. In order to obtain lower temperature values preserving the behaviour of the mean multiplicity and rapidity distribution, we should discard the assumption of the isotropic fireball decay. In fact, the assumption of isotropicity of statistical objects immediately after the first period of oscillation of substrings might have been too restrictive. They might still keep some longitudinal collective motion. Such an effect certainly lowers the temperature without increasing the mean multiplicity of hadrons from these fireballs and does not affects the behaviour of the pseudorapidity distributions except for the very central region.

In *fig*.3, both the experimental points[7] and calculated curve for $\sqrt{s} = 20GeV$ are *rapidity* distributions and not pseudorapidity distributions. In order to see the difference between the rapidity and pseudorapidity distributions, we compare them in *fig*.7. We see from *fig*.7 that for higher incident energies, their differences at $y = 0$ is $\approx 10\%$, whereas this difference becomes around 20% for lower incident energy.

We conclude that the phenomenological fire-tube model applied in a impact parameter overlap integral approach gives a good description of multiparticle production process. Despite its great simplicity, the calculated results are satisfactory with respect to those quantities which are most important in the future application of our model to the analysis of nucleus-nucleus collision.

In our model, the property of increasing total cross section with $\sqrt{s}$ is incorporated into the string tension by the geometrical overlap area of two colliding protons. In fact, this procedure is fundamental to reproduce the energy dependence of mean multiplicity and other quantities. Thus the geometrical factor plays some essential role for various observables as was discussed in ref. 13,14. Further studies on this point are in progress.

## Appendix A - string fragmentation probability

Let w be the probability of string breaking per unit time and unit lenght. Then, the probability $P_n$ that the original string breakes up into exactly n+1 substrings is given by[4],

$$P_n = \int \dots \int d^2x_1\, d^2x_2 \dots d^2x_n\, w^n\, e^{-wA} \tag{A.1}$$

where $A = A(x_1, x_2, \dots, x_n)$ is the area of hatched domain in $fig.2$. The integration in $x's$ should be done with the condition that all $x's$ are causally disconnected from each other. The probability $P_n$ is a Lorentz scalar and a function of $w, k, and M$, where M is the initial $CM$ energy $(\sqrt{s})$; $P_n = P_n(M; w, k)$.
Eq. A.1 can be rewritten as

$$P_n = \int d^2x_n\, w\, e^{-wA_1} \int \dots \int d^2x_1 \dots d^2x_{n-1}\, w^{n-1}\, e^{-wA_2} \tag{A.2}$$

where $A_1$ is the area of rectangle indicated in $fig.2$ and $A_2 = A - A_1$. Froe Lorentz invariance, it is easy to recognize that the second part in the right hand side of eq. A.2 is equal to $P_{n-1}(M'; w, k)$ where $M' = M'(x_1, x_2, \dots, x_{n-1})$ is the invariant mass of the part of the string corresponding to the area $A_2$. Therefore, eq. A.2 forms a recursion formula,

$$P_n(M; w, k) = \int d^2x_n\, w\, e^{-wA_1}\, P_{n-1}(M'; w, k) \tag{A.3}$$

To handle further eq.A.3, it is convinient to introduce the light-cone variables,

$$u = \frac{k}{M}(x + t) \qquad v = k/M\,(t - x) \tag{A.4}$$

In the *c.m.* system of the original string, we can express $A_1$ and $M'$ explicitly as,

$$A_1 = M^2/2k^2\, u_n \qquad M' = M\sqrt{(1-u_n)\,v_n} \tag{A.5}$$

so that eq. A.3 becomes,

$$P_n(M; w, k) = \frac{wM^2}{2k^2}\int_0^1\int_0^1 du\, dv\, \exp\left(-\frac{wM^2}{2k^2}\,u\right) P_n(M(1-u)v; w, k) \tag{A.6}$$

We can see from eq. A.6 that $P_i\,(M; w, k)$ scales as a function of $z = wM^2/2k^2$, viz,

$$P_n(M; w, k) = P_n(z) \tag{A.7}$$

and after some calculations we find that eq. A.6 can be cast into,

$$P_n(z) = \int_0^1 dt\, k(z,t)\, P_{n-1}(zt) \tag{A.8}$$

where,

$$k(z,t) = z\, e^{-z} \int_0^1 \frac{1}{s}\, e^{zs}\, ds \tag{A.9}$$

Eq. A.8 can also be written as,

$$P_n(z) = e^{-z} \int_0^z dt\, \frac{1}{t} e^t \int_0^t ds\, P_{n-1}(s) \tag{A.10}$$

The recursion relation eq. A.8 or eq. A.10 can be solved with the starting function,

$$P_0(z) = e^{-z} \tag{A.11}$$

Now, the multiplicity distribution of fireballs is determined by functions $P_n(z)$ which satisfy the recursion formula eq. A.8 or eq. A.10 with the initial condition eq. A.11. The recursion formula can be solved explicitly in the form of power series in $z$. First, we calculate $P_1(z)$ as,

$$P_1(z) = e^{-z} \int_0^z dt\, \frac{1}{t} e^t\, (1 - e^{-t}) = e^{-z} \sum_{r=1}^{\infty} \frac{1}{r r!}\, z^r \tag{A.12}$$

We now then calculate the integral of $P_1$ as,

$$\begin{aligned} \int_0^t ds\, P_1(s) &= \sum_{r=1}^{\infty} \frac{1}{rr!} \int_0^t e^{-s_\circ r}\, ds \\ &= \sum_{r=1}^{\infty} \frac{1}{rr!} e^{-t} \sum_{m=0}^{\infty} \frac{t^{r+1+m}}{(r+1)...(r+1+m)} \\ &= e^{-t} \sum_{r=2}^{\infty} \frac{C(r,2)}{r!}\, t^k \end{aligned} \tag{A.13}$$

where,

$$C(r,2) = \sum_{j=1}^{r-1} \frac{1}{j} \tag{A.14}$$

Substituting eq. A.13 into eq. A.12 with $n = 2$, we get,

$$P_2(z) = e^{-z} \sum_{r=2}^{\infty} \frac{C(r,2)}{rr!}\, z^r \tag{A.15}$$

Repeating the procedure, we obtain, for general $n$,

$$P_n(z) = e^{-z} \sum_{r=n}^{\infty} \frac{C(r,n)}{rr!}\, z^r \tag{A.16}$$

where $C(r,n)$ can be determined by the following recursion formula,

$$C(r,n) = \sum_{k=n-1}^{r-1} \frac{1}{k}\, C(k,n-1) = C(r-1,n) + \frac{C(r-1,n-1)}{r-1} \tag{A.17}$$

The above equation holds for $r > n > 2$. For $n = 1$,

$$C(r,1) = 1 \qquad for\ all\ r > 1 \tag{A.18}$$

In particular, we have,

$$C(n,n) = \frac{1}{(n-1)!} \tag{A.19}$$

From eqs. A.17 and A.18, it is easy to prove that,

$$\sum_{n=1}^{r} C(r,n) = r \tag{A.20}$$

This reflects in eq. A.16 as,

$$\sum_{n=1}^{\infty} P_n(z) = 1 - e^{-z} \tag{A.21}$$

With eq. A.11, we obtain,

$$\sum_{n=0}^{\infty} P_n(z) = 1 \tag{A.22}$$

which is the expected result since the left-hand side of eq. A.22 is the total probability for a string to break into any number of pieces including the case of no breaking. We show in fig.A1, the multiplicity distribution of fireballs for several values of $z$.For very large values of $z$ ($\ln(z) \gg 1$), it can be shown that $P_n(z)$ approaches asymptotically as

$$P_n(z) \rightarrow 1/z \ln(z)^{n-1}/(n-1)! \qquad (A.23)$$

## Appendix B - Rapidity and Mass Di: tribution

The rapidity and mass distribution of fireballs is given as the probability to find in the final configuration of string fragmentation a fireball whose rapidity and mass are specified. In our picture, this is given as the probability to have two adjacent breaking points $(x_1, t_1)$ and $(x_2, t_2)$ which satisfy,

$$k^2[(x_1 - x_2)^2 - (t_1 - t_2)^2] = m^2 \tag{B.1}$$

and,

$$(t_1 - t_2) / (x_1 - x_2) = tanh\, y \tag{B.2}$$

where $m$ and $y$ are mass and rapidity of a fireball, respectively. In terms of $u$ and $v$ light-cones variables eq. A.4, we can express $m$ and $y$ as,

$$\begin{aligned} u_2 - u_1 &= \frac{m}{M} e^{y} \\ v_1 - v_2 &= \frac{m}{M} e^{-y} \end{aligned} \tag{B.3}$$

For endpoint fireballs, the mass and rapidity distribution is calculated as,

$$\left(\frac{d^2 P}{dmdy}\right)_{ep} = \frac{mw}{k^2} \exp\left(-\frac{zm}{M} e^{\mp y}\right) \Theta(\mp y + ln(M/m)) \tag{B.4}$$

where the minus sign of $y$ is for the fireball from the right end-point, and the plus sign is for the left end-point. $\Theta(x)$ is the Heaviside step function. The average number and mass of these fireballs are calculated as,

$$< n >_{ep} = 2(1 - e^{-z}) \tag{B.5}$$

and

$$\begin{aligned} < m >_{ep} &= M\, z \int\int du\, dv\, (1-u)\, v\, e^{-zv} \\ &= 2/3\, M/\sqrt{z}\, \gamma(3/2, z) \end{aligned} \tag{B.6}$$

where $\gamma(p,z)$ is the incomplete gamma function,

$$\gamma(p,z) = \int_0^z t^{p-1}\, e^{-t}\, dt \tag{B.7}$$

For large values of $z >> 1$, $< m >_{ep}$ rapidly converges to,

$$< m >_{ep} \rightarrow \frac{\sqrt{2\pi}}{3} \left(\frac{k^2}{w}\right)^{\frac{1}{2}} \tag{B.8}$$

On the other hand, the rapidity and mass spectrum of non-endpoint fireballs can be calculated as,

$$\left(\frac{d^2P}{dmdy}\right)_{n.ep} = z^2 \int_0^1 \int_0^1 du_1 dv_1 \int_{u_1}^1 du_2 \int_0^{v_1} dv_2$$
$$\delta(m - M\sqrt{(u_2-u_1)(v_1-v_2)})\, \delta\left[y - \frac{1}{2} ln\left(\frac{(u_2-u_1)}{(v_1-v_2)}\right)\right] e^{-zv_1u_2}$$
$$= m\left(\frac{w}{k^2}\right)\left[Ein(z) - Ein\left(\frac{zm}{Me^y}\right) - Ein\left(\frac{zm}{Me^{-y}}\right) + Ein\left(\frac{zm^2}{M^2}\right)\right] \tag{B.9}$$
$$\Theta\left[ln\left(\frac{M}{m}\right) - y\right] \Theta\left[y + ln\left(\frac{M}{m}\right)\right]$$

where $Ein(x)$ is defined as,

$$Ein(x) = \int_0^x \frac{(1-e^{-t})}{t}\, dt = E_1(x) + ln(x) + \gamma \tag{B.10}$$

with $\gamma = 0.57721...$(Euler's constant) and $E_1(x)$ is the exponential integral function. It turns out to be convinient to use a dimensionless variable $\overline{m} = m\sqrt{w/2k^2}$. Then eq. B.9 can be rewritten as,

$$\left(\frac{d^2P}{d\overline{m}dy}\right)_{n.ep} = 2\overline{m}\Big[E_1(z) - E_1(\overline{m}^2 e^{y_{max}-y})$$
$$- E_1(\overline{m}^2 e^{y_{max}+y}) + E_1(\overline{m}^2)\Big]\, \Theta(y_{max}^2 - y^2) \tag{B.11}$$

for $-y_{max} < y < y_{max}$ , with $y_{max} = ln(z/\overline{m}^2)/2$. Since $E_1(x)$ is a rapidly decreasing function in $x$, we can see from eq. B.11 that the rapidity distribution for very large $z$ values tends to have a plateau in the central region (see fig.B1), whose central value is given by,

$$\left(\frac{d^2P}{dyd\overline{m}}\right)_{n.ep} |_{y=0} \rightarrow 2\,\overline{m}\,E_1(\overline{m}^2) \tag{B.12}$$

The mean multiplicity of non-endpoint fireballs is calculated as,

$$\begin{aligned} <n>_{n.ep} &= \int d\overline{m} \int dy \frac{d^2P}{d\overline{m}dy} = z^2 \int du \int dv\, uve^{-zuv} \\ &= e^{-z} - 1 + Ein(z) \end{aligned} \tag{B.13}$$

The probability of forming only one fireball is given by eq. A.11. Thus, the total mean multiplicity of fireballs $<n>$ is given as,

$$\begin{aligned} <n> &= e^{-z} + <n>_{ep} + <n>_{n.ep} \\ &= 1 + e^{-z} + Ein(z) \end{aligned} \tag{B.14}$$

For large values of $z$, this increases asymptotically as,

$$<n> \rightarrow ln(z) + 1.57721 + O(e^{-z}/z) \tag{B.15}$$

If we integrate eq. B.11 in $y$ fixing $m$, we obtain the normalized mass distribution of non-endpoint fireballs as,

$$\frac{1}{P}\left(\frac{dP}{d\overline{m}}\right)_{n.ep} = \frac{2\overline{m}}{<n>_{n.ep}} \int_{\overline{m}^2}^{z} dt\, ln\left(\frac{t}{z}\right) ln\left(\frac{t}{2\overline{m}^2}\right) e^{-t} \tag{B.16}$$

For large $z$ (see fig.B2), eq. B.16 tends to,

$$\frac{1}{P}\left(\frac{dP}{d\overline{m}}\right)_{n.ep} \rightarrow 2\overline{m}\int_{\overline{m}^2}^{\infty} dt\, ln\left(\frac{t}{\overline{m}^2}\right)\, e^{-t} \qquad (B.17)$$

We also can calculate the average mass of non-endpoint fireballs as follows,

$$\begin{aligned} <m>_{n.ep} &= \frac{1}{<n>_{n.ep}}\int\int dm\, dy\, m\, \left(\frac{d^2P}{dmdy}\right)_{n.ep} \\ &= \frac{Mz^2}{<n>_{n.ep}}\int_0^1 du_1\int_0^1 dv_1\int_{u_1}^1 du_2\int_0^{v_1} dv_2\sqrt{(u_2-u_1)(v_1-v_2)}e^{-zu_2v_1} \\ &= \frac{M}{<n>_{n.ep}}\left(\frac{2}{3}\right)^2\left(\frac{w}{2k^2}\right)^{-1/2}[ln(z)\gamma(5/2,z)\ -\ \gamma_{ln}(5/2,z)] \end{aligned} \qquad (B.18)$$

where we have introduced a new function $\gamma_{ln}(p,z)$ by,

$$\gamma_{ln}(p,z)\ =\ \int_0^z dt\, t^{p-1}\, ln(t)\, e^{-t} \qquad (B.19)$$

For $z >> 1$, this average mass converges to,

$$<m>_{n.ep} \rightarrow \frac{\sqrt{2\pi}}{3}\left(\frac{k^2}{w}\right)^{1/2}\frac{ln(z)\ -\ 0.7031}{ln(z)\ -\ 0.4228} \qquad (B.20)$$

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**Figure Captions**

**Fig.1** String breaking. When a string breaks at a space-time point P, two substrings are generated.

**Fig.2** Formation of fireballs. Whenever two endpoints of a substring coinside, the substring is considered to turn into a fireball. The integrals in eq. 2.1 should be done inside the rectangle $S_o$.

**Fig.3** black squares → UA5 NSD experimental pseudorapidity data: ref.8-10. Open squares → experimental rapidity data: ref.7.

Solid curves → pseudorapidity distributions from our model. Dashed curve → rapidity distribution from our model.

**Fig.4** Central density $\rho(0) = \frac{dN}{d\eta}|_{\eta=0}$ as a function of CM energy. black squares → UA5 NSD (ref.10). Dashed curve → our calculations.

**Fig.5** Fireball temperature as a function of fireball mass.

**Fig.6** Pion multiplicity as a function of fireball mass.

**Fig.7** Rapidity (solid curves) and Pseudorapidity (dashed) calculations.

**Fig.A1** Multiplicity distribution of fireballs.

**Fig.B1** Rapidity spectra of fireballs for various massas $\bar{m}$. Solid curves are for $z = 50$ and dotted curves are for $z = \infty$.

**Fig.B2** Mass Spectra of fireballs as function of $\bar{m}$.

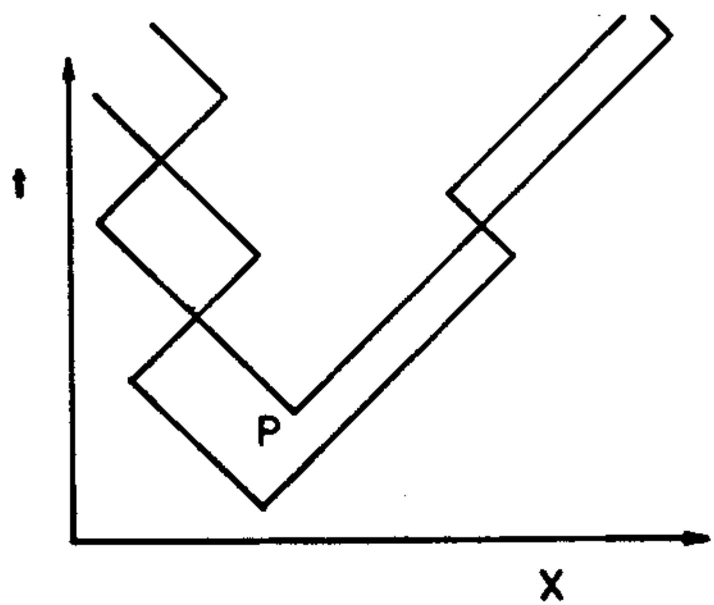


Fig. 1

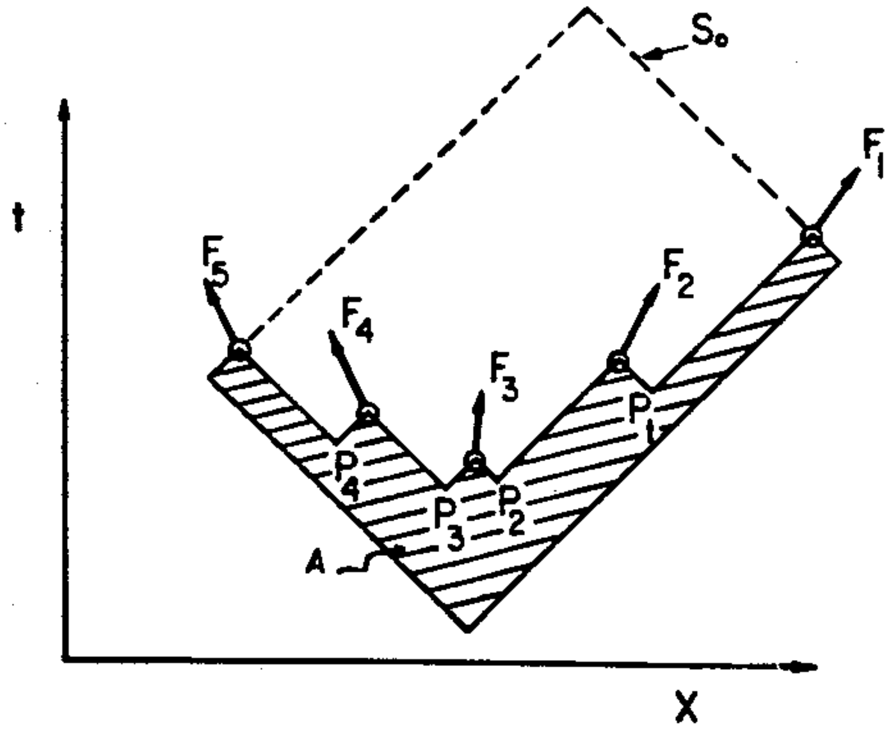


Fig. 2

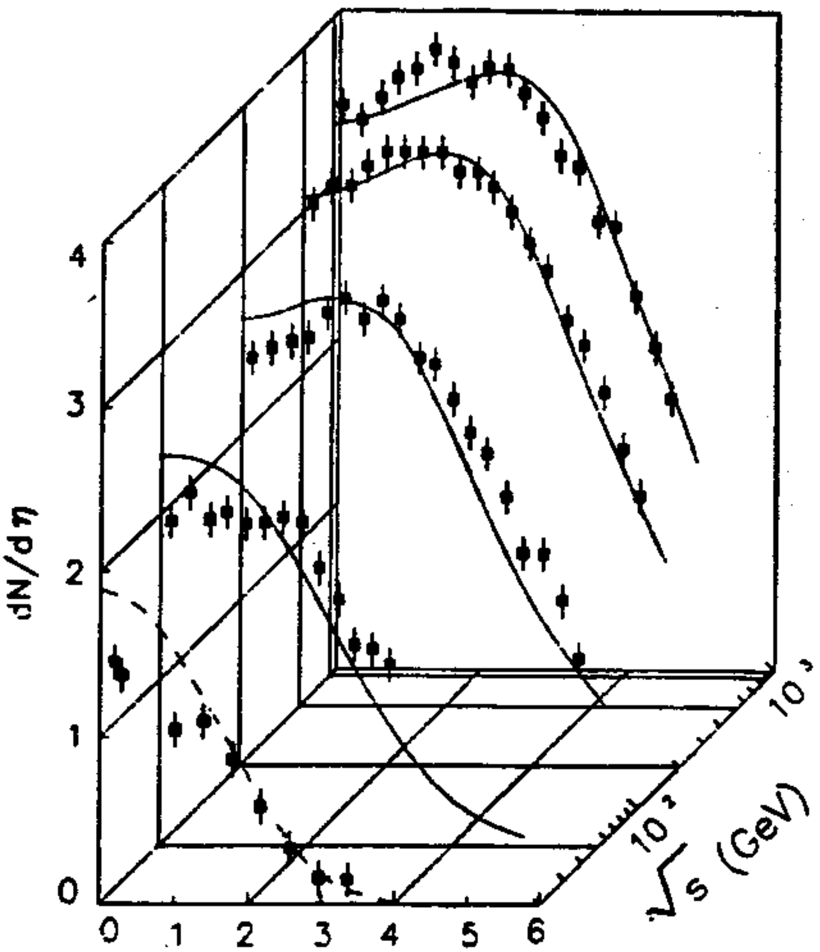


Fig. 3

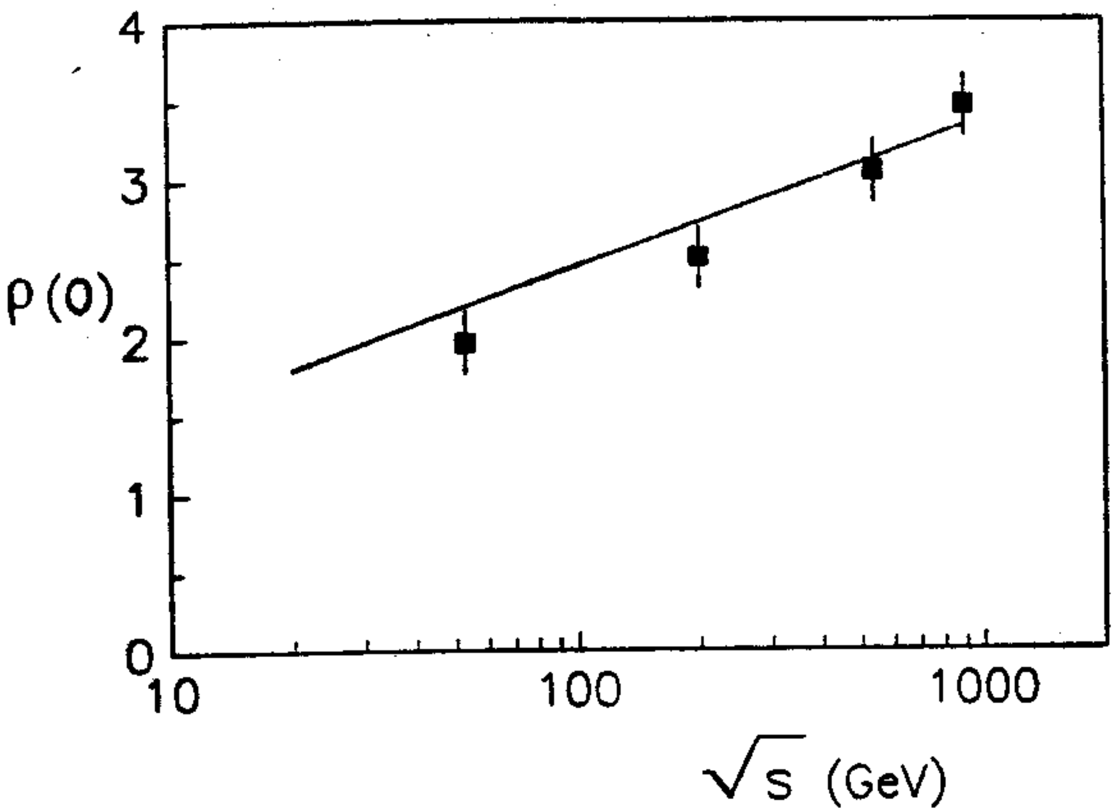


Fig. 4

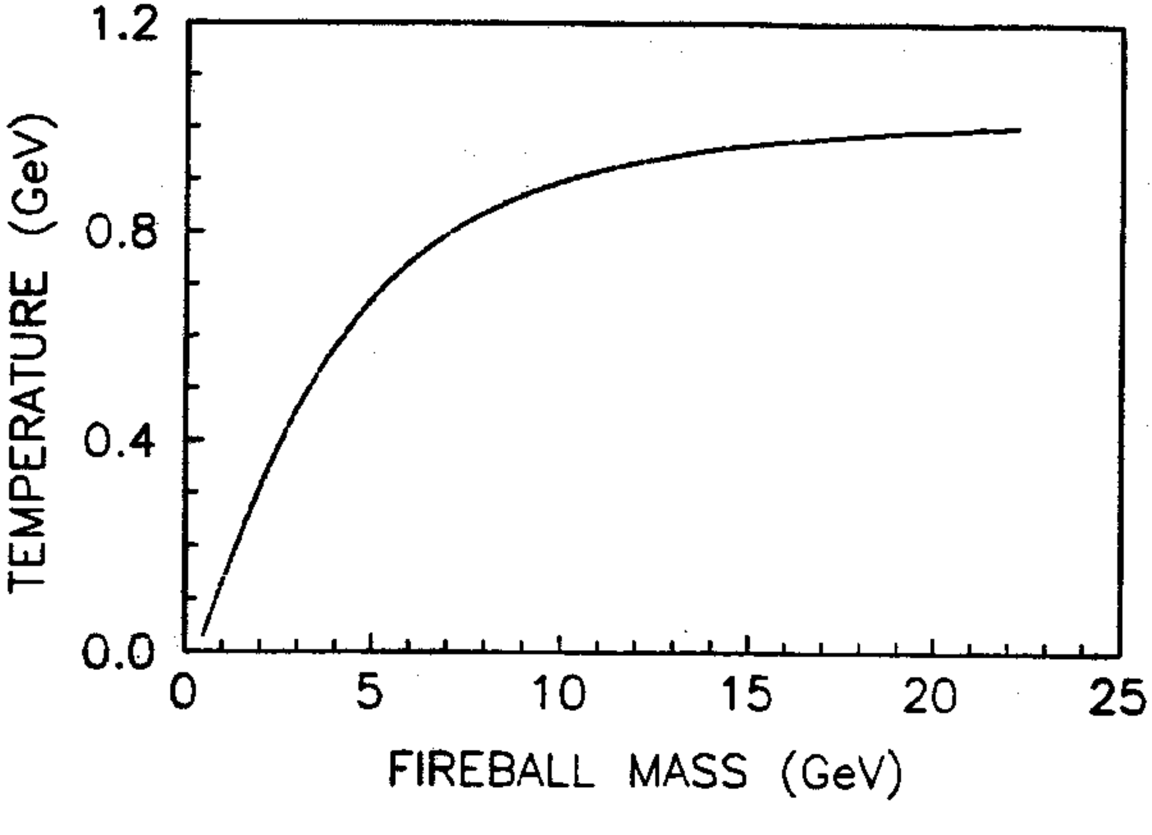


Fig. 5

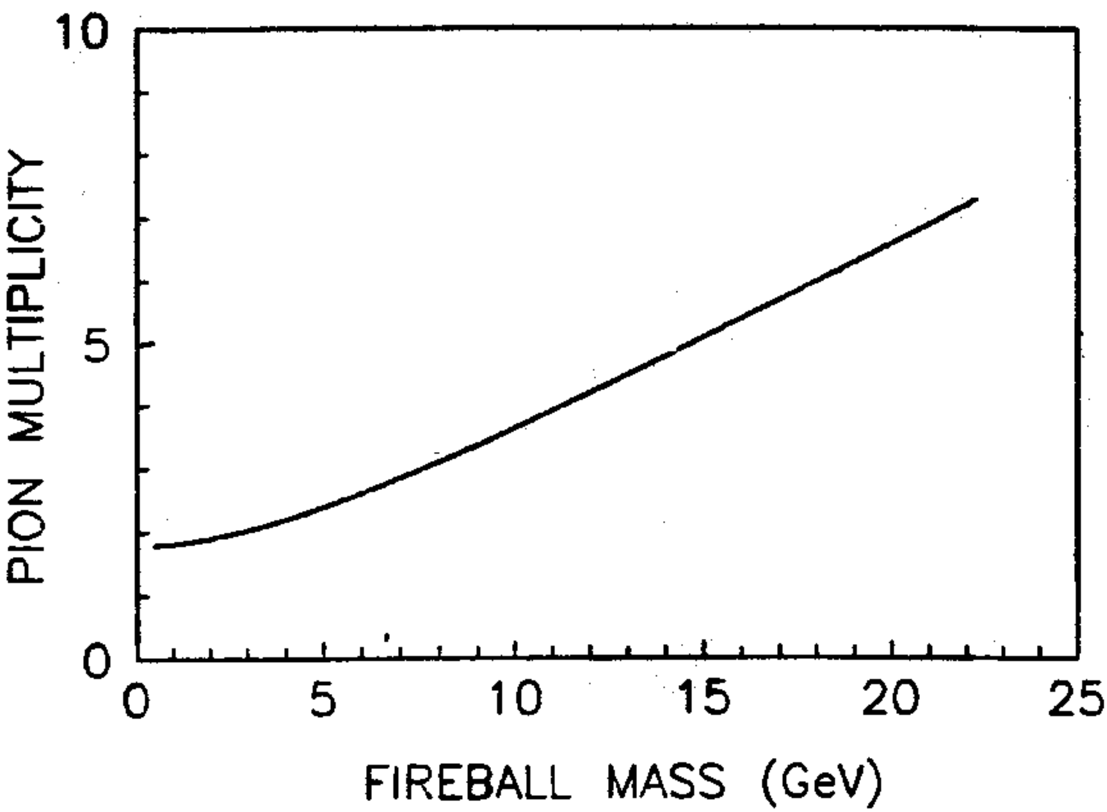


Fig. 6

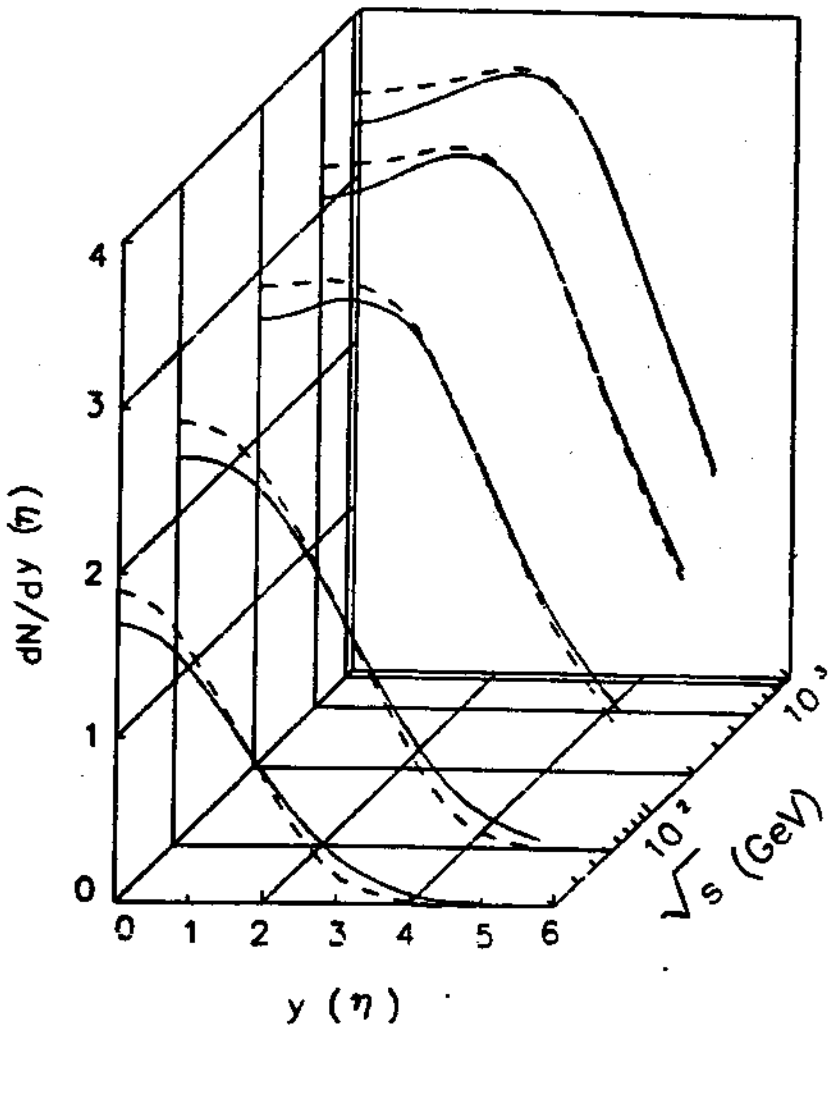


Fig. 7

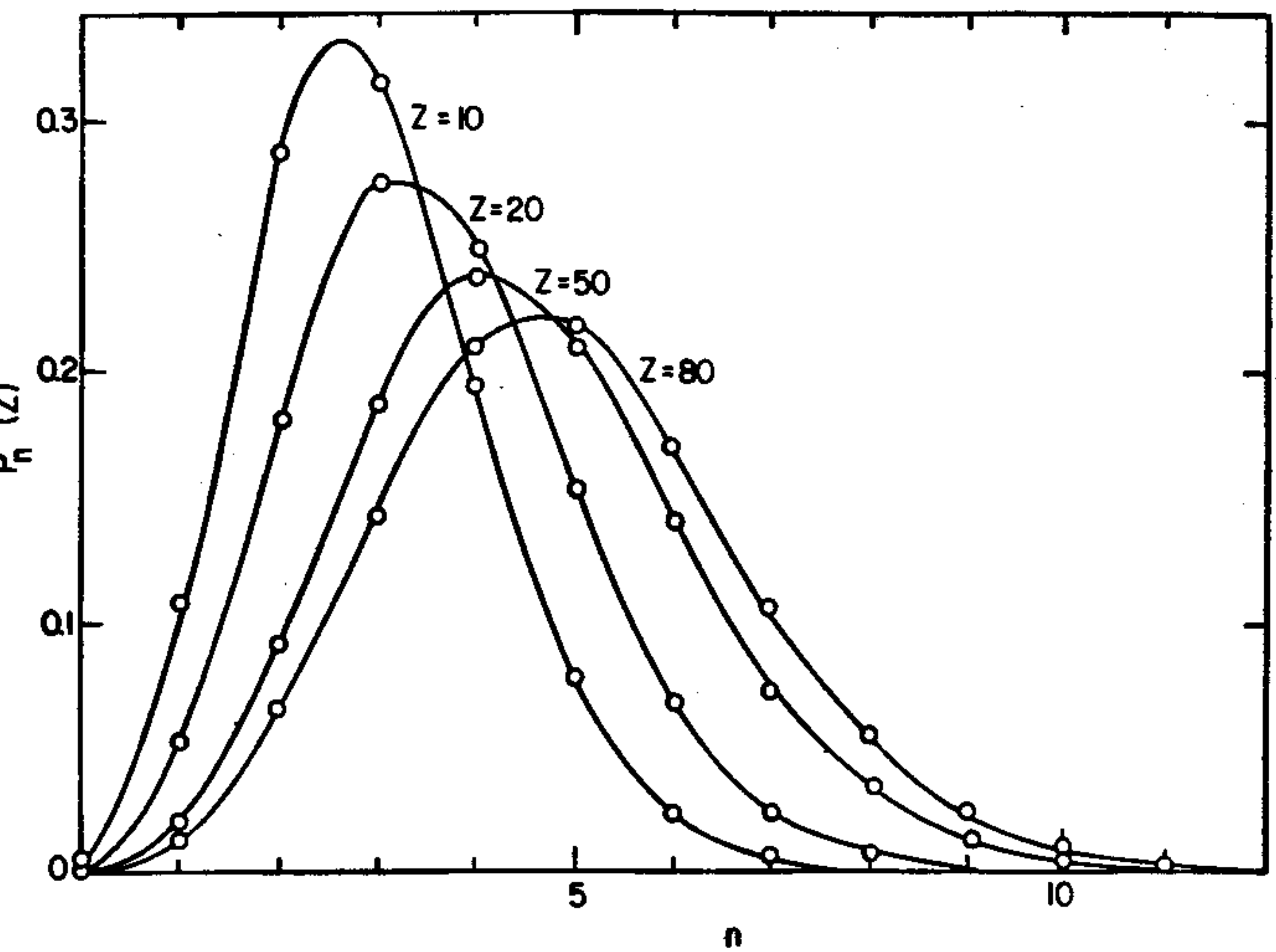


Fig. A1

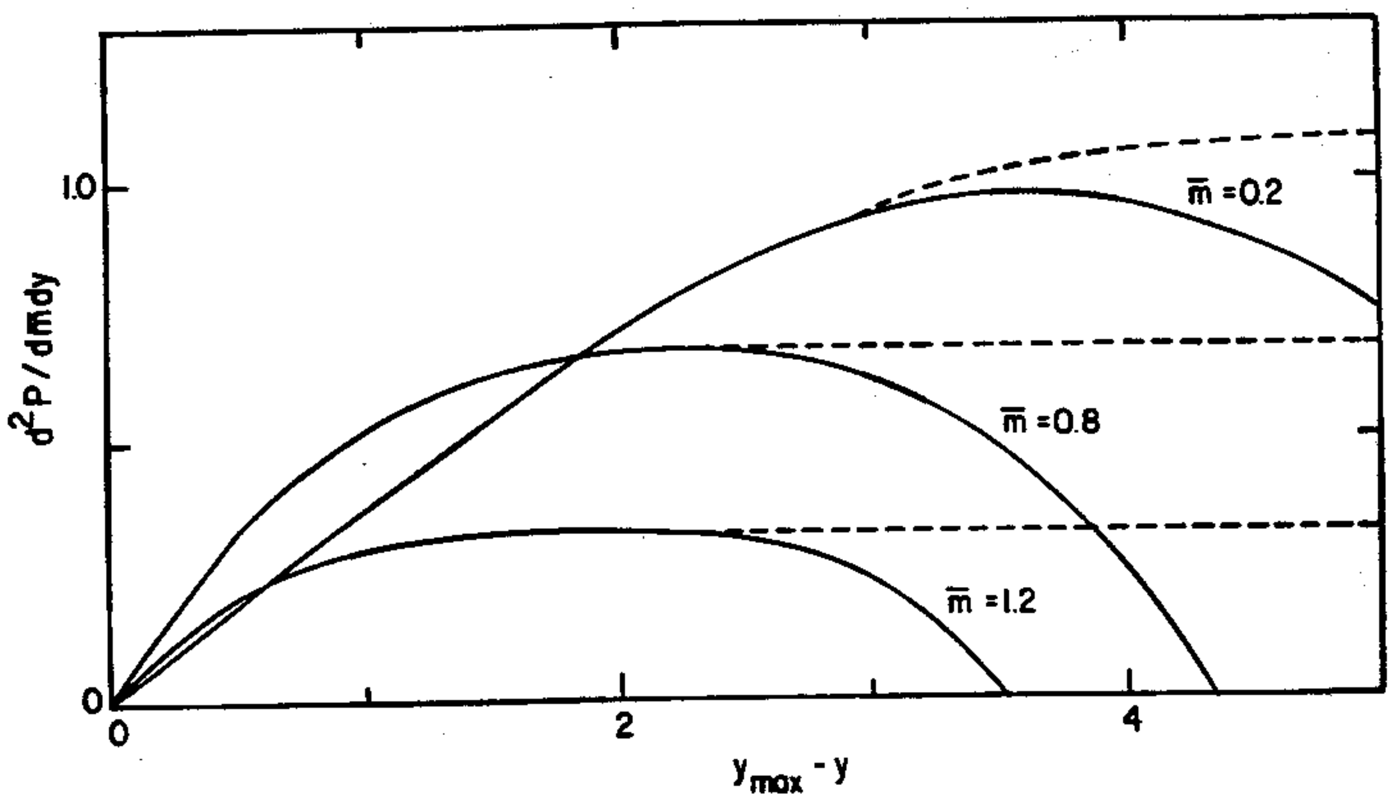


Fig. B1

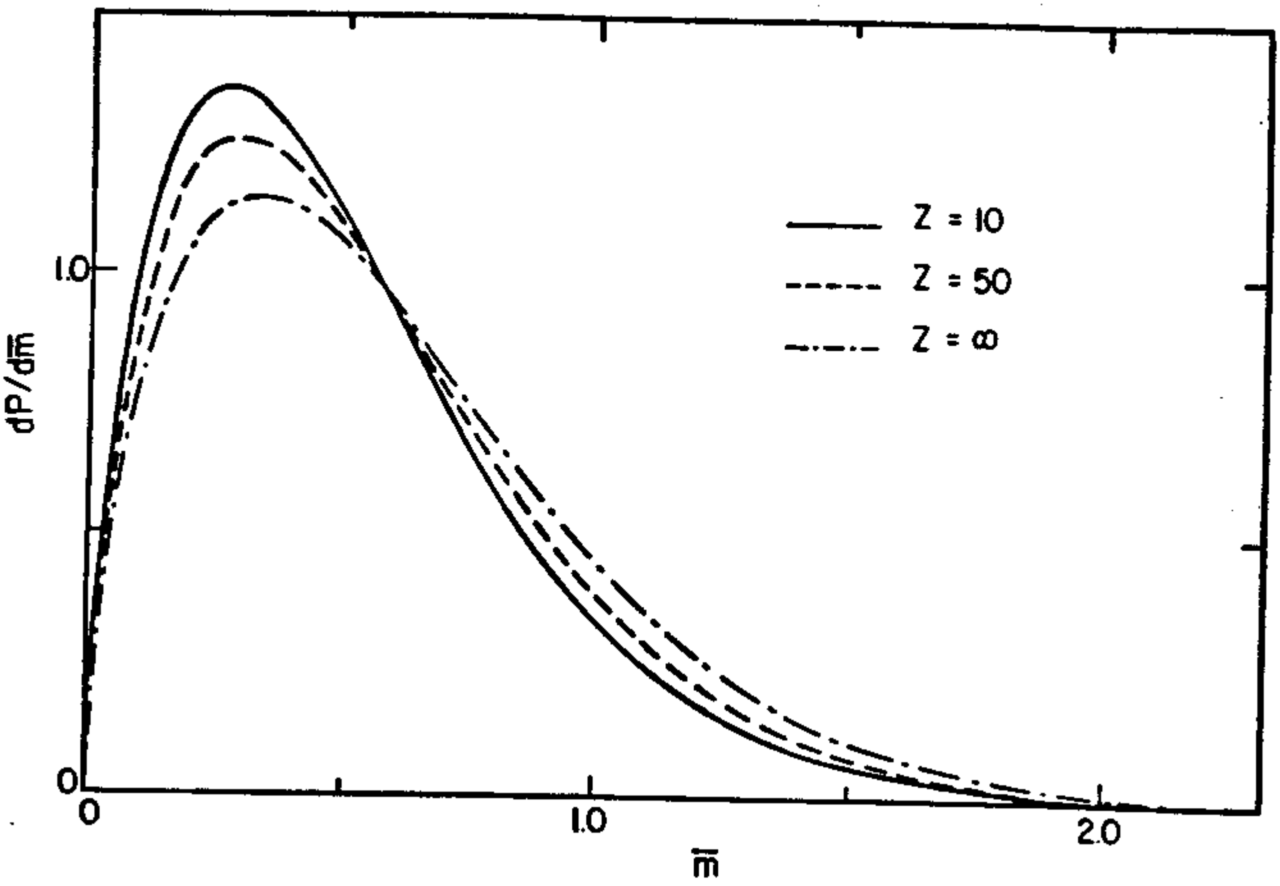


Fig. B2

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