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ON POSSIBLE MEASUREMENT OF GRAVITATIONAL INTERACTION PARAMETERS ON BOARD A SATELLITE

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Abstract

The recently suggested SEE (Satellite Energy Exchange) method of measuring the gravitational constant G, possible equivalence principle violation (measured by the Eötvös parameter η) and the hypothetic 5th force parameters α and λ on board a drag-free Earth's satellite is discussed and further developed. Various particle trajectories near a heavy ball are numerically simulated. Some basic sources of error are analysed. The G measurement procedure is modelled by noise insertion to a "true" trajectory. It is concluded that the present knowledge of G, α (for $\lambda \geq 1$ m) and η can be improved by at least two orders of magnitude.

Key-words: Space experiments; Measurement of G.

The gravitational constant G is at present the least accurately measured fundamental physical constant: the error $\delta G/G$ is about 10^{-4} , while the other constants are known up to 10^{-6} or better [1-4]. Despite the repeated suggestions of laboratory G measurements at the level of 10^{-5} not a single group has penetrated beyond 10^{-4}; moreover, three of the four best absolute G determinations are at variance with each other at their accuracy levels. There also exist some geophysical data on G which disagree with the laboratory ones [3].

Apparently suggestions to measure G and other gravitational interaction parameters in space, by precision tracking the motion of artificial bodies ([5,6] and others), are more promising: one can avoid environmental influences difficult to account for and create such conditions that a particle be not subject to forces much greater than those under study.

The approach of Ref.6 is to study the relative motion of two bodies on board a drag-free Earth's satellite using the horseshoe type trajectories [7]: the lighter body ("particle"), moving along a lower orbit than the heavier one ("shepherd"), overtakes it and, due to their gravitational interaction, gains energy, passes to a higher orbit and begins to lag behind (the Satellite Energy Exchange, or SEE method). The interaction phase can be studied within a drag-free capsule (a cylinder 20 m long, about 1 m in diameter) where the particle can remain as long as 10^5 seconds. By [6], particle trajectory measurements enable one to improve the existing knowledge of G by 2 orders of magnitude. Moreover, the 5th force parameter α for a certain range of interaction lengths λ and the possible equivalence principle (EP) violation parameter (the Eötvös parameter η) can be also measured with an unprecedented accuracy. Ref.[6] contains a number of details of the proposed experiment, in particular, it is shown that optimum orbital heights H range from 1390 to 3330 km.

We have carried out a further study of the SEE method. As compared with [6], a wider range of particle trajectories has been investigated, various sources of error have been studied and some new estimates concerning the capabilities of the method have been obtained. The results are as follows.

1. The particle motion is governed by tidal and inertial forces and by interaction with the shepherd. Estimates of influence of different factors on particle motion are given in Table 1. The upper bounds of displacements are estimated as $\delta l = at^2/2$ assuming that an acceleration a acts in the same direction for the time t (either 10^4 s, or half orbital period, i.e., about 1 hour, for external tidal forces whose influence is actually periodical). For definiteness we assumed that the orbital radius is $a = 8000$ km and the particle-shepherd distance is 10 m (half length of the capsule). The value of δl is of particular interest since it is the particle position that is actually measured.

Table 1
Contributions to particle motion dynamics

Factor	Acceleration created (cm/s^2)	Resulting displacement for $t \sim 10^4$s
1. Quadrupole tidal forces	$\sim 10^{-8}$	~ 10
2. Higher geopotential harmonics	$\sim 10^{-12}$	$\sim 10^{-4}$
3. Solar tides	$\sim 7 \cdot 10^{-11}$	$\sim 3 \cdot 10^{-4}$
4. Lunar tides	$\sim 3 \cdot 10^{-10}$	$\sim 10^{-3}$
5. Jovian tides	$\sim 5 \cdot 10^{-16}$	$\sim 2 \cdot 10^{-8}$
6. Lunar nonsphericity	$\sim 5 \cdot 10^{-18}$	$\sim 2 \cdot 10^{-10}$
7. Relativistic tides	$\sim 10^{-12}$	$\sim 3 \cdot 10^{-5}$
8. Uncertainty of shepherd's orbit	$\sim 3 \cdot 10^{-13}$	$\sim 10^{-5}$
9. Possible EP violation ($\eta = 10^{-13}$)	$\sim 7 \cdot 10^{-11}$	$\sim 3 \cdot 10^{-3}$

Assuming that the measurement error is no less than 10^{-6} cm (about 1/50 of the visible light wavelengths), the factors 5 and 6 from the table are manifestly negligible, like many others of similar origin. The factors 2,3,4,7 are to be included in the computer routine of an actual experiment but can be neglected at the planning stage aimed at working out the experiment strategy.

Effects changing the satellite orbit are not included since the actual orbit is assumed to be known from radar or laser measurements. However, the corresponding (possibly systematic) error implies tidal acceleration uncertainties as reflected in line 8 of the table. One has to conclude that this uncertainty is a key factor for the experiment viability since a better accuracy than that to $\Delta R \sim 1$ cm is not expected in the coming years and even 1 cm is questionable. On the other hand, it makes no sense to measure particle positions up to a certain δl for such a period t that the above uncertainty is greater than δl. For instance, if $\Delta R = 1$ cm and $\delta l = 10^{-6}$ cm, a particle trajectory measurement should not last longer than ~ 3000 s ~ 1 hour.

2. We considered the equations of particle motion with respect to the shepherd for arbitrary satellite orbits and arbitrary capsule orientations, including linear and quadratic terms in the ratio s/R where s is the shepherd-particle distance and R is the shepherd's separation from the Earth's centre, which provided the required calculation accuracy. It has proved to be impossible to find even approximate analytic solutions, even for the simplest situation of particle motion in the plane of a circular orbit of the shepherd in the spherically symmetric Newtonian field of the Earth when the equations are

$$\ddot{x} - 2\omega\dot{y} = 3\omega^2 xy/a + (M+m)(x/s)dU/ds \tag{1}$$

$$\ddot{y} + 2\omega\dot{x} = 3\omega^2 y + 3\omega^2(x^2 - 2y^2)/(2a) + (M+m)(y/s)dU/ds. \tag{2}$$

Here x is a backward along-track coordinate, y is directed from the Earth along the geocentric radius vector and $\omega = (GM_E/a^3)^{1/2}$ is the orbital frequency (M_E is the Earth's mass). The potential $U(s)$ can include, along with the Newtonian term G/s, the 5th force potential $(G\alpha/s)\exp(-s/\lambda)$ or several terms of this sort.

A possible EP violation at distances of the order of the Earth's radius leads to emergence of an additional term of the form $-\eta\omega^2 a$ at the right-hand side of (2).

Elliptic satellite orbits and (or) inclusion of the Earth's quadrupole gravitational potential lead to certain complications in the equations of motion.

3. In our computer simulations we solved the particle equations of motion for the following shepherd orbits in the Earth's Newtonian gravitational field: (i) circular in spherical field (Eqs. (1) and (2)); (ii) circular equatorial, in spherical plus quadrupole field; (iii) elliptic with eccentricities up to 0.05 in spherical field. The rational extrapolation method was used, with a variable integration step and accuracy control. In some cases parallel calculations were performed by the Runge-Kutta method, by the 5th order Adams method and by calculations with time reversal (from the finish to the start of the same trajectory). It was concluded that the computational error was within 10^{-10} cm, far beyond the achievable measurement accuracies.

4. Part of the simulations used the so-called standard initial data (SID), i.e., those corresponding to particle motion along a nearby circular orbit, or, in case (iii), an elliptic one with the same eccentricity.

Typical families of trajectories for the case (i) with SID are shown in Fig.1 for $H = 1500$ km. As expected, the paths are approximately U-shaped and the travel times are about 10^5 s for initial separation $x_0 \approx 18$ m and depend on H and initial particle position. The U-shaped paths exist in a narrow range of "impact parameters" y_0 connected with the natural length scale along the y axis, the separation Δ between the libration points L_1 and L_2 (unstable equilibrium points situated "over" and "under" the shepherd):

$$|y_0| \leq \Delta \approx 2a[G(M+m)/(3GM_E)]^{1/3}. \tag{3}$$

The trajectories are slightly asymmetric: the lower half is nearly straight while the upper one contains a significant sinusoidal component with the shepherd's orbital period and the amplitudes $a_{\sin}$ depending on H and the initial data. Thus, for $H = 1500$ km, $x_0 = 18$ m, $y_0 = -25$ cm the amplitude $a_{\sin}$ is about 2 mm. The x_0 and y_0 dependence of $a_{\sin}$ shows that the origin of the oscillatory component can be connected with the nature of SID as "switching on" the shepherd-particle interaction at the starting position. Such a "cutoff" should result in a path different from a perfect horseshoe orbit near its turning point.

Simulations with different initial velocities v_{x0} confirm this conclusion: for v_{x0} faintly different from SID the value $a_{\sin}$ varies. The oscillations can occur at one or both branches of the trajectory; for v_{x0} smaller than at SID they exist only at the lower branch. Larger deflections from SID lead to larger $a_{\sin}$; for sufficiently large $|v_{x0}|$ the trajectories contain loops.

The initial velocity range providing a sufficiently long particle travel within the capsule, is rather narrow and depends on H and y_0. In particular, for $H = 1500$ km and $y_0 = -25$ cm the allowed initial velocity values are

$v_{y0} < 0.025$ cm/s, -0.0425 cm/s $< v_{x0} < -0.028$ cm/s.

The trajectories proved to be stable under variations of the initial position (x_0, y_0).

5. Trajectory dependences on the values of G (the product GM_E, known with a good accuracy, remaining invariable), the 5th force parameter α for λ of the order of meters, and possible EP violation (η) have been studied. As the variations $\delta x(t)$ turned out to be significantly greater than $\delta y(t)$, we speak only of δx. The main results are:

(a) $\delta x(\delta G)$ and $\delta x(\delta\alpha)$ grow with growing H: they are approximately doubled when $H = 1500$ km is changed for $H = 3000$ km.

(b) y_0-dependence: $\delta x(\delta G)$ and $\delta x(\delta\alpha)$ are the greatest for $y_0 \approx -(1/3)\Delta(\approx -18$ cm for $H = 3000$ km).

(c) For U-shaped trajectories $\delta x(\delta G)$ is the greatest near the turning point.

(d) For looped trajectories the maximum values of $\delta x(\delta G)$ are about an order of magnitude greater and those of $\delta x(\delta\alpha)$ are nearly tripled as compared with the U-shaped paths; the dependence $\delta x(\eta)$ remains practically the same. Thus in general the looped trajectories are more promising from the experimental viewpoint.

(e) Numerically, the maximum variations δx are:

$\sim 10^{-3}$ cm for $\delta G/G \sim 10^{-6}$,

$\sim 5 \cdot 10^{-3}$ cm for $\delta\alpha \sim 10^{-5}$ ($\lambda \sim 1$ m),

$\sim 2 \cdot 10^{-3}$ cm for $\eta \sim 10^{-14}$.

These estimates confirm the viability of the proposed experiment.

(f) The variations δx behave both qualitatively and quantitatively different at different parts of the trajectories under variations of G, α and η, allowing one to hope that these effects can be separated in an actual experiment.

6. It has been found that the quadrupole component of the Earth's potential causes a common displacement of the trajectories within about 12 cm (for $y_0 = -25$ cm and $H = 1500$ km) while all the effects connected with G, α and η variations remain practically the same as those with the purely spherical potential.

7. The above basic features of particle motion are preserved when the shepherd moves along elliptic orbits with small eccentricities e but some new features appear.

With nonzero e the sinusoidal component of particle trajectories becomes unavoidable and $a_{\sin}$ grows with growing e; when $e > 0.01$, loops inevitably appear. As before, $a_{\sin}$ grows when v_{x0} deflects from SID: loops either appear or increase in number.

Unlike the circular orbit case, the loops become tilted and (of possible interest for an actual experiment) increased v_{x0} lead to trajectory squeezing in the y direction, providing its confinement inside the capsule and creating a hope to use orbits with high eccentricities. However, simultaneously the turning points of the trajectories become remoter from the shepherd. The sensitivity of trajectories under gravitational interaction parameter variations are practically the same as that for circular orbits of the shepherd.

8. Among the possible sources of error, we examined shepherd nonsphericity and inhomogeneity by using multipole expansions of its gravitational field. We concluded that for a measurement of G up to 1 ppm the shepherd nonsphericity $\delta R_0/R_0$ (R_0 being its radius) should not exceed 80 ppm, or about $1.6 \cdot 10^{-3}$ cm. Large-scale density inhomogeneities (of the order of R_0) must be within $1.5 \cdot 10^{-3}$ and small-scale ones (smaller that $R_0/10$) within 0.07. All these requirements are easily met by modern technology.

9. Particle trajectory measurements are carried out with respect to capsule walls where the instruments are placed. The capsule and other bodies are sources of many

sorts of noise, including fundamentally unavoidable, like thermal ones, which thus restrict the measurement accuracy. We considered the following basic sources of thermal noise:

(a) radial oscillations of the shepherd's surface;

(b) longitudinal oscillations of the capsule;

(c) transversal oscillations of the capsule.

Spectral analysis of thermal noises with the aid of the fluctuation-dissipation theorem [8] has shown that the maximum noise-induced measurement error does not exceed $2.5 \cdot 10^{-12}$ cm, much smaller than the expected measurement error.

10. The gravitational constant measurement procedure was modelled for U-shaped trajectories by three methods of G determination with the aid of Eqs.(1, 2): (i) the differential method, directly using the equations, (ii) the two-point method, employing their first integral, and (iii) the integral method, comparing an empirical trajectory with a calculated one and fitting them by varying G.

The first method has the advantage of measuring G at any small part of the trajectory, irrespective of the initial data, to obtain a large set of independent estimates and to use averaging methods to improve their accuracy. Its shortages are connected with relatively low accuracies with which accelerations and velocities can be determined. Thus, if lengths are measured up to 10^{-6} cm, the error is $\delta G/G \sim 3 \cdot 10^{-5}$.

11. The two-point method employs the first integral of Eqs.(1, 2)

$$\dot{x}^2 + \dot{y}^2 - \frac{2G}{s}(M+m) - 3\omega^2 y^2 + \frac{\omega^2}{a} y(2y^2 - 3x^2) = \text{const.} \tag{4}$$

The constant G is estimated by two points with known coordinate and velocity values, for instance, the starting and turning points. In the latter the velocity v and the coordinate y are zero, thus removing two sources of error.

An analysis shows that G is best of all found from a set of independent estimates in the vicinity of the turning point. The achievable accuracy at the best trajectories (those with the turning point at 1.55-2.35 m from the shepherd) is to $\delta G/G \approx 4 \cdot 10^{-6}$ if lenghts are measured up to $\delta l \sim 10^{-6}$ cm.

12. In the integral method, the most powerful one, G is evaluated from the minimum of the functional

$$S(G) = \sum_{k=1}^{n} [(x_k^{\mathrm{e}} - x_k)^2 + (y_k^{\mathrm{e}} - y_k)^2] \tag{5}$$

measuring a "distance" between the two trajectories: the calculated one, $\{x(t), y(t)\}$, with a prescribed value of G taken for true, and an "empirical" one, $\{x^{\mathrm{e}}(t), y^{\mathrm{e}}(t)\}$, with a Gaussian noise corresponding to the measurement error δl inserted at all "observation" points separated by equal time intervals Δt. This enabled us to estimate the bias $(6 \cdot 10^{-9})$ and random $(4 \cdot 10^{-8})$ errors $\delta G/G$ (at best) for $\delta l = 10^{-6}$ cm.

At the present stage of the study the achievable G determination accuracy by the integral method can be estimated as $\delta G/G \sim 10^{-7}$ for $\delta l \sim 10^{-6}$ cm and $\delta G/G \sim 10^{-6}$ for $\delta l \sim 10^{-4}$ cm.

The latter estimate is of particular significance due to the orbit uncertainty effect (see Table 1): evidently one can measure G within 10^{-6} by tracking either small segments of particle trajectories for times ~ 1 hour with $\delta l \sim 10^{-6}$ cm, or larger segments for times ~ 10 hours with $\delta l \sim 10^{-4}$ cm.

Both the two-point and integral methods admit improvements of the experimental data processing algorithms. In particular, in the integral method the bias error can be in principle entirely eliminated.

A general conclusion is that the SEE experiment, if realized, can improve our present knowledge of G, α (for certain λ) and η by at least two orders of magnitude.

More details are presented in a series of papers to appear in Izmeritelnaya Tekhnika (Russia) [9]. An alternative class of particle trajectories (elliptic and hyperbolic ones near the libration points over and under the shepherd) is analyzed in Ref.[10] (see also preprint CBPF-NF-022/93).

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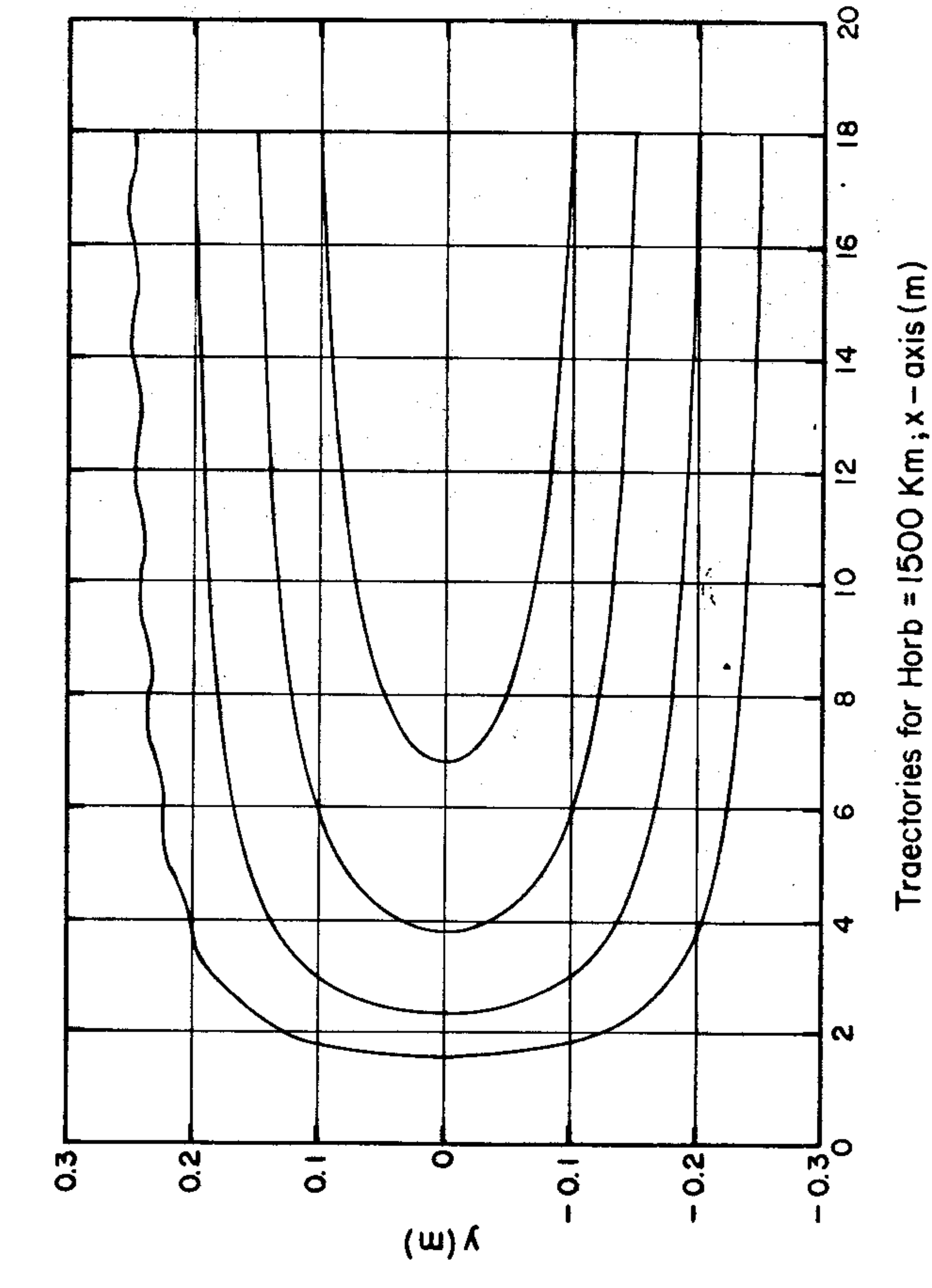
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y (m)
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2
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6
8
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14
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18
20
Traectories for Horb = 1500 Km ; x – axis (m)

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