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DYNAMICAL EFFECTS IN THE COULOMB EXPANSION FOLLOWING NUCLEAR FRAGMENTATION

by

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ABSTRACT

The effects of the Coulomb expansion on the fragment kinetic energy spectrum for a fragmentating hot nuclear system is investigated. In particular, ^{12}C-fragment spectra are calculated and compared with those predicted by the uniform expansion approximation. The results indicate that the energy spectra of fragments are quite sensitive to the details of the Coulomb expansion treatment.

Key-words: Fragmentation; Coulomb expansion; Uniform expansion.

I. INTRODUCTION

One of the most interesting processes taking place in a heavy ion collision is the fragmentation of the nuclear system in many pieces. Although its study has drawn considerable attention, this reaction mechanism is far from being fully understood. Very different models, which assume that the fragmentation arises from mechanical instabilities (cold fragmentation[1]), thermal instabilities[2-4], sequential evaporation[5], and several others, are able to describe the observed mass distribution of fragments.

Although it cannot by itself pinpoint the reaction mechanism involved in the nuclear disassembly, the study of the kinetic energy spectrum of fragments provides some important clues. Hirsch et al.[6] have measured the energy spectra of several fragments, especially carbon and oxigen isotopes, produced in high-energy proton collisions with krypton and xenon targets. These authors analyse their data by assuming that fragmentation takes place as a liquid-gas phase transition at the critical point of the target remnant, the fragments being simultaneously produced and then dispersed via Coulomb repulsion. The treatment of the Coulomb expansion will thus be an essential ingredient of the analysis of energy spectra, and the conclusions reached through such an analysis will depend very strongly on the details of the expansion procedure.

In the present paper we show how the uniform expansion hypothesis considered by Hirsch et al. in their analysis of the

kinetic energy spectra of fragments leads to a large overestimate of these energies, or, reciprocally, to a considerable underestimate of the excitation energies of the fragmentating pieces of nuclear matter. A short description of the fragmentation process along the lines of the statistical multifragmentation model of Ref.4 is given in Sec.II. An alternative treatment of the Coulomb expansion process is discussed in Sec.III. Finally, the results obtained are presented and discussed in Sec.IV.

II. FRAGMENTATION MODEL

We consider a portion of expanding hot nuclear matter at the point when it disassembles into several fragments. The system is assumed to be characterized by definite nucleon and proton numbers, A_R and Z_R respectively, energy E_R and also by the volume V_R at which this breakup occurs. Furthermore, if we consider the different fragments (A, Z) into which the system splits, i.e., a given partition of the system, baryon number and charge conservation imply that

$$\sum_{Z,A} N_{Z,A}\, A = A_R \quad , \qquad \sum_{Z,A} N_{Z,A}\, Z = Z_R \tag{1}$$

where $N_{Z,A}$ is the number of fragments with a given Z and A.

The fragment energies $E_{Z,A}$ of these fragments must satisfy the energy conservation relation

$$\sum_{Z,A} N_{Z,A}\, E_{Z,A} = E_R \quad . \tag{2}$$

In order to describe the breakup process, additional information about the system is required. In what follows we will consider the statistical multifragmentation model of Ref.4, which is consistent with the conservation laws indicated by Eqs.(1) and (2), and besides makes the following assumptions:

i) at the disassembly stage the system is in thermal and chemical equilibrium,

ii) except for the lightest fragments ($A \leq 4$) the fragment internal energies are calculated using a finite temperature liquid drop model.

For our study of the expansion process we will consider as initial systems partitions of the original nucleus which are obtained through a Monte Carlo sampling procedure based on the statistical fragmentation model as described in Ref.4. At the point where the expanding nuclear matter disassembles, we assume that the interfragment distances are sufficiently large so that we may safely consider the expansion to be governed mostly by the Coulomb forces. In the following section we will discuss the evolution of the system in the expansion stage.

III. COULOMB EXPANSION PROCESS

If one makes the simplifying assumption that the expansion

is uniform (i.e., the distance between any two fragments is altered in the same way as the radius of the system) then the final kinetic energies of the different fragments may be directly evaluated as the sum of their initial kinetic and potential energies at the breakup stage. This procedure was considered by Hirsch et al. in order to analyze their fragmentation data and, in particular, to deduce the temperature of the fragmentating system from the kinetic energy spectra of the resulting fragments. They assume that the system possesses spherical symmetry, mass A_R, charge Z_R and radius R_R. A fragment of mass A_f, charge Z_f and radius R_f at a distance R from the center of the assumedly uniform charge distribution acquires a kinetic energy due to Coulomb repulsion equal to

$$E_{Coul} = \frac{Z_f Z_R e^2}{R_R^3} (R^2 + 3R R_f)(1 - A_f/A_R)^2 \qquad (3)$$

where the last factor $(1 - A_f/A_R)^2$ is included so as to take into consideration linear momentum conservation.

We have noticed that in deriving this expression Hirsch et al. have implicitly assumed that the fragment radius also increases during the expansion, in direct relation to the system's size. This assumption is clearly unreasonable, and may lead to a large systematic error, especially for large fragments. If one instead leaves the fragment radius fixed at its initial value, so that as the expansion proceeds the remaining charge distribution develops a hole around it, we arrive at the following Coulomb contribution to the final kinetic energy

$$E_{Coul} = \frac{Z_f Z_R e^2}{R_R^3} R^2 \ (1 - A_f/A_R)^2 \qquad (4)$$

which does not contain the term $3R\,R_f$ of Eq.(3). In the case of large fragments, this is an important contribution, so that Eq.(3) is expected to overestimate the fragment kinetic energies.

We observe that in the uniform expansion approximation the contribution from the Coulomb repulsion to the kinetic energy of each fragment depends exclusively on the position of that fragment at the beginning of the expansion. This is obviously an oversimplification. Since the fragments already possess thermal velocities at breakup, their relative positions are expected to shift during the expansion. These initial velocities depend on the average temperature and the fragment masses. Therefore we expect the deviations from the uniform expansion to depend on the total energy of the system as well as on the mass of the particular fragment which energy spectrum is being observed.

In the uniform expansion the total fragment kinetic energy is calculated as the sum of the thermal and Coulomb energies. Let us instead consider a dynamical description of the Coulomb expansion. The classical Hamiltonian H is given in this case by

$$H = \sum_i \left\{ \frac{\vec{p}_i^{\,2}}{2m_i} \right\} + \sum_{i<j} \left\{ \frac{Z_i Z_j e^2}{|\vec{r}_i - \vec{r}_j|} \right\} \qquad (5)$$

where m_i, $\vec{r}_i$ and $\vec{p}_i$ are, respectively, the mass, position and momentum of the ith fragment. The coupled differential equations

of motion obtained from Eq.(5) are then integrated using standard numerical methods. The initial values of the positions and momenta are obtained using a Monte Carlo procedure. The positions $\vec{r}_i$ are randomly selected from a uniform spherical distribution of volume V_R, making sure that there is no overlap among fragments. The value of V_R depends on the system size and excitation energy, as described in Ref.4. The initial momenta $\vec{p}_i$ are obtained from the Maxwell-Boltzmann distribution associated to the excitation energy of the system.

The time evolution of the Coulomb expansion process is continued until the total energy of each fragment ceases to show appreciable changes. These final total energies are then stored and the procedure repeated until the statistics are satisfactory.

IV. RESULTS AND DISCUSSION

In Fig.1, we show the calculated primary ^{12}C-fragment energy spectra for the case of a piece of hot nuclear matter of mass $A_R = 100$ excited to an energy per nucleon equal to 4 MeV/A. In the three cases depicted in that figure the most probable partitions were selected according to the statistical fragmentation model. Their evolution was studied according to the uniform expansion approximation of Eq.(3) (dotted curve), the modification to that approximation proposed in Eq.(4) (dashed curve), and the dynamical expansion calculation just described (full line). The differences among these results are quite noticeable. The uniform expansion

in the approximation of Eq.(3) predicts much higher energies than the other two calculations. This is due to the superfluous term that was shown to appear in that expression.

The improved uniform expansion of Eq.(4) yields, in this case, results which approach closely the dynamical ones. However, if one considers a higher excitation energy (Fig.2), this approximation differs more from the classical dynamics calculation, although still much less than that of Eq.(3). The results tend to peak at a slightly higher energy and that peak is noticeably thinner than in the dynamical calculation. Simple considerations may help to explain why the energy distribution becomes broader when the relative fragment positions are allowed to shift in their relative positions during the expansion. Indeed, if one fragment has above (below) average kinetic energy, its radial velocity is also expected to be above (below) average. Thus, as the expansion process develops it will feel an increasing (decreasing) region of charged nuclear matter repelling it. In this way the Coulomb repulsion among fragments tends to broaden their final energy spectra.

One should remark that these results should not be directly compared with experiment since the secondary decay stage has not been included. As discussed in Ref.8 the decay of the primary fragments strongly affects the energy spectra. It is also shown in that reference that the characteristic times for the Coulomb expansion and the evaporation processes are quite comparable. This makes almost mandatory a treatment in which both mechanisms operate simultaneously. This is beyond the aim of the present work,

which was to draw attention to the fact that the expansion process should be carefully treated in order not to radically distort the information extracted from the fragmentation data.

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FIGURE CAPTIONS

Fig.1: Calculated primary ^{12}C-fragment energy spectra for the case of a nuclear system of size A_R = 100 with an excitation energy of 4 MeV per nucleon. The results were calculated with a Monte Carlo sample size of 1000 runs. See text for additional details.

Fig.2: Same as Fig.1 for an excitation energy of 7 MeV per nucleon.

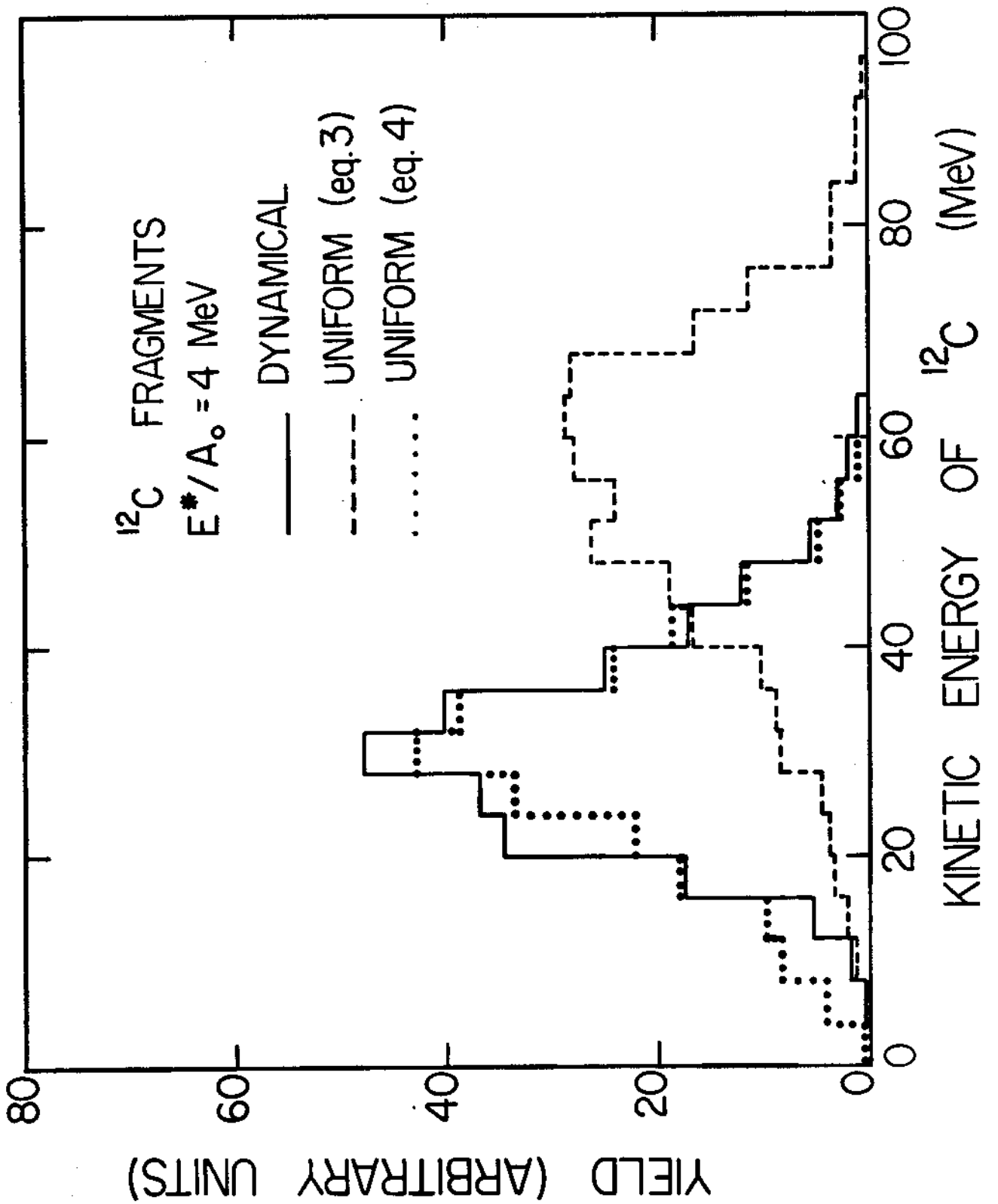
^{12}C FRAGMENTS
$E^*/A_o = 4$ MeV
DYNAMICAL
UNIFORM (eq.3)
UNIFORM (eq.4)
YIELD (ARBITRARY UNITS)
KINETIC ENERGY OF ^{12}C (MeV)
0
20
40
60
80
100

Fig. 1

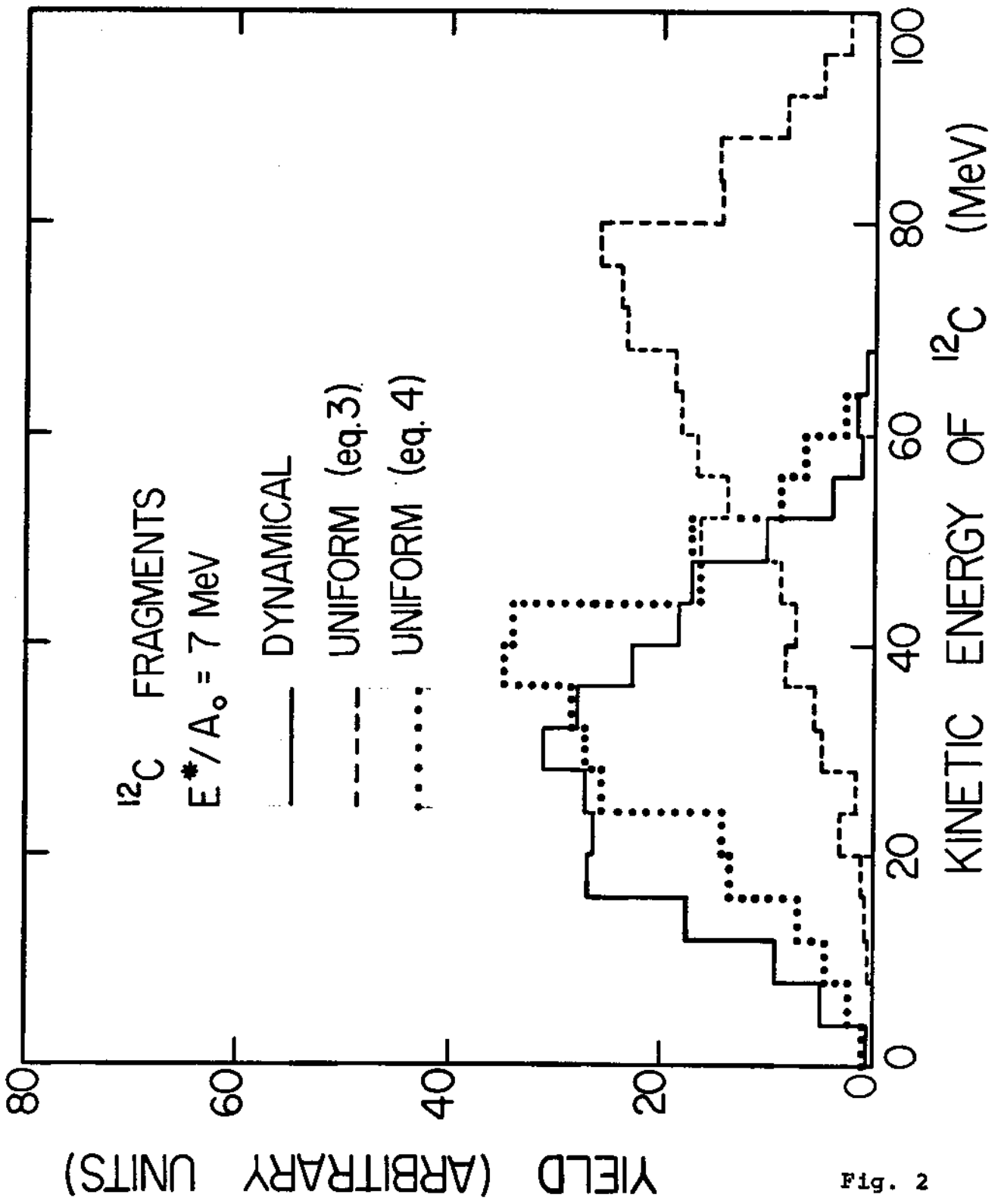
^{12}C FRAGMENTS
E^*/A_o = 7 MeV
DYNAMICAL
UNIFORM (eq.3)
UNIFORM (eq.4)
YIELD (ARBITRARY UNITS)
KINETIC ENERGY OF ^{12}C (MeV)
0
20
40
60
80
100

Fig. 2

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