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A NEW SHORT RANGE GRAVITATIONAL FORCE IN THE LEPTONIC WORLD?

by

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Abstract

We use Grishchuk's approach to introduce Gravitation via a rank-two symmetric tensor field in a flat space-time. We then examine the consequences of a non-identical coupling of gravity with electrons and neutrinos. We show that a local counterpart of the gravitational interaction must occur, mediated by massive spin-two tensor bosons. A universal Higgs mechanism thus determine the mass of these bosons.

Key-words: Gravitational interaction; New short range force; Leptonic world.

1. Introduction

One of the most interesting advance in general relativity in the last years has been given in the Grishchuk et al[1] paper. These authors have shown that it is completely equivalent to treat Einstein's General Relativity in terms of a rank two symmetric tensor field $\varphi^{\mu\nu}$ in an arbitrary space-time manifold. Such a $\varphi^{\mu\nu}$ tensor field has the same properties of all usual standard physical fields approached in the ordinary Minskowski space-time.

In the first part of this paper we give a short review of the main features of the GPP (Grishchuk, Petrov, Popova)[1] approach as to convince that the $\varphi^{\mu\nu}$ tensor field may lead to the usual equations of General Relativity. We shall give the bridge formula which allows to pass from $\varphi^{\mu\nu}$ variables to Einstein's geometric ones.

In the second part we use this formulation to couple Gravity to the Fermionic world. For the sake of simplicity we restrict ourselves to leptons. Actually we deal only with the first generation, the electron and its corresponding neutrino, the generalization for other leptons being in principle straightforward.

Leptons interact with photons, the intermediate vector bosons of the Weak Interaction and Gravity. We use the Standard Model of the $SU(2)_L \times U(1)$ gauge theory to describe the Electroweak Interaction. In order to preserve the symmetry properties of such a theory we are thus led to introduce new charged and neutral spin two fields which are the companion in the $SU(2)$ scheme of the gravitational field.

In a quite natural way it appears that such fields become massive after a spontaneous symmetry breaking via a Higgs mechanism. Such a feature will be described in the third part of our paper.

The overview of our paper is that the only different coupling of electron and neutrino to Gravity sets in evidence the existence of a new short range force which can be interpreted as the local counterpart of Gravity in the same way that the Weak force can be interpreted as the local counterpart of Electromagnetic interaction.

2. The GPP variables

Let us consider a symmetric rank two tensor field $\varphi^{\mu\nu}$ defined in a flat Minkowski Space-Time (MST). Let be $\gamma^{\mu\nu}$ the metric tensor of (MST) given in an arbitrary coordinate system. The covariant derivative of an arbitrary vector A^μ is defined in the standard way

$$A^{\mu}{}_{;\nu} = A^{\mu}{}_{,\nu} + \gamma^{\mu}_{\epsilon\nu}\ A^{\epsilon} \tag{1}$$

where

$$\gamma^{\mu}_{\epsilon\nu} = \frac{1}{2}\,\gamma^{\mu\alpha}\,\left(\gamma_{\alpha\epsilon,\nu} + \gamma_{\alpha\nu,\epsilon} - \gamma_{\epsilon\nu,\alpha}\right) \tag{2}$$

is the usual Christoffel symbol.

As we do work with a flat space-time we could choose a coordinate system in which the $\gamma^{\alpha}_{\mu\nu}$ Christoffel symbol all vanish. However we decide to work in an arbitrary coordinate system in order to preserve the general covariance of the theory.

The gravitational field is represented by $\varphi^{\mu\nu}$ and for reasons which will be clear later on, we deal with a functional $K^{\alpha}_{\mu\nu}$ of $\varphi^{\mu\nu}$ which contains up to first order the derivatives of $\varphi^{\mu\nu}$. We then introduce a $\mathcal{L}g$ lagrangian

$$-2\sqrt{\kappa_E}\ \mathcal{L}g = \sqrt{-\gamma}\,\left(\gamma^{\mu\nu} + \sqrt{\kappa_E}\ \varphi^{\mu\nu}\right)\left[K^{\alpha}_{\mu\nu;\alpha} - K_{\mu;\nu} + K^{\alpha}_{\mu\nu}\ K_{\alpha} - K^{\alpha}_{\mu\beta}\ K^{\beta}_{\nu\alpha}\right] \tag{3}$$

where $\kappa_E = \frac{8\pi G}{c^4}$ is the Einstein's constant ($\kappa_E \simeq 10^{-37}\ GeV^{-2}$)

and

$$K^{\cdot\alpha}_{\mu\nu} = K^{\cdot\alpha}_{\nu\mu} \tag{4}$$

while

$$K_{\alpha} = K^{\lambda}_{\alpha\lambda} \tag{5}$$

Note that we have made the standard choice $dim\varphi^{\mu\nu} \equiv [\varphi^{\mu\nu}] = E^{1/2}\ L^{-1/2}$ and used natural units $\hbar = c = 1$.

For simplicity sake we define

$$(KK)_{\mu\nu} = K^{\alpha}_{\mu\nu}\ K_{\alpha} - K^{\alpha}_{\mu\beta}\ K^{\beta}_{\nu\alpha} \tag{6}$$

and the $\mathcal{L}g$ lagrangian now becomes

$$-2\sqrt{\kappa_E}\ \mathcal{L}g = \sqrt{-\gamma}\ (\gamma^{\mu\nu} + \sqrt{\kappa_E}\ \varphi^{\mu\nu})\ [K^{\alpha}_{\mu\nu;\alpha} - K_{\mu;\nu} + (KK)_{\mu\nu}] \tag{7}$$

We now follow Grishchuk et al[1] and consider independent variations of $\varphi^{\mu\nu}$ and $K^{\alpha}_{\mu\nu}$. The Euler-Lagrange dynamic equations for a $\varphi^{\mu\nu}$ variation give

$$K^{\alpha}_{\mu\nu;\alpha} - \frac{1}{2}\ (K_{\mu;\nu} + K_{\nu;\mu}) + (KK)_{\mu\nu} = 0 \tag{8}$$

while variation of $K^{\alpha}_{\mu\nu}$ give the functional relation between $K^{\alpha}_{\mu\nu}$ and $\varphi^{\mu\nu}$

$$\varphi^{\mu\nu}_{;\alpha} - (\gamma^{\mu\nu} + \varphi^{\mu\nu})\ K_{\alpha} + (\gamma^{\mu\lambda} + \varphi^{\mu\lambda})\ K^{\nu}_{\lambda\alpha} + (\gamma^{\nu\lambda} + \varphi^{\nu\lambda})\ K^{\mu}_{\alpha\lambda} = 0 \tag{9}$$

We extract K from this equation and bring it into (8) giving a rather complicated set of equations

$$\Box\ \varphi_{\mu\nu} - \varphi^{\tau}_{(\mu;\nu);\tau} + \varphi^{\alpha\beta}_{\ \ ;\alpha;\beta}\ \gamma_{\mu\nu} = -2\ (KK)_{\mu\nu} + (KK)\ \gamma_{\mu\nu} + 2Q^{\lambda}_{\mu\nu;\lambda} \tag{10}$$

in which $Q^{\lambda}_{\mu\nu}$ is given by

$$2Q^{\lambda}_{\mu\nu} = K_{\nu\epsilon\mu}\ \varphi^{\epsilon\lambda} + K_{\mu\epsilon\nu}\ \varphi^{\epsilon\lambda} - K_{\mu\epsilon}{}^{\lambda}\ \varphi^{\epsilon}{}_{\nu} - K_{\nu\epsilon}{}^{\lambda}\ \varphi^{\epsilon}{}_{\mu} + K^{\lambda}{}_{\epsilon\mu}\ \varphi^{\epsilon}{}_{\nu}$$

$$+K^{\lambda}_{\epsilon\nu}\ \varphi^{\epsilon}_{\mu} - K_{\mu}\ \varphi^{\lambda}_{\nu} - K_{\nu}\ \varphi^{\lambda}_{\mu} + K^{\lambda}\ \varphi_{\mu\nu} - K^{\lambda}{}_{\alpha\beta}\ \varphi^{\alpha\beta}\ \gamma_{\mu\nu} \tag{11}$$

These equations can now be interpreted in a beautiful and simple manner using Einstein's geometrical scheme.

It can be accomplished through the identification of a new metric tensor $g^{\mu\nu}$ defined in terms of $\gamma^{\mu\nu}$ and of the gravitational $\varphi^{\mu\nu}$ field by

$$\sqrt{-g}\ g^{\mu\nu} = \sqrt{-\gamma}\ (\gamma^{\mu\nu} + \sqrt{\kappa_E}\ \varphi^{\mu\nu}) \tag{12}$$

where $g = det\ g_{\mu\nu}$ and $\gamma = det\ \gamma_{\mu\nu}$.

We introduce the object $(g^{\mu\nu})^{-1}$ as the inverse of $g^{\mu\nu}$ by the definition relation

$$(g^{\mu\nu})^{-1}\ g^{\nu\lambda} = g_{\mu\nu}\ g^{\nu\lambda} = \delta^{\lambda}_{\mu} \tag{13}$$

and we shall stress that for all tensors lowering and rising of indices is made by the flat $\gamma^{\mu\nu}$ metric tensor. However with the given definition of $g^{\mu\nu}$ its inverse $(g^{\mu\nu})^{-1}$ is different from $\gamma_{\mu\alpha}\ \gamma_{\nu\lambda}\ g^{\alpha\lambda}$.

From now we will not write $(g^{\mu\nu})^{-1}$ but simply $g_{\mu\nu}$ and we define the $\Gamma^{\alpha}_{\mu\nu}$ Christoffel symbol associated with the $g^{\mu\nu}$ metric tensor in the standard way

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2}\ g^{\alpha\lambda}\ (g_{\mu\lambda,\nu} + g_{\nu\lambda,\mu} - g_{\mu\nu,\lambda}) \tag{14}$$

When one brings in that definition equation (12) one gets after a straightforward calculation

$$\Gamma^{\alpha}_{\mu\nu} = \gamma^{\alpha}_{\mu\nu} + \sqrt{\kappa_E}\ K^{\alpha}_{\mu\nu} \tag{15}$$

where

$$\sqrt{\kappa_E}\ K^{\alpha}_{\mu\nu} = \frac{1}{2}\ g^{\alpha\beta}\ (g_{\beta\mu;\nu} + g_{\beta\nu;\mu} - g_{\mu\nu;\beta}) \tag{16}$$

We then recognize in $K^{\alpha}_{\mu\nu}$ the tensor introduced in the $\mathcal{L}g$ lagrangian. We insist on the fact that even if $K^{\alpha}_{\mu\nu}$ is defined as the difference between two connections, it is a true tensor as shown on its definition (16).

We can now use definition (12) and properties (15) and (16) to get the Ricci curvature tensor associated to $g^{\mu\nu}$

$$\frac{1}{\sqrt{\kappa_E}}\ R_{\mu\nu} = K_{\mu;\nu} - K^{\alpha}_{\mu\nu;\alpha} - (KK)_{\mu\nu} \tag{17}$$

It is thus evident that the Euler-Lagrange equation (8) reads

$$R_{\mu\nu} = 0 \tag{18}$$

Moreover the $\mathcal{L}g$ lagrangian (7) is now expressible in terms of the curvature tensor

$$-2\kappa_E \ \mathcal{L}g = \sqrt{-g} \ R_{\mu\nu} \ g^{\mu\nu} \tag{19}$$

which shows the equivalence of the description of the gravitational field either in the coordinates $\varphi^{\mu\nu}$ or in Einstein's geometrical variables.

Now comes the question : which one of the $\gamma^{\mu\nu}$ or $g^{\mu\nu}$ metric are observable quantities? This depends on how matter couple to Gravity. Universality of Gravity implies that only $g^{\mu\nu}$ metric is the observable quantity while $\gamma^{\mu\nu}$ is a fictitious auxiliary metric.

In order to prove it, let us suppose that Φ^A represents all existing matter (A is a collective index) and the dynamics of which is given by the lagrangian $\mathcal{L}_M$ $(\Phi^A \ \Phi^A_{;\lambda})$ (we remind that the covariant derivative (;) is defined through the flat connection $\gamma^{\alpha}_{\mu\nu}$). In order to couple the field matter with gravity the rule extracted from (GPP) is to introduce the $g^{\mu\nu}$ tensor through a correspondence principle which modifies the metric $\gamma^{\mu\nu}$ and its associated Christoffel symbol $\gamma^{\alpha}_{\mu\nu}$ in terms of $g^{\mu\nu}$ and $K^{\alpha}_{\mu\nu}$

$$\sqrt{-\gamma} \ \gamma^{\mu\nu} \rightarrow \sqrt{-\gamma} \ (\gamma^{\mu\nu} + \sqrt{\kappa_E} \ \varphi^{\mu\nu}) = \sqrt{-g} \ g^{\mu\nu} \tag{20}$$

$$\gamma^{\alpha}_{\mu\nu} \rightarrow \gamma^{\alpha}_{\mu\nu} + \sqrt{\kappa_E} \ K^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\mu\nu} \tag{21}$$

It follows then that in presence of matter $R_{\mu\nu}$ has to be replaced by

$$R_{\mu\nu} = -\kappa_E \ (T_{\mu\nu} - \frac{1}{2} \ T \ g_{\mu\nu}) \tag{22}$$

which is nothing but Einstein's equation of motion in presence of matter.

We have thus achieved to show that it becomes completely equivalent to work with the only symmetric rank-two tensor $\varphi^{\mu\nu}$ in a fictitious flat espace-time background, or with the geometric variables $g^{\mu\nu}$ and $K^{\alpha}_{\mu\nu}$. Both choices lead to the same exact Einstein General Relativity Theory.

3. Gravitation in the leptonic world

Let us review briefly the Weinberg-Salam Standard theory of Electroweak interactions[2-5] since we shall use it as a guide to couple Gravity to leptonic matter.

The description of electroweak interactions induces us to treat electrons and neutrinos by means of an isodoublet

$$L = \frac{1}{2}\,(1-\gamma_5)\begin{pmatrix}\nu_e\\ e^-\end{pmatrix} \tag{23}$$

and a singlet

$$R = \frac{1}{2}\,(1+\gamma_5)\,e^- \tag{24}$$

where γ_5 represents the usual Dirac's matrix $\gamma_5 = i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3$.

The gauge group $SU(2)_L \times U(1)$ beeing a local symmetry group it follows that the usual derivatives ∂_μ have to be replaced by the covariant derivatives

$$D_\mu = \partial_\mu - ig\,\frac{\vec{\tau}}{2}.\vec{W}_\mu - \frac{i}{2}\,g'\,B_\mu \tag{25}$$

where $\vec{W}_\mu$ and B_μ are the gauge fields (connection of the symmetry group).

The dynamics of these fields is given by the Standard Electroweak theory through the $\mathcal{L}_{GSW}$ lagrangian

$$\mathcal{L}_{GSW} = i\,[\overline{L}\,\gamma_\mu\,D_\nu\,L\,\gamma^{\mu\nu} + \overline{R}\,\gamma_\mu\,D_\nu\,R\,\gamma^{\mu\nu}] - \frac{1}{4}\,B_{\mu\nu}\,B^{\mu\nu} - \frac{1}{4}\,\vec{W}_{\mu\nu}.\vec{W}^{\mu\nu} \tag{26}$$

where

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu\,B_\nu - \partial_\nu\,B_\mu \\ W_{\mu\nu} &= \partial_\mu\,W_\nu - \partial_\nu\,W_\mu + g\,\vec{W}_\mu \wedge \vec{W}_\nu \end{aligned} \tag{27}$$

and as throughout this paper, $\gamma^{\mu\nu}$ represents the flat Minkowski metric written in an arbitrary coordinate system.

In order to set altogether the interactions of leptons with the fields in which they are immersed we have first to schwitch on gravity.

The standard way to couple gravity $\Phi^{\mu\nu}$ with leptons is through their energy-momentum tensor $T_{\alpha\beta}(\ell)$ by means of the lagrangian

$$\mathcal{L}_{int} = \sqrt{\kappa_E}\; T_{\alpha\beta}(\ell)\; \Phi^{\alpha\beta} \tag{28}$$

or after separation of leptons into electron and neutrino

$$\mathcal{L}_{int} = \sqrt{\kappa_E}\; (T_{\alpha\beta}(e) + T_{\alpha\beta}(\nu))\; \Phi^{\alpha\beta} \tag{29}$$

For a given fermion f we recall that

$$T_{\alpha\beta}(f) = \overline{f}\; \gamma_{(\mu}\; D_{\nu)}\; f + h.c. = \overline{f}\; (\gamma_\mu\; D_\nu + \gamma_\nu\; D_\mu)\; f + h.c. \tag{30}$$

so that one obtains for the energy-momentum tensor of leptons

$$T_{\alpha\beta}(e) + T_{\alpha\beta}(\nu) = \overline{L}\; \gamma_{(\alpha}\; D_{\beta)}\; L + \overline{R}\; \gamma_{(\alpha}\; D_{\beta)}\; R + h.c. \tag{31}$$

For the moment, there is no evidence that the coupling of Gravity with electron and neutrino are exactly identical.

To let such a question open, let us introduce a ξ weight between the two energy-momentum tensors

$$\mathcal{L}_{int} = \sqrt{\kappa_E}\; (T_{\alpha\beta}(e) + \xi\; T_{\alpha\beta}(\nu))\; \Phi^{\alpha\beta} \tag{32}$$

It is clear that ξ must be close to unity since the observations of the SN1987 A Supernovae[6] has given $\xi = 1 + 0\ (< 10^{-3})$.

After some simple algebraic manipulations one can show that

$$T_{\alpha\beta}(e) + \xi\ T_{\alpha\beta}(\nu) = \frac{\xi+1}{2}\ \overline{L}\gamma_{(\alpha}\ D_{\beta)}\ L + \overline{R}\ \gamma_{(\alpha}\ D_{\beta)}\ R + \frac{\xi-1}{2}\ \overline{L}\ \gamma_{(\alpha}\ D_{\beta)}\ \tau_3\ L + h.c. \quad (33)$$

The important fact is that a τ_3 operator occurs now. To complete the algebra we have to introduce the charged tensor currents $\overline{L}\ \gamma_\mu\ D_\nu\ \tau^\pm\ L$. Due to charge conservation these new currents do not couple directly to gravity.

In order to preserve the $SU(2)_L \times U(1)$ gauge symmetry even in presence of tensorial coupling, let us introduce a triplet of tensors $\varphi^{(i)}_{\mu\nu}$ and a singlet $\Psi_{\mu\nu}$. Now leptons L and R couple to the intermediate vector bosons $\varphi^{(i)}_{\mu\nu}$ and $\Psi_{\mu\nu}$. These tensor bosons are true vectors in the $SU(2)$ algebra and not connections. In this sense they are not gauge fields, so that we can note that gravity in our approach is not treated as a gauge field. By the way, there is no need to treat gravity as a gauge field.

As we have shown in the previous section the interaction lagrangian with the gravity can be written using the flat space time tensor field $\Phi^{\mu\nu}$ as

$$\mathcal{L}_{int} = \sqrt{\kappa_E}\ [\frac{\xi+1}{2}\ \overline{L}\ \gamma_{(\mu}\ D_{\nu)}\ L + \overline{R}\ \gamma_{(\mu}\ D_{\nu)}\ R + \frac{\xi-1}{2}\ \overline{L}\ \gamma_{(\mu}\ D_{\nu)}\ \tau_3\ L]\ \Phi^{\mu\nu} \qquad (34)$$

When one introduces the $\Phi^{(i)}_{\mu\nu}$ and $\Psi_{\mu\nu}$ to restore the $SU(2) \times U(1)$ symmetry it appears two arbitrary constants κ_a and κ_b and one transforms the above lagrangian into the following

$$\mathcal{L}_{int} = -\frac{1}{2}\ \sqrt{\kappa_a}\ \overline{L}\ \gamma_{(\mu}\ D_{\nu)}\ \vec{\tau}\ L.\vec{\Phi}^{\mu\nu} + \frac{1}{2}\ \sqrt{\kappa_b}\ \overline{L}\ \gamma_{(\mu}\ D_{\nu)}\ L\ \Psi^{\mu\nu}$$

$$+\frac{1}{1+\xi}\ \sqrt{\kappa_b}\ \overline{R}\ \gamma_{(\mu}\ D_{\nu)}\ R\ \Psi^{\mu\nu} \qquad (35)$$

The point of contact with observation is made through the fact that gravity couples to the electron and the neutrino by the terms $\sqrt{\kappa_E}\ T_{\mu\nu}(e)\ \Phi^{\mu\nu}$ and $\xi\ \sqrt{\kappa_E}\ T_{\mu\nu}(\nu)\ \Phi^{\mu\nu}$ and by the fact that gravity like electromagnetism (EM) conserves parity.

The gravitational field $\Phi^{\mu\nu}$ is to be identified with a definite linear combination of $\varphi^{\mu\nu(3)}$ and $\Psi^{\mu\nu}$ which can be written in a standard way as a particular rotation with a η mixing angle

$$\begin{pmatrix} \varphi^{\mu\nu(3)} \\ \Psi^{\mu\nu} \end{pmatrix} = \begin{pmatrix} cos\ \eta & -sin\ \eta \\ sin\ \eta & cos\ \eta \end{pmatrix} \begin{pmatrix} Z^{\mu\nu} \\ \Phi^{\mu\nu} \end{pmatrix} \tag{36}$$

We thus obtain for the gravity field $\Phi^{\mu\nu}$ the expression

$$\Phi^{\mu\nu} = -sin\ \eta\ \varphi^{\mu\nu(3)} + cos\ \eta\ \Psi^{\mu\nu} \tag{37}$$

The same rotation applied to the lagrangian leads to the following relations

$$cos\ \eta = (\xi + 1)\ \sqrt{\frac{\kappa_E}{\kappa_b}} \tag{38}$$

$$sin\ \eta = (\xi - 1)\ \sqrt{\frac{\kappa_E}{\kappa_a}} \tag{39}$$

and we can extract the κ_E Einstein's coefficient in terms of the coupling constants κ_a and κ_b and the ξ coefficient

$$\kappa_E = \frac{\kappa_a\ \kappa_b}{(\xi + 1)^2 \kappa_a + (\xi - 1)^2 \kappa_b} \tag{40}$$

These relations may be expressed in perfect analogy with the case of the electroweak unification in which

$$e = g\ sin\ \theta_W = g'\ cos\ \theta_W \tag{41}$$

In our case too it thus appears with (38) and (39) a short range gravity-like force as the local counterpart of the gravity

$$\sqrt{\kappa_E} = \frac{\sqrt{\kappa_a}}{\xi - 1}\, sin\ \eta = \frac{\sqrt{\kappa_b}}{\xi + 1}\, cos\ \eta \tag{42}$$

It thus appears an evident correspondence between the electroweak constants of the Weinberg-Salam standard model and the constants emerging in our theory (Table 1).

What conclusion can we extract up to now ? Firstly, when dealing with the classical Electromagnetic (EM) and gravitational fields, we get infinite range forces mediated by null-mass bosons. The enlarging of EM to Electro-Weak (EW) is made after taking into account the existence of a peculiar fermion, the neutrino, which interacts quite differently with EM (no interaction at all) and with W interaction. It leads to the necessary introduction of a local counterpart of the EM which is precisely the W interaction. Since we have treated gravity with the same $SU(2) \times U(1)$ standard procedure, the fact that the neutrino may interact differently than the electron with gravitation has obliged us to introduce a local counterpart of gravitation that is a short-range gravity-like force mediated by massive spin two tensor bosons. We have now to determine the origin of the mass of such bosons.

4. The Higgs mechanism

The most economic way to give mass to vector and tensor bosons is by introducing a unique Higgs's doublet

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^o \end{pmatrix}$$

of complex scalar field, one electrically charged the other being neutral.

The total lagrangian will then include a term which couples the gauge bosons (GB) to the scalar field S

$$\mathcal{L}_{GB-S} = |D_\mu\ \Phi|^2 - V\ (\Phi^+\ \Phi) \tag{43}$$

where D_μ is the covariant derivative operator defined in (25) and $V\ (\Phi^+\ \Phi)$ the Higgs potential

$$V\ (\Phi^+\ \Phi) = \mu^2\ (\Phi^+\ \Phi) + h\ (\Phi^+\ \Phi)^2, \quad h > 0 \quad \mu^2 < 0 \tag{44}$$

Let us now introduce a Lagrangian which couples the tensor bosons (TB) with the scalar field

$$\mathcal{L}_{TB-S} = |(A\ \vec{\tau}.\vec{\varphi}^{\mu\nu} + B\ \Psi^{\mu\nu})\ \Phi|^2 \tag{45}$$

The A and B coefficients must be adimensional numbers.

The conventional spontaneous symmetry breaking mechanism uses the minimum value of Φ

$$<\Phi>= \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \text{with} \quad v^2 = -\frac{\mu^2}{h} \tag{46}$$

and by replacing Φ^+ by zero and Φ^o by $\frac{1}{\sqrt{2}}\ (v + \chi(x))$ some mediating bosons become massive. Using the relation (36) and expanding $\mathcal{L}_{TB-S}$ one obtains after the symmetry breaking the following results

$$m^2 = m^2\ (\varphi^{\mu\nu\pm}) = v^2\ A^2 \tag{47}$$

$$M^2 = m^2\ (Z^{\mu\nu}) = v^2\ (A\ cos\ \eta - B\ sin\ \eta)^2 \tag{48}$$

in which we use the conventional notation $\varphi^{\mu\nu}_{\pm} = \varphi^{\mu\nu}_{(1)} \pm i\ \varphi^{\mu\nu}_{(2)}$.

The condition that $\varphi^{\mu\nu}$ given by equation (37) is identified with the long range gravitational field imposes to its mass to be zero, so that

$$A\ sin\ \eta + B\ cos\ \eta = 0 \tag{49}$$

One thus gets the constraint

$$\frac{B}{A} = -tg\ \eta \tag{50}$$

and (38) and (39) lead to the value

$$\frac{B}{A} = \frac{1-\xi}{1+\xi}\sqrt{\frac{\kappa_b}{\kappa_a}} \tag{51}$$

One can then extract from (47) and (48) the mass of the tensor bosons in the following form

$$m = v\ A \tag{52}$$

$$M = \frac{v\ A}{cos\ \eta} = \frac{m}{cos\ \eta} \tag{53}$$

Comparing these results to the mass of the intermediate bosons of the Electroweak interaction

$$\begin{aligned} M_W &= \frac{1}{2}\ g\ v \\ M_Z &= \frac{M_W}{cos\ \theta_W} \end{aligned} \tag{54}$$

One finds exactly an analogous relation as expected by our analogy (Table 1).

It makes natural then to set $\frac{1}{2}\ g \longleftrightarrow A$ and thus fix the A and B coefficients in terms of κ_a and ξ

$$A = \frac{1}{2}\ \frac{\sqrt{\kappa_a/\kappa_E}}{\xi - 1} = \frac{1}{2}\ \frac{1}{sin\ \eta} \tag{55}$$

and thus

$$B = -\frac{1}{2}\ \frac{\sqrt{\kappa_b/\kappa_E}}{\xi + 1} = -\frac{1}{2}\ \frac{1}{cos\ \eta} \tag{56}$$

We check that obviously such values fulfill the constraint condition (49). We now get the mass of the tensor bosons

$$m = m\ (\varphi^{\mu\nu\pm}) = \frac{v}{2\ sin\ \eta} \tag{57}$$

$$M = m\ (Z^{\mu\nu}) = \frac{v}{sin\ 2\eta} \tag{58}$$

The universality of Higgs mechanism that we are using fix the vacuum expectation value $v \simeq 246\ GeV/c^2$, so that the determined masses depend on the only η mixing angle parameter.

We have to wait for future observations in order to fix experimentally the value of the η angle. This new Short Range Force induces many new decay processes as the consequence of the above lagrangian and the full properties of this force can be obtained only through experiments. For a mean value of $\eta = \frac{\pi}{4}$ one finds that $m \simeq 174\ GeV/c^2$ and $M = 246\ GeV/c^2$.

5. Conclusion

In this paper we have presented a model in which Gravity can be introduced into the EW unified scheme. This has been made possible because of the previous GPP results which allow to treat gravity as a conventional rank-two symmetrical tensor field in ordinary Minkowski space time. We stress the fact that the GPP approach is not an approximation but it is the exact complete general relativity Einstein's theory. Then we are obliged to introduce charged spin-two fields in order to deal with the group structure of the EW gauge theory, under the hypothesis that gravity does not break this symmetry and couples differently with the electron and the neutrino. This has the first important consequence that there must exist a new Short Range weak interaction which becomes the local counterpart of Gravity in the same way as the W interaction can be thaught as the local counterpart of the EM field. This new force is mediated by massive spin two tensor bosons[7].

Following the conventional procedure the mass of these particles is provided by a Higgs mechanism. The universality of a spontaneous symmetry breaking process is proposed meaning that the same Higgs field gives mass to the vector bosons of the E.W theory and gives in the same way mass to these tensor bosons. It means that we have only one fundamental vacuum responsible for the mass creation.

The success of the standard EW $SU(2) \times U(1)$ is due to the fact that this scheme presents a unique description of two well-known forces, a long range (EM) and a short range one (W force). In our case we have been conducted to add to the long range gravitational force a new Short Range Gravity force in order to preserve this unified scheme. It is now a matter of experimentation to put into evidence the existence of such a new force and of its mediating tensor two bosons and determine the η mixing angle.

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Table 1

Correspondence between the coupling constant and the Electroweak angle θ_W to the constants emerging in our Short-Range Gravity - Gravity unification

EW	$SRG - G$	$SRG - G$ *normalized*
θ_W	η	η
g	$\sqrt{\kappa_a}/\xi - 1$	$\sqrt{\kappa_a/\kappa_E}/\xi - 1$
g'	$\sqrt{\kappa_b}/\xi + 1$	$\sqrt{\kappa_b/\kappa_E}/\xi + 1$
e	$\sqrt{\kappa_E}$	1

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