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QUANTIZATION OF SELF-DUAL FIELD REVISITED

by

Prem P. SRIVASTAVA

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

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ABSTRACT:

Self-dual field is described by the Lagrangian for ordinary scalar field with a term added to it to take care of the self-duality constraint. A self-consistent Hamiltonian formulation is obtained using Dirac's method. The constraints are second class, the auxiliary field drops out of the Hamiltonian and the quantized theory does not show any violation of causality .

Key-words: Chiral boson; Quantization; Gauge theory.

Self-dual fields in two dimensions, sometimes called chiral bosons, are basic ingredients in the formulation of the Heterotic string[1]. The quantization of scalar self-dual field has been much discussed recently[2,3]. Siegel's theory[4] with a local symmetry seems to be equivalent[5] to the dimension zero field formulation of Floreanini and Jackiw[2]. However, the Euler-Lagrange eqns. for this field lead to the result that it is the space derivative of the field which satisfies the self-duality condition and not the field itself. The Hamilton equations on the other hand being linear do result in self-duality condition for the field. Moreover, this field violates microcausality postulate[6]. No completely satisfactory quantized theory of self-dual field seems available[2,3,6].

We propose here to study the quantization by Dirac's method[7] of the self-dual field described by the Lagrangian for ordinary scalar field with a term added to it to take care of the self-duality constraint. The motivation for such a study derives from an analogous situation in Yang-Mills theory[8]: The time component of the vector potential A^o appears here as an auxiliary (Lagrange multiplier) field. If we decide to choose the gauge $A^o=0$ before varying the action we miss the first class Gauss' law constraint. The Lagrange eqns. of motion do, however, lead to the vanishing of time derivative of this constraint and we are required to impose an appropriate boundry condition to work in the right sector. Keeping A^o term allows us to derive Gauss' law constraint from the Lagrangian and a self-consistent Hamiltonian formulation is obtained by following Dirac's procedure where if we wish we may eliminate[8] A^o by a choice of gauge. For the action of scalar self-dual field proposed here a similar situation is obtained except for that the constraints are second class and the auxiliary field is removed from the reduced Hamiltonian via Dirac bracket. We show that a self-consistent Hamiltonian formulation can be developed and no violation of causality in the quantized theory occurs.

The second order Lagrangian for scalar self-dual field ϕ with a quadratic term for it will be taken to be

$$\mathcal{L} = (1/2)(\partial^{\mu}\phi)(\partial_{\mu}\phi) + \lambda_{\mu}(\epsilon^{\mu\nu} + \eta^{\mu\nu})\partial_{\nu}\phi \tag{1}$$

The auxiliary vector field λ_{μ} appears linearly and the resulting Lagrange eqns. of motion are

$$\partial^{\mu}\partial_{\mu}\phi + (\epsilon^{\mu\nu} + \eta^{\mu\nu})\partial_{\nu}\lambda_{\mu} = 0 \tag{2}$$

$$(\epsilon^{\mu\nu} + \eta^{\mu\nu})\partial_{\nu}\phi = 0 \tag{3}$$

From (3) we derive $\partial^{\mu}\partial_{\mu}\phi = 0$ and consequently from (2) we obtain $(\epsilon^{\mu\nu} + \eta^{\mu\nu})\partial_{\nu}\lambda_{\mu} = 0$, which does not lead to $\partial^{\mu}\partial_{\mu}\lambda_{\nu} = 0$ for *all* the components of λ_{μ}. It is convenient, without any loss of generality, to rewrite (1) as#, $(\lambda = \lambda_0 + \lambda_1)$,

$$\mathcal{L} = (1/2)[(\partial_0\phi)^2 - (\partial_1\phi)^2] + \lambda(\partial_0 - \partial_1)\phi \tag{4}$$

The eqns. of motion then read as $(\partial_0-\partial_1)\phi=0$, $(\partial_0-\partial_1)\lambda=0$ etc. and they are decoupled. No eqn. for $(\lambda_0 - \lambda_1)$ is obtained. Denoting by p_λ and $\Pi\equiv\Pi^0=\partial_0\phi+\lambda$ (where $\Pi^{\nu}=\partial\mathcal{L}/\partial(\partial_{\nu}\phi)$) the canonical momenta corresponding to λ and ϕ respectively, the primary constraint is $p_\lambda \approx 0$. The canonical Hamiltonian is obtained to be

$$\mathcal{H}_c = (1/2)(\Pi-\lambda)^2 + (1/2)(\partial_1\phi)^2 + \lambda\,\partial_1\phi \tag{5}$$

On requiring the persistency in time of the primary constraint the secondary constraint follows to be $\Phi \equiv \Pi-\partial_1\phi-\lambda \approx 0$ and the extended Hamiltonian $\mathcal{H}' = \mathcal{H}_c + u\, p_\lambda + v\,\Phi$, where u and v are arbitrary functionals, gives rise to $d\Phi/dt \equiv \{\Phi, \mathcal{H}'\} = \partial_1\lambda - (u - 2\partial_1 v)$ which allows us to assure through an appropriate choice of u and v that the constraint Φ is preserved in time and no additional constraints arise in the theory. The two constraints are second class as is evident from their Poisson brackets

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$\eta^{\mu\nu}=\mathrm{diag}(1,-1)$, $\epsilon^{01}=-1$ and under Lorentz transformation $\lambda=\lambda_+\equiv(\lambda_0+\lambda_1)\to e^{\beta}\lambda_+$, $\lambda_-\equiv(\lambda_0-\lambda_1)\to e^{-\beta}\lambda_-$.

$$\{\phi(x,t),\ \phi(y,t)\} = -2\,\partial_x\delta(x-y)\ ,\qquad \{p_\lambda(x,t),\ p_\lambda(y,t)\} = 0$$

$$\{\phi(x,t),\ p_\lambda(y,t)\} = -\delta(x-y) \qquad (6)$$

The Dirac bracket with respect to these constraints is easily found to be

$$\{f(x,t),g(y,t)\}^* = \{f,g\} + 2\int\int dz dz'\,\partial_z\delta(z-z')\ \{f,p_\lambda(z,t)\}$$

$$\{p_\lambda(z',t),g\} + \int dz\ [\{f,p_\lambda(z,t)\}\ \{\phi(z,t),g\} - (\phi\leftrightarrow p_\lambda)] \qquad (7)$$

and we can implement the weak (second class constraints) now as strong relations, e.g., $p_\lambda = 0$ and $\Pi = \partial_1\phi+\lambda$, e.g., $\Pi_- = \Pi_0 - \Pi_1 = 0$.

The Dirac brackets for the self-dual field are found to coincide with the standard Poisson brackets

$$\{\phi(x,t),\Pi(y,t)\}^* = \delta(x-y)$$

$$\{\phi(x,t),\phi(y,t)\}^* = \{\Pi(x,t),\Pi(y,t)\}^* = 0 \qquad (8)$$

The reduced Hamiltonian is found to be

$$\mathcal{H} = \Pi\ \partial_1\phi \qquad (9)$$

which leads to the eqns. of motion $\partial_0\phi = \partial_1\phi$, $\partial_0\Pi = \partial_1\Pi$ on using (8). These are consistent with the Lagrangian formulation and no problem with the causality arises on performing the canonical quantization , $\{f,g\}^* \to -i[f_{op},g_{op}]$. The Lagrangian in the first order formulation may then be written as ($\Pi_- = 0$)

$$\mathcal{L} = (1/2)\Pi_+\partial_-\phi$$

$$= (1/2)\Pi_\mu(\eta^{\mu\nu}+\varepsilon^{\mu\nu})\partial_\nu\phi \qquad (10)$$

which is also conformal invariant. For the anti-self-dual field satisfying $\partial_0\phi=-\partial_1\phi$ we find $L=(1/2)\Pi_-\partial_+\phi$. The action for the ordinary field may not in general be written as the sum of the actions of these two self-dual fields. In the prescription of canonical quantization[7] operator ordering and hermiticity of quantized operators must be taken care of. We may alternatively use path integral formalism due to Batalin-Fradkin-Vilkowski[9] for theory with second class constraints. It is also posible to introduce[10] a Wess-Zumino term to the action (1) so that the undesired mode is cancelled and we obtain a gauge theory with only first class constraints. These discussion may also be extended[10] to some Kähler manifolds as well.

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