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TOTAL CROSS SECTION OF ULTRA-RELATIVISTIC HEAVY ION COLLISIONS*

by

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ABSTRACT

A possible increase of nuclear cross section at ultra-relativistic energies is suggested. Such an increase is expectred to start much earlier than in the case of proton-proton reactions due to more diffused nuclear surface compared to that of proton. Experimental data seem to be consistent with this picture.

Key-words: Ultra-relativistic; Heavy ion; Total cross section; Geometric cross section.

1. Introduction

The negative energy sea states of nucleons and virtual meson fields are known to play very important roles in relativistic field theoretical formulations of nuclear physics. The aim of this note is to call attention to possible dynamical aspects of these degrees of freedom if the nuclear system is really a strongly interacting relativistic system as suggested by the QHD.

First, let us recall the well-known fact that the total cross section of proton-(anti)proton collision process increases as a function of energy at very high energies. This fact was first discussed by Froissart in 60's from the view point of unitarity and analyticity of S-matrix, and since then there have been many theoretical works on this subject. The most intuitive physical picture for such a behavior of the cross section is that actually the radius of matter distribution increases as a function of energy(e.g. geometrical scaling model, see Ref.[1] and references there in). In order to illustrate how such mechanism can occur, let us consider the following picture. In a quantum bound system of n-particles, there exist many virtual particle-antiparticle pairs as a short living bubbles (Fig.1-a). For nonrelativistic energies, the roles of these virtual pairs are mainly modifying the mass and coupling constant of constituent particles. Now, let us boost this system up to near the velocity of light (Fig.1-b). In this case, all the particle world-lines are inclined to 45 degrees, and so the bubbles of virtual pairs. In additiion, the bubbles are elongated by a factor γ due to the Lorentz dilation of time. Therefore, within a finite time interval τ of the interaction, an observer encounters with many non-closed bubbles, i.e, nearly real particles and antiparticles. That is, for this observer the system behaves as if an assembly of many particles and many antiparticles. In fact, in relativistic quantum mechanics, the number of particles is not an invariant quantity with respect to a Lorentz boost, but the difference of particles and antiparticles is.

In order to relate the above consideration to the cross section, it is convenient to take the eikonal picture for the collision process. The total cross section in the eikonal picture is expressed as

$$\sigma_{tot} = 2\int d^2\mathbf{b}(1 - e^{-\chi(\mathbf{b})}). \tag{1}$$

where $\chi(\mathbf{b})$ is the usual eikonal phase. The inelastic cross section is

$$\sigma_{inel} = \int d^2\mathbf{b}(1 - e^{-2\chi(\mathbf{b})}). \tag{2}$$

In the Glauber picture, the eikonal is related to the thickness function of the object. For the collision of two composite system whose particle densities are $\rho_P(\mathbf{r})$ and $\rho_T(\mathbf{r})$ for projectile and target, respectively, we have

$$\chi(\mathbf{b}, \sqrt{s}) \approx \int d^3\mathbf{r} f(\sqrt{s})\rho_P(\mathbf{r} - \mathbf{b})\rho_T(\mathbf{r}). \tag{3}$$

where $f(\sqrt{s})$ is the scattering amplitude of two constituent particles with particle-particle CM energy $\sqrt{s}$. If the number density of constituent particles is independent of energy, then the only source of energy dependence of cross section comes from the energy dependence of this scattering amplitude. However, according to the intuitive picture mentioned above, it is reasonable to consider that the effective number of constituent particles in a strongly interacting system may increase at relativistic energies. In such a case, the energy dependence of the eikonal would have the kinematical origin, too. In Fig.2, we illustrated the relation between χ and the differential cross section with respect to the impact parameter b. If $\chi(b, \sqrt{s}) \gg 1$, then $\frac{d^2\sigma}{db^2} \cong 1$. Therefore, when χ increases with energy, the energy dependence of σ comes from the surface area where $\frac{d^2\sigma}{db^2}$ is still not saturated to the value 1 ("darkening" of the halo). We may define the effective radius $b_{eff}(\sqrt{s})$ by

$$\chi(b_{eff}, \sqrt{s}) \approx \ln 2. \tag{4}$$

In such case, the total (inelastic) cross section and the effective radius of the "matter" distribution are related as

$$\sigma_{inel} \simeq \pi b_{eff}^2(\sqrt{s}) \tag{5}$$

In the case of proton-proton collision process, the constituent particles can be identified as partons, and in fact the number of partons that one can "see" effectively increases with the incident energy[2].

However, such increase of cross section becomes only appreciable for very high energy region, viz, $\sqrt{s}$ > several tens of GeV. Therefore, if a nucleus is essentially an aggregation of nucleons, the total nucleus-nucleus cross section also stays constant in the energy region where the nucleon-nucleon cross section is constant. On the other hand, there are two basic features we should remember for the nucleus-nucleus collisions. First, if a nucleus is really a strongly interacting system of nucleons as is suggested by the QHD, where the virtual meson fields and negative energy sea nucleons seem to play an important role, then they may serve to increase the effective number of constituents in the relativistic energy domain as pointed out. Second, the nuclear surface is much more diffused compared to the nucleon surface. We show in the following that these two features in fact cause the earlier onset of halo darkening of nuclei compared to the case of nucleon-nucleon collision process.

2. Kinematical Model

In the first place, let us see only from the kinematical argument how the halo darkening may be related to the geometry of the system as a function of energy. Consider two nuclei A and B colliding with an inpact parameter b, energy E_{in}/A (See Fig.3). Similar to the firestreak model, let us consider a tube of unit area parallel to z-axis located at (x,y) in x-y plane. The projectile matter in this tube collides with the target matter in it. The CM kinetic energy (per unit area) in this tube is given by

$$T_{CM}(x,y;b)=\frac{2(\gamma-1)e_P(x,y;b)e_T(x,y)}{\sqrt{2(\gamma-1)e_P(x,y;b)e_T(x,y)+e_Pe_T}}, \tag{6}$$

where $e_{P(T)}(x,y,z)$ is energy density distribution of projectile (target) nucleus in its rest frame. Let κ be the inelasticity. Then κT_{cm} is the available energy for inelastic processes to take place in this tube. We may compare this to the threshold energy E^* for the inelastic channel. Thus, we may consider that whenever

$$\kappa T_{cm}(x,y;b,\sqrt{s})\geq E^*, \tag{7}$$

some inelastic process takes place in this particular tube. In other words, for a given impact parameter b, if there exist (x,y) values which satisfy the above inequality, an inelastic process must take place. Let b_{max} be the maximum value of b above which there's no tube of any (x,y) satisfying the above inequality. Then the total inelastic cross section is

$$\sigma_{inel}\approx\pi b_{max}^2(\sqrt{s}). \tag{8}$$

For the collision of two equal Gaussian energy distil ution, ie,

$$\rho_P(\mathbf{r})=\rho_T(\mathbf{r})=\epsilon_0 e^{-\alpha r^2}, \tag{9}$$

the ratio of inelastic cross section to the geometrical one, $\sigma_{inel}/\sigma_{geo}$ is calculated to be

$$\frac{\sigma_{inel}}{\sigma_{geo}}=\frac{1}{3}\ln\left(\frac{2\epsilon_0^2\pi}{m_p}\left(\frac{\kappa}{E^*}\right)^2\frac{T_{lab}}{\alpha}\right), \tag{10}$$

where

$$\sigma_{geo}\equiv\pi(2\sqrt{<r^2>})^2=6\pi/\alpha. \tag{11}$$

From Eq.(10), we see that the inelastic cross secion increases as $ln(E_{lab}/\alpha)$. That is, the greater the surface thickness (smaller α), the earlier the onset of halo darkening occurs. In Fig.4, we show the calculated ratio of inelastic cross section to the geometrical one for more realistic energy distribution functions for $S+Pb$[3] for several values of $\xi \equiv E^*/\kappa$. From this figure, we see that for some reasonable values of E^* and κ, the nuclear cross section may increase substantially at energy regions $E_{in}^{lab} \approx$ several tens of GeV .

3. A Simple Field Theoretical Model

Naturally, the argument of the previous chapter only indicates the kinematical limit to the cross secction. There's nothing to guarantee that κ stays constant as a function of energy. In the following we show that such a mechanism can be realized in a very simple field theoretical model for inelastic processes.

For the sake of simplicity, we assume that whole inelastic cross section can be represented by production of some massive mesons by the collision of projectile and target matter. Let $\varphi_\mu(t,\mathbf{r})$ be a field operator of such a meson with mass μ. The equation of motion of this field is given by

$$(\partial^\nu \partial_\nu + \mu^2)\varphi_\mu(t,\mathbf{r}) = J(t,\mathbf{r};b), \tag{12}$$

where $J(t,\mathbf{r};b)$ is the source of this field which describes the time-dependent matter density in nucleus-nucleus collision with impact parameter b. If the source is an external one, this problem is exactly soluble, and the S-matrix for the field φ is given as

$$S = e^{i\int \frac{d^3k}{\sqrt{2\omega_k}} j(k;b) a^\dagger_{in,k}} e^{i\int \frac{d^3k}{\sqrt{2\omega_k}} j^*(k;b) a_{in,k}} e^{-\frac{1}{2}\int \frac{d^3k}{\omega_k} |j(k;b)|^2} \tag{13}$$

where $a_{in,k}$ is the aniquilation operator of the meson with momentum k. and j(k:b) is the Fourier transform of $J(t,\mathbf{r};b)$. From this, the eikonal for the production of mesons with mass $\mu, \chi_\mu(b)$ is obtained as

$$\chi_\mu(b) = e^{-\frac{1}{2}\int\frac{d^3k}{\omega_k}|j(k;b)|^2}. \tag{14}$$

Now, let us assume that the source density is given by

$$J(t,\mathbf{r};b) = g|\beta_p - \beta_t|\gamma_p\gamma_t\rho_p(x,y-b,\gamma_p(z-\beta_p t))\rho_t(x,y,\gamma_t(z-\beta_t t)) \tag{15}$$

which is a Lorentz scalar and representing two colliding matter densities with respective velocities.

If the matter density in the rest frame is given by the Gaussian distribution, Eq.(9), then the eikonal function is found to be

$$\chi_\mu(b;\sqrt{s}) \approx \frac{\pi^2 g^2 \epsilon^2}{64\alpha^4} e^{-\alpha b^2} e^{\frac{1}{4\alpha}\mu^2} \int\limits_{\mu^2}^{\infty} dy e^{-\frac{1}{4\alpha}y} K_0(\frac{y}{4\alpha\gamma_{cm}^2}), \tag{16}$$

where γ_{cm} is the Lorentz factor of the colliding system in their CM frame, and K_0 is the modified Bessel function. In the above equation, we have kept only the leading term in $\frac{1}{\gamma_{cm}^2}$.

The total inelastic cross section is calculated from the total eikonal, i.e., the sum of χ_μ's over all massive mesons of different μ. Let $f(\mu^2)$ be the spectrum of these massive mesons. Then the total eikonal is given by

$$\chi_{tot}(b;\sqrt{s}) = \int\limits_0^\infty d\mu^2 f(\mu^2)\chi_\mu(b;\sqrt{s}). \tag{17}$$

For simplicity, let us assume $f(\mu^2) = const$. We get,

$$\chi_{tot}(b;\sqrt{s}) \propto e^{-\alpha b^2}(\frac{\sqrt{s}}{\alpha})^2 \tag{18}$$

upto the leading term in $1/\sqrt{s}$. From this, we have the effective interaction radius as

$$b_{eff}(\sqrt{s}) \sim \frac{1}{\sqrt{\alpha}}(ln(\frac{\sqrt{s}}{\alpha})+ const.). \tag{19}$$

Thus, we again see that the halo darkening starts earlier for smaller α, i.e., the larger surface diffuseness.

4. Comparison with Experiment

There are still few data available for the total inelastic cross sections for heavy ion reactions at ultra-relativistic energy region. In Fig.5, we compared the recent CERN data of O + Pb reactions at $E_{in}^{lab}/A = 60$ and 200 GeV[4] to the geometrical value,

$$\sigma_{geo} = \pi R_0^2(A_P^{\frac{1}{3}} + A_T^{\frac{1}{3}} - \delta)^2, \tag{20}$$

with $R_0 = 1.48 fm$ and $\delta = 1.32$. At low energies, this formula is well reproduces the inelastic cross section data[5]. As we cas see from this figure, the CERN data are clearly consistent with the $ln\sqrt{s}$ increase. We should remind that the nucleon-nucleon cross section is almost constant in this energy region ($\sqrt{s}_{nucleon-nucleon} \sim 20GeV$). In this figure, we also plotted the $\alpha+\alpha$ data at $\sqrt{s} = 126$ GeV[6]. Although the error bar is big, they are consistent with the increasing cross section.

In Fig.6, data for one-neutron removal cross section, $\sigma(^{197}Au(^{16}O, X)^{196}Au)$ due to Hill et al.[7]. In this figure, the curve $\sigma^{exp} - \sigma_{geo}$ represents the measured total cross section for one-neutron removal process subtracted by the contribution due to the nuclear interaction estimated with the limiting fragmentation model[7]. If there's no additional nuclear contribution than the conventional geometrical one, this curve should coincide with the theoretical curve for the electromagnetic excitation processes. The discrepancy as a function of energy suggests that the nuclear contribution increases with energy. In particular, the mechanism proposed here indicates that such an increase comes mainly from the surface region of matter distribution. Therefore we expect that the peripheral processes like one-particle emmission are especially enhanced.

5. Conclusion

We argued that if a nucleus is a strongly interacting relativistic system of nucleons and mesons as suggested in the QHD, the nuclear inelastic cross section should inc ease much earlier with the incident energy than expected from the behaviour of nucleon-nucleon cross section. That is, the relativistic heavy ions look much larger at ultra-relativistic energy region than those in nonrelativistic energy. We may call such a phenomena as Relativistic Ion Overgrowth (RIO) effect[3]. Although data at present moment are not yet conclusive, they are consistent with this picture. If it is true, the RIO effect indicates that the virtual mesons and negative energy sea nucleons play also important roles in dynamical processes. It is desireble to confirm experimentally the RIO effect suggested here. One possible way to see is to check the target-projectile mass number dependence of these total cross sections, in order to distinguish from the Coulomb fragmentation effects.

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FIGURE CAPTIONS

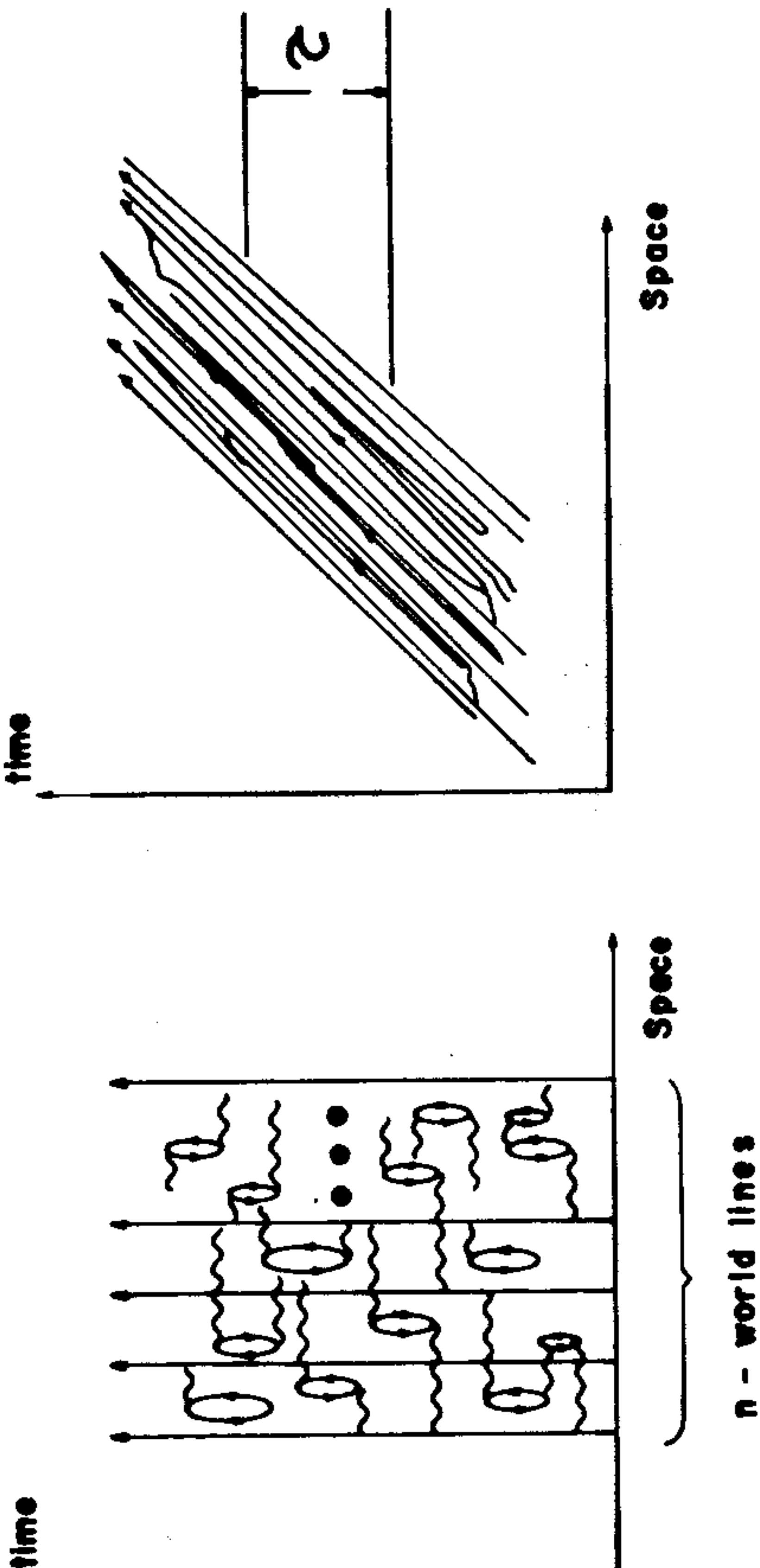
1) a) World-lines of particles and associated virtual particle-hole pairs.
b) Boosted system at a ultra-relativistic energy. Virtual pairs are elongated and behaves as if real particle and anti-particles.

2) The relation between $\chi(b)$ and $\frac{d^2\sigma}{db}$ as functions of impact parameter b. If the curve $\chi(b)$ increases, then $\frac{d^2\sigma}{db}$ extends to larger b.

3) Two Colliding nuclei with an impact parameter b. We consider the collision kinematics of projectile and target matter inside a tube parallel to the z-axis located at (x,y) position indicated.

4) Total inelastic cross sections as function of laboratory incident energy E_{in}^{lab}/A. Data are taken from Refs.[4],[6].

5) One-neutron removal cross section of Au for $^{197}Au + ^{16}O$ reaction[7].



(a)

(b)

FIG.1

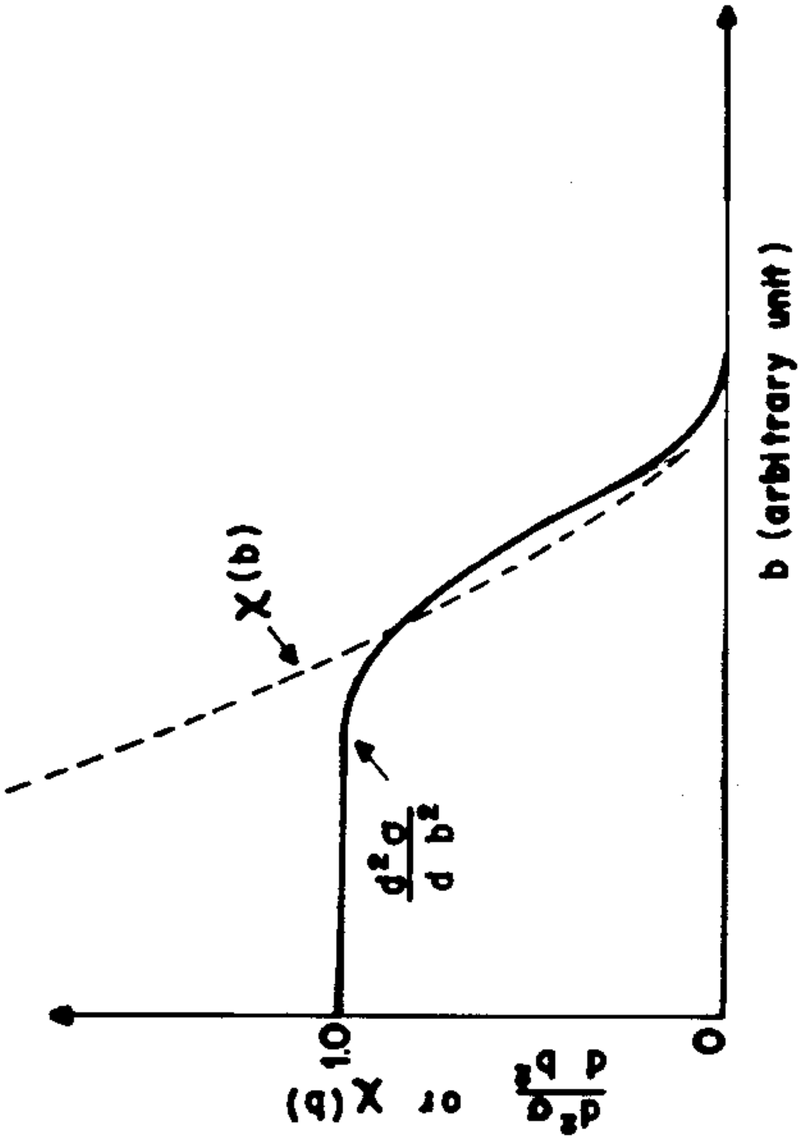


FIG.2

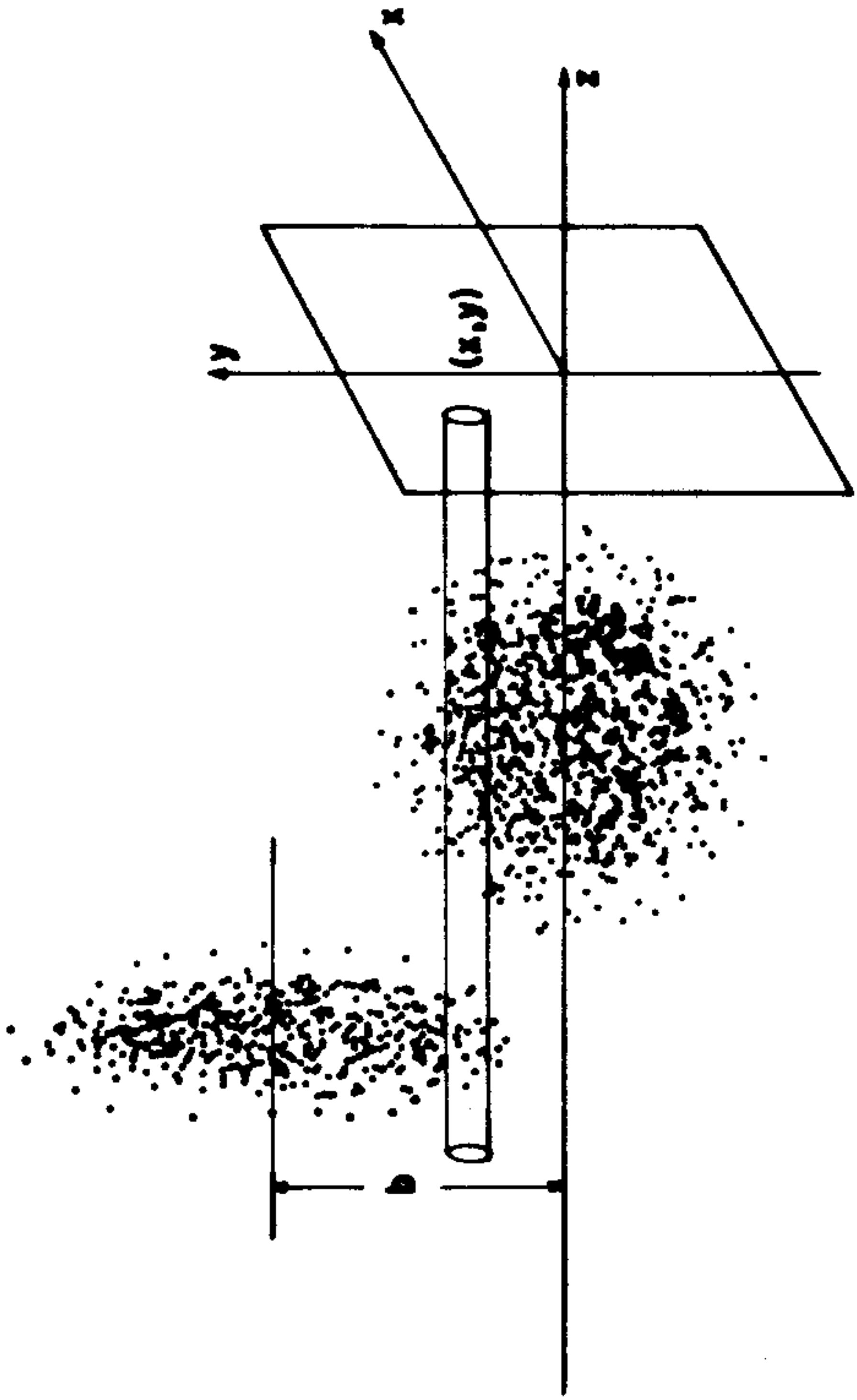
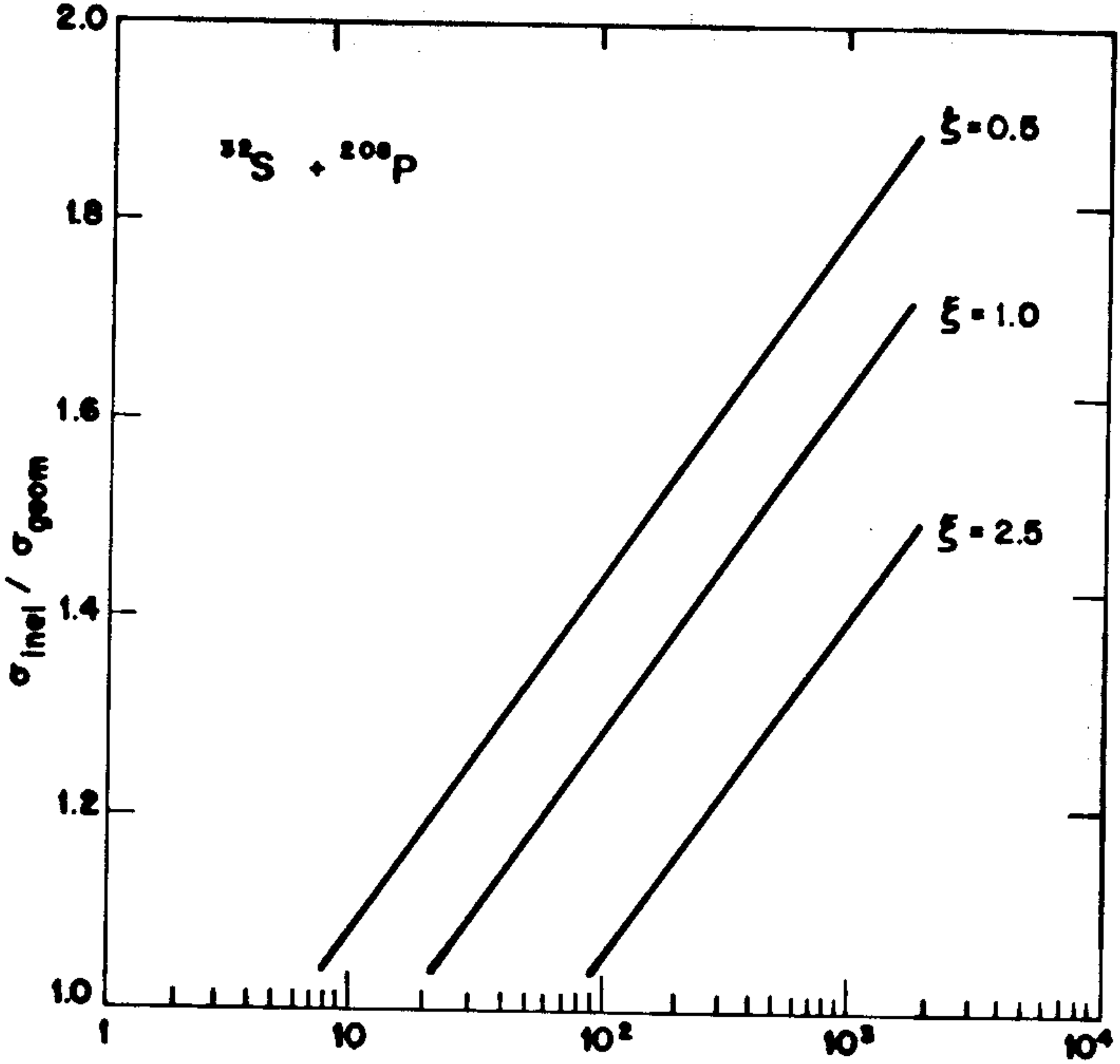
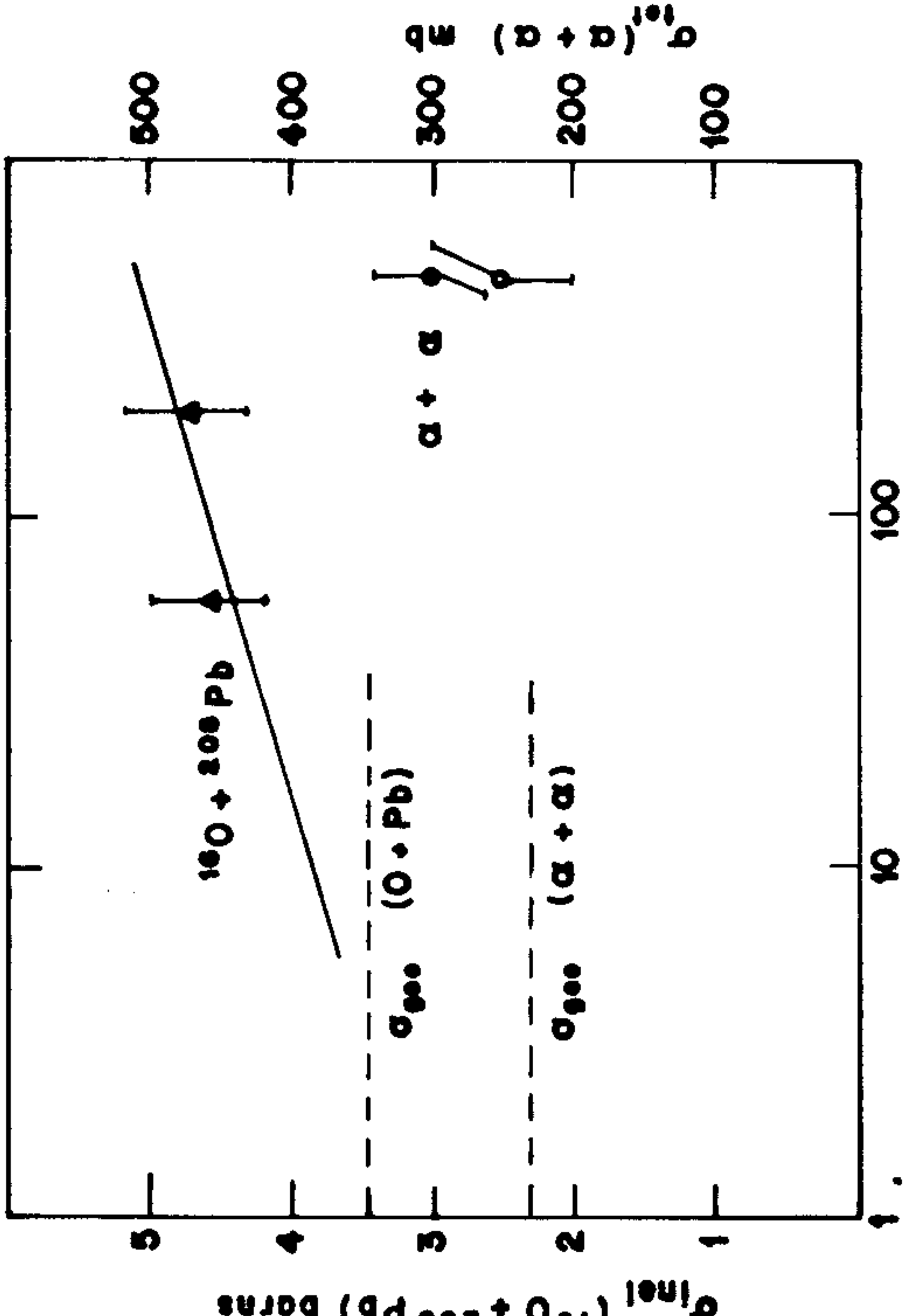


FIG. 3



2.0
1.8
1.6
1.4
1.2
1.0
σ inel / σ geom
^{32}S + ^{208}P
ξ = 0.5
ξ = 1.0
ξ = 2.5
1
10
10^2
10^3
10^4
E inc/A GeV

FIG. 4



σ_{inel} ($^{16}O + {}^{208}Pb$) barns
σ_{el} ($\alpha + \alpha$) mb
E_{in}^{Lab}/A GeV
$^{16}O + {}^{208}Pb$
$\alpha + \alpha$
σ_{geo} (O + Pb)
σ_{geo} ($\alpha + \alpha$)
1
2
3
4
5
100
200
300
400
500
1
10
100

FIG. 5

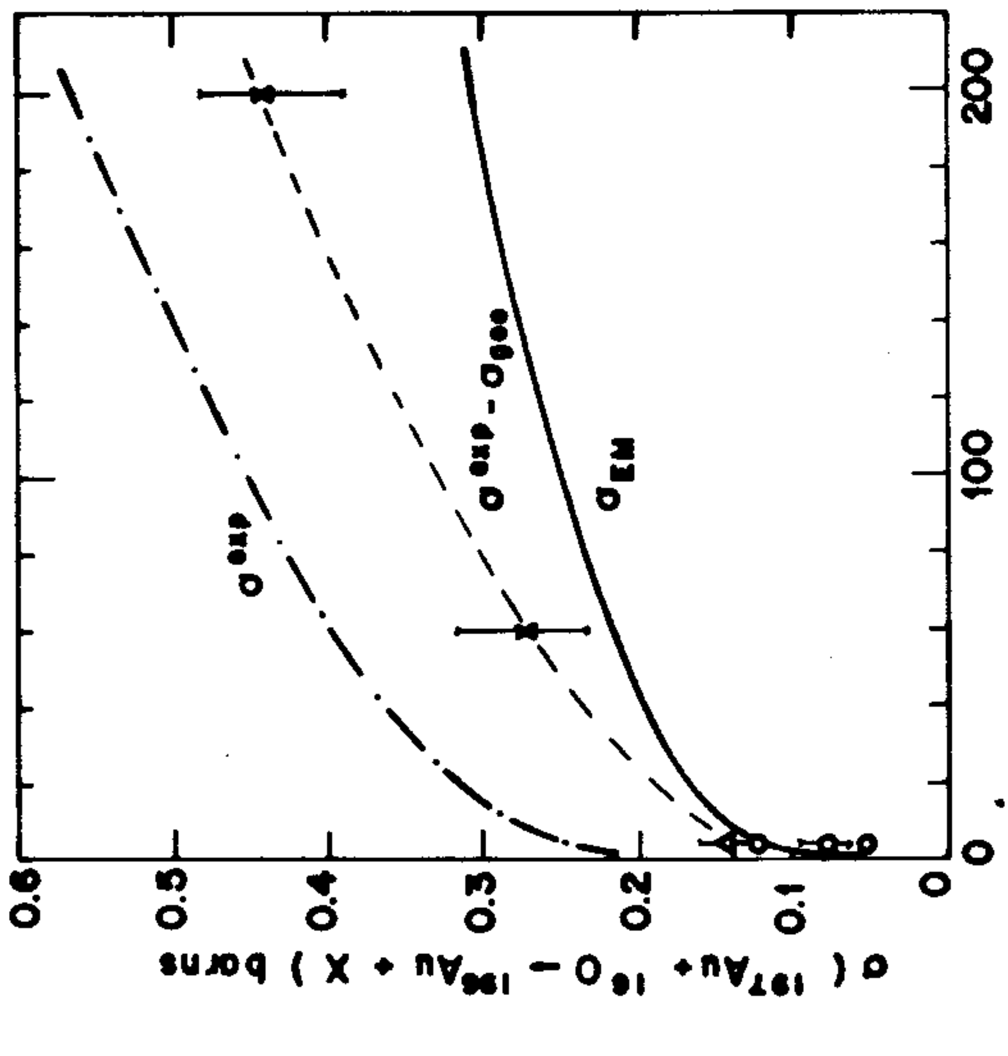

0.6
0.5
0.4
0.3
0.2
0.1
0
σ (197Au + 16O → 196Au + X) barns
0
100
200
E_in^Lab/A GeV
σ^exp
σ^exp - σ_geo
σ_EM

FIG.6

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