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ON THE DIFFUSION OF THE HADRONIC COMPONENT OF THE COSMIC RAYS AND SEMI-GROUPS

by

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Abstract: Semigroup theory is used to integrate the diffusion of hadrons in the atmosphere.

Key-words: Semi-groups; Hadronic diffusion; Atmosphere.

"It is a pleasure to recognise old things of a new point of view"

R. Feinmann

The diffusion of an hadronic component of the cosmic radiation in the atmosphere can be described approximately by some special case of the following equation

$$\frac{\partial F(x,E)}{\partial x} = - \frac{F(x,E)}{\lambda(E)} + \int_0^1 \frac{F(\eta)}{\eta} \frac{F(x,E/\eta)}{\lambda(E/\eta)} d\eta + P(x,E) \qquad (1)$$

with the prescribed value $F(0,E)$, for $x = 0$.

1) The nucleonic component corresponds to the homogeneous equation which results from (1) putting $P(x,E) = 0$ and $F(0,E)dE = G(E)dE$ which is the differential spectrum of the primary cosmic nucleons at the top of the atmosphere (mostly protons).

2) The distribution of the pionic charged component of the first generation (pions produced by the nucleonic component) is described by equation (1) where the function $P(x,E)$ is supposed to be known and depends on the model assumed to describe the pion's production. In this case $F(0,E) = 0$.

The function $F(x,E)$ represents the flux (per cm^2.ster.s) of the hadronic component, at the atmospheric depth $x(g/cm^2)$, with energy in the range E, E+dE, $f(\eta)d\eta$ is the distribution of the elasticity and $\lambda(E)$ is the mean interaction length in (g/cm^2) of the hadron's interaction with an atomic nucleus of a chemical component of the atmosphere. The method that has been used to solve eq. (1) is the method of the Mellin transform, following the example of Landau and Rumer [1] who used it to integrate the differential equation of the electromagnetic cascade of the cosmic radiation.

The solution thus obtained is represented by a contour integral (in the complex domain) which only in very fez particular cases can be evaluated excatly and some approximate method must be used to estimate it as for example the saddle point method.

About ten years ago we have [2] introduced a symbolic method to deal with a special case of equation (1) assuming $\lambda(E) = \lambda_0$ and $\eta = \eta_0$ to be constants independent of E. For this purpose we defined an operator

$$\sigma_\eta F(x,E) = \frac{1}{\eta} F(x,E/\eta) \qquad \eta \geq \eta_{min} > 0 \qquad (2)$$

for $x \geq 0$ which operates only on E and has for dominium and range the set of positive functions $F(x,E)$, bounded and continuous respect to E, in the interval $0 < a \leq E < \infty$, and such that the integral

$$\int_E^\infty F(0,E)dE$$

exists.

If we suppose that $1/\lambda(E)$ belongs to the dominium of σ_η and we introduce this operator in the equation (1) it is reduced to

$$\frac{\partial F(x,E)}{\partial x} = A\,F(x,E) + P(x,E) \tag{3}$$

where the operator A

$$A = \left(-1 + \int_0^1 f(\eta)\,d\eta\sigma_\eta\right)\frac{1}{\lambda(E)} \tag{4}$$

is independent of x.

Provided A is bounded the solution of the equation (1) is

$$F(X,E) = e^{xA}\,F(0,E) + \int_0^x e^{(x-\xi)A}\,P(\xi,E)\,d\xi \tag{5}$$

The operators e^{xA}, for $x \geq 0$, are the elements of a semi-group $G_x = e^{xA}$, which is

$$G_x G_y = G(x+y) \qquad x,y \geq 0$$
$$G(0) = I$$

I is the identity operator and A is the generator of the semi--group.

It is interesting to note that the operators σ_α satisfy the following relations

$$\sigma_\alpha \sigma_\beta = \sigma_{\alpha\beta} \qquad \alpha,\beta > 0$$
$$\sigma_1 = I$$

The operator A is the sum of two operators

$$\alpha = -\frac{1}{\lambda(E)} \quad \text{and} \quad \beta = \left(\int_0^1 F(\eta)\,d\eta\sigma_\eta\right)\frac{1}{\lambda(E)}$$

If λ is a constant independent of E, the operators α and β commute and we can write

$$e^{\alpha+\beta} = e^{\alpha}.e^{\beta}$$

but when n depends on E, α and β do not commute and

$$e^{\alpha+\beta} \neq e^{\alpha}e^{\beta}$$

Therefore in this case we must consider the order of the factors in the development of the exponential

$$e^{xA} = e^{x(\alpha+\beta)} = \sum_{n=0}^{\infty} \frac{x^n}{n!} (\alpha+\beta)^{(n)}$$

The theory of semi-groups has been succesfully applied to the integration of certain equations of Physics such as the Schrödinger equation and the heat conduction equation [3]. We show here that the case of the diffusion equation of the hadronic components of the cosmic radiation is another example.

<u>Note</u> - A similar formal operator A was used recently by Bellandi et al.[4] to deal with the nucleonic component.

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