Mass Tunneling in Brane World and Formation of Charged Black Holes

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Solutions of Einstein-Maxwell field equations suggest that charged black holes should exist. However, no physical process has been identified as to effectively violate the charge neutrality in a collapsing star. *In this essay* we demonstrate that such a process is possible in theories with an infinite extra dimension, where free massive particles localized on the brane can leak into the extra space. Because of color confinement only electrons, rather than protons, can escape. This leakage generates an electric charge asymmetry on initially neutral brane matter. Although the effect is quite small, it could be enhanced on large densities as in astrophysical objects. At the supernova collapse of a massive star, the residual charge in the imploding matter must be inherited by its remnant giving origin to a Reissner-Nordström black hole.

The Reissner-Nordström line-element describes the gravitational field of a collapsed and charged object: a *charged* black hole (BH). For the BH spherical symmetry (t, r, θ, ϕ), it reads:

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \qquad (1)$$

where M is the BH mass, and Q its charge. This solution is obtained by solving the field equations with Maxwell's energy-momentum tensor in source-free regions written as

$$T_{\mu\nu} = \frac{1}{4\pi}(-g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}), \qquad (2)$$

where $g_{\mu\nu}$ is the space-time metric, and $F_{\mu\nu}$ is the electromagnetic field strength.

Charged BHs (CHBs) have been invoked as a central ingredient in an uncountable number of physical and astrophysical theories of the more wide spectrum. Nonetheless, it has been emphasized that as such the Reissner-Nordström solution is just a mathematical solution with no *conclusive* astrophysical foundations for its realization in nature. The question on the origin of BH's non-zero electric charged, as implied by the solution, is a matter of concerns since then, essentially because most stars; the prospective progenitors of BHs, are shown to be neutral objects. Up today no consistent theory has been provided to demonstrate the appearance of an electric charge asymmetry in stellar processes. Thus the issue is one of the long-standing open problems in relativistic astrophysics.

We address this puzzle and show that there exists a *natural solution* to this fundamental enigma in the context of brane world physics: Charged BHs can be formed in supernovae collapses since charge leaks into the infinite extra-dimension [1] over the whole life of a progenitor star. Our result also stresses that most stars should be electrically charged instead of neutral, as conventional wisdom used to claim. The implications for physics of this conclusion are still to be unraveled.

Extra dimensions could exist in nature and yet remain hidden to the current experiments. Motivated by the old ideas of Kaluza and Klein and the modern string theory one has usually believed that such extra dimensions should be compact. Recently, however, it has been suggested the possibility that such extra dimensions can be rather infinite [2,4] and still be hidden to our eyes. The explanation of why it is so comes from the concept of localization of particles [4–7] in a higher dimensional space (the bulk) around a fixed point that defines a four dimensional hypersurface (the brane). Thus, at low energy, observed particles would behave just as four dimensional fields. A simple realization of this scenario appears on five dimensional theories where the background space is given by the Randall-Sundrum (RS) metric [2]

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \; ; \qquad (3)$$

where the parameter k relates the fundamental five dimensional gravity scale, M_*, with the now effective Planck scale, $M_P \sim 10^{19}$ GeV, through $M_*^3 = k\ M_P^2$.

The above metric induces an effective potential for gravitons that has the form of a vulcano barrier [5] around the brane (located at $y = 0$). The corresponding spectrum of gravitational perturbations have a massless bound state localized at the brane and a continuum of bulk modes with suppressed couplings to brane fields. The bound state is the true graviton that mediates the standard four dimensional gravity interaction. Bulk modes exchange induces on the brane deviations from the four dimensional Newton's law of gravity only at distances smaller than k^{-1}. Thus, from the current experimental limits on short distance gravity interactions [8] one has $k \gtrsim 10^{-3}$ eV. That means $M_* \gtrsim 10^8$ GeV.

True localization, however, takes place only for massless fields [10,11]. In the massive case the bound state becomes metastable and able to leak out into the bulk of extra space. For charged particles, their lost into the bulk would account for an apparent violation of conservation of charge in the 4-dim world [1] through the process $e^- \rightarrow$ *nothing*. For hadrons, opposedly, confinement of color interactions prevents the escape of a single colored component [1]. Confinement acts as a strong attractive force that grows with the distance, making the trapping on the brane even stronger. This also forbids processes that may induce proton decay, and the sudden creation of colored composed states from uncolored ones. Thus the asymmetric nature between electrons and protons, translated into the only desappearance of electrons from the brane, will induce a tiny excess of charge (protons), which may be accumulated in any actual brane matter density, otherwise neutral. The associated escaping rate can be calculated in specific models for localization of free fields. It usually depends on the mass of the fundamental particle [10]. The process is bounded by the electron desappearance (life)time: $\tau_{e\rightarrow nothing} > 4.2 \cdot 10^{24}$ yrs [12]. This implies a escaping rate for electrons [15]

$$\Gamma \lesssim \Gamma_{exp} \equiv 1/\tau_{e\rightarrow nothing} = 4.96 \cdot 10^{-57}\ \mathrm{GeV}. \tag{4}$$

The curve on the parameter space associated to the saturation of this bound has been plotted on Fig. 1. The constraint on the charge asymmetry at the Big Bang Nucleosynthesis (BBN) era reads [13–15]:

$$\Gamma \lesssim \Gamma_{BBN} = 6.58 \cdot 10^{-58}\ \mathrm{GeV}\ . \tag{5}$$

Both these bounds can be used to constrain the parameters of brane world models, as shown in Fig. 1.

Charge asymmetry from particle leaking.– Most of the particles we know are in fact massive. However, the sole existence of mass will lift the bound state on the spectrum. Therefore, our formerly localized mode will no longer be the lightest state, since the continuum of modes will remain unaffected. Thus, the matter fields would rather be metastable states. Indeed, there would be a non zero probability for the escaping of brane fields into the bulk [10,1]. Obviously, as massless particles cannot leak out, the escaping rate should be directly dependent on the mass of the particle. One can check this by considering the localization mechanism of a massive fermion, for which the general form of the Dirac equation in terms of chiral components $\Psi_{L,R}$ reads [10,15]

$$e^{k|y|}\gamma^\mu p_\mu \Psi_{L,R} \pm \partial_5 \Psi_{R,L} - \left(\phi(y)\sigma_1 + \mu\sigma_2\right)\Psi_{R,L} = 0\ .$$

As shown in Refs. [10,15], the above equation has a continuum of massive modes that starts at $m = 0$, for $m^2 = p_\mu p^\mu$, and a "localized" massive mode (that has radiation boundary conditions at $y \rightarrow \pm\infty$) for which the mass eigenvalue is complex: $m = m_0 + i\Gamma$. The presense of an imaginary part on m reflects the metastable nature of such a state. Γ is then interpreted as the "life time" of the particle from the brane point of view.

In the case where the physical mass, m_0, is much smaller than $k < v$ the probability of escaping goes as

$$\Gamma = m_0 \cdot \left(\frac{m_0}{2k}\right)^{2v/k-1} \cdot \frac{\pi}{[\Gamma(v/k+\frac{1}{2})]^2}, \tag{6}$$

where $m_0 = (1 - k/2v)\mu$. This case ($v > k$) will probe to be the most interesting one on our discusion below. By taking $k \gg m_e = 5.11 \cdot 10^{-4}$ GeV, we state our interest to models where $M_* > 10^{12}$ GeV. Then, the theory of escaping matter is described by the following parameters: the mass of the particle, m_0; the ϕ vacuum, v; and the fundamental scale, M_*, encoded in the metric parameter k. It is consistently supplemented with the bounds given above: Eqs.(4-5).

Charge accumulation in Stars and CBH formation.– Though the charge asymmetry might be too small as to be directly detected in terrestrial laboratories, it could be relevant in longevous astrophysical objects which contain a large number of charged particles. Though it is actually difficult to calculate this effect (recall that gravity tends to strongly tie star material to the brane) [15], a simple way to address this problem is to assume that the particles may escape only during those short instants when they move as free (along a mean free path distance). Therefore, out of a certain time t, there is an effective freedom time, t_f, that measures the amount of time over which the particle behaves effectively free. Obviously $t_f \lesssim t$. It is useful to define the ratio: $x \equiv \frac{t_f}{t}$, that we call the *freedom rate*, so that by definition $x \lesssim 1$.

Now, the source of any reduction on the value of x, as for the mean free path, is the presence of matter surrounding the particle. As denser the surrounding medium as smaller the value of x. Therefore one should take $x = x(\eta)$, for η the volumetric density of particles, which for any realistic medium is function of position and time. Time dependence comes, in a stable medium, only due to the leakage of particles. Thus, as in any decay process one can write at zero order $\eta(\vec{r},t) \approx \eta(\vec{r})e^{-x(\vec{r})\,\Gamma\,t}$. From this equation one notices that the quantity $x\Gamma$ has the physical meaning of being the effective escaping rate, as modified by the presence of interactions. So, the freedom rate plays the role of a form factor that encodes the internal forces of the medium to which the particle is subjected. Thus, the total number of remaining electrons at any given time is given as: $N_e(t) = \int dV\, \eta(\vec{r})e^{-x(\vec{r})\,\Gamma\,t}$. Therefore, due to the smallness of Γ, the induced charge asymmetry, defined by: $\Delta q(t) \equiv \frac{N_p(t)-N_e(t)}{N_e^0} = 1 - \frac{N_e(t)}{N_e^0}$, where N_e^0 is the initial number of electrons, can now be approximated to the simple form: $\Delta q(t) \approx \bar{x}\,\Gamma\,t$; (‡) with $\bar{x}$ the volume average of freedom rate: $\bar{x} \equiv \frac{\int dV\ x(\vec{r})\eta(\vec{r})}{\int dV\ \eta(\vec{r})}$. Hence, $\bar{x}$ becomes just a phenomenological parameter, yet to be calculated for realistic cases.

Let us now consider a typical star to estimate how much of asymmetry it can accumulate over its long life, which may be almost as large as the age of the Universe (U). Thus, taking $t_*^{\rm U} = 10^{10}$ yr, and $t_*^{\rm ms} \sim 10^7$ yr for massive stars (ms: $\sim 30 - 40{\rm M}_\odot$ [17]), into Eq. (‡), combined with the constraints from Eqs. (4) and (5), we get the upper bounds

$$\Delta q(t_*) \lesssim 10^{-14,\ -17}\ . \qquad (7)$$

Such a tiny asymmetry would account for the generation of small electromagnetic fields which, in some cases, can be made manifest by some astrophysical processes as a supernova core collapse, where tiny induced electromagnetic fields can be substantially magnified [18]. The non zero total charge accumulated over the massive star life could trigger the formation of a charged black hole when the type II supernova ensues. As shown in Ref. [15], this charge asymmetry matchs the requirements [15] for Ruffini et al.'s mechanism for gamma-ray bursts to work [19]. Consequently, CBHs could indeed be the central engines of those giant cosmic explosions. Thence, we conclude that the brane world mass (charge) leaking mechanism may naturally solve this fundamental puzzle about the CBH formation.

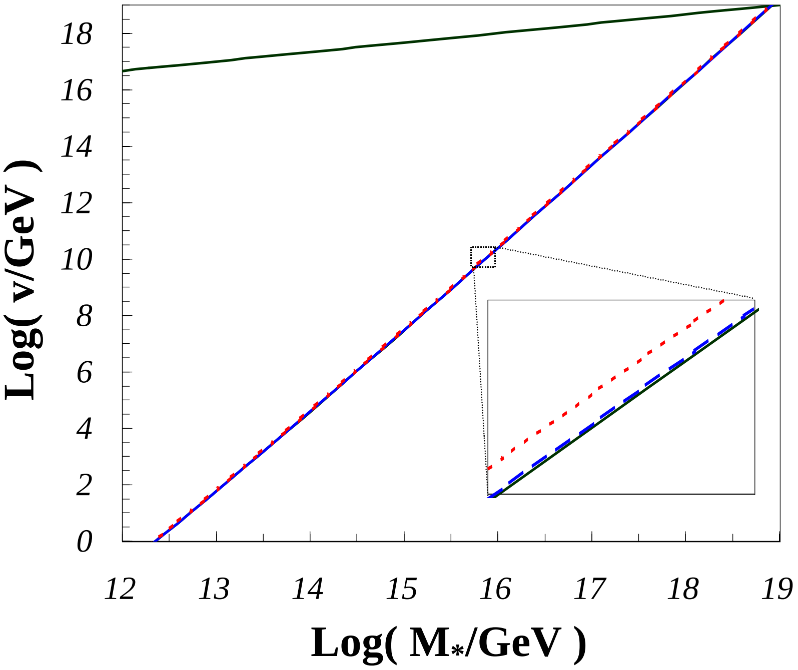

FIG. 1. Allowed parameter space for brane world models. Upper line stands for $v = M_*$. Lower line is the limit imposed by Γ_{exp} (continuous line) and Γ_{BBN} (dashed line) as on the text. Dotted line on the amplified zone pictures a hypothetical future limit on electron disappearance of about 10^{33} yr.

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