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ON THE EQUIVALENCE BETWEEN THE SCHWINGER AND AXIAL MODELS

by

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Abstract: We show the equivalence between the Schwinger and axial models, in the sense that all Green's functions of one model can be obtained from those of the other, and that both models have the same effective Lagrangian density (and so they have equal partition functions associated with them). In particular, we show that the two models have the same chiral anomaly. Finally it is demonstrated that the Schwinger model can keep gauge invariance for an arbitrary mass, dispensing with an additional gauge group integration.

Key-words: Two-dimensional models; Schwinger model; Axial model.

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I)Introduction:

Since the Schwinger model (SM) was introduced[1] , many other two-dimensional exactly solvable models have appeared, such as the Thirring model[2], describing fermions with quartic self-interaction; the axial model (AM)[3], which describes a massive pseudoscalar field interacting with massless fermions, the chiral-Schwinger model[4], etc. The great appeal of these models in 1 + 1 dimensions, is that they provide the possibility to study problems as confinement, asymptotic freedom, renormalizability, etc., in models which can be exactly solved.

Some years ago Rothe and Stamatescu[3] presented the axial model, solving it in the operatorial formalism. Many other works have appeared since then[5-12], discussing several aspects of the model. In this paper we intend to show that, in fact, this model is equivalent to the Schwinger model . We do this by obtaining the rules to map the Green's functions of the models. The material is organized as follows: in section II we relate the Lagrangian density of the SM with that of a pseudoscalar field with higher derivative terms interacting with massless fermions, establishing their connection and demonstrating the relation between the boson self-energies of the SM and of the AM, which appears as an observation in the original work of Rothe and Stamatescu[3]; in section III, we obtain the anomaly of the SM from that calculated for the AM, deriving the rules to pass from AM Green's function to that of the SM; then in section IV we show the equivalence of the effective Lagrangian densities of the models, by solving exactly the SM; section V is devoted to see the equivalence via the method of point-splitting; in section VI it is shown that the SM has a bosonic effective action that is manifestly gauge invariant, with an arbitrary mass for the gauge boson. This is done without integrating in the gauge volume as it happens in Harada-Tsutsui's work[13] and this is interpreted. Finally, in section VII we present our conclusions.

II) The SM as a pseudoscalar model:

The Lagrangian density of the SM is given by

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu})\psi - F_{\mu\nu}F^{\mu\nu}/4 + J_{\mu}A^{\mu} + \bar{\psi}\,\theta + \bar{\theta}\,\psi \qquad (1)$$

where we have included the field sources, necessary to obtain correlation functions in the functional formalism. Throughout the work we use

$$\{\gamma^{\mu},\gamma^{\nu}\} = 2\delta^{\mu\nu} \ , \qquad (2)$$

and in two dimensions and in Euclidean space is valid that

$$\gamma_5 = i\gamma^0\gamma^1 \ , \ \gamma_5\gamma^{\mu} = i\epsilon^{\mu\nu}\gamma_{\nu} \ , \ \epsilon^{01} = +1. \qquad (3)$$

Now we can use the vectorial identity, valid in 2D:

$$eA_{\mu} = g\Big[\partial_{\mu}\eta + \epsilon_{\mu\nu}\partial^{\nu}\chi \Big]. \qquad (4)$$

In this section we will choose $\eta = 0$, as can always be done in the Lorentz gauge[14]. In section IV we shall see that, in fact, this corresponds to the use of identity (4) putting at the end the gauge parameter α equal to zero (Landau gauge). Now, it is easy to rewrite the Lagrangian density as

$$\mathcal{L} = \bar{\psi}\gamma^{\mu}(i\partial_{\mu} - g\partial_{\mu}\eta - ig\gamma_{5}\partial_{\mu}\chi)\psi + (g^{2}/2e^{2})\chi\Box\Box\chi +$$

$$+ (g/e)J^{\mu}\partial_{\mu}\eta + (g/e)\ \epsilon_{\mu\nu}J^{\mu}\partial^{\nu}\chi + \bar{\psi}\theta + \bar{\theta}\psi, \qquad (5)$$

where we have used $\epsilon_{\mu\nu}\gamma_{\mu}\partial_{\nu} = i\gamma_{\mu}\gamma_{5}\partial_{\mu}$, and

$$\epsilon^{\mu\nu}\epsilon^{\sigma\rho} = -\ \delta^{\mu\sigma}\delta^{\nu\rho} + \delta^{\mu\rho}\delta^{\nu\sigma}. \qquad (6)$$

The bosonic correlation functions are obtained by taking functional derivatives with respect to the source $J^{\mu}(x)$. To each derivative in this source we obtain a term of the type

$$(1/i)\delta Z/\delta J_{\mu}(x) = N^{-1}\int \mathcal{D}\bar{\psi}\ \mathcal{D}\psi\ \mathcal{D}\chi\Big((g/e)\epsilon^{\mu\nu}\partial_{\nu}\Big)\chi(x)\ \exp\Big(iS_{\kappa}\Big) =$$

$$= (g/e)\epsilon^{\mu\nu}\partial_{\nu}(1/i)\delta\bar{Z}/\delta K(x), \qquad (7)$$

where

$$\overline{Z}[\overline{\theta},\theta,K] = N^{-1}\int \mathcal{D}\overline{\psi}\ \mathcal{D}\psi\ \mathcal{D}\chi\ \exp\left(i\overline{S}_{\kappa}\right), \tag{8a}$$

and $\overline{S}_{\kappa}$ comes from:

$$\overline{\mathcal{L}}_{\kappa} = \overline{\psi}\gamma^{\mu}(i\partial_{\mu} - ig\gamma_5\partial_{\mu}\chi)\psi + (g^2/2e^2)\chi\Box\Box\chi + \overline{\psi}\theta + \overline{\theta}\psi + K\chi. \tag{8b}$$

On the other hand, the Lagrangian density of the AM is given by

$$\mathcal{L}_{AM} = \overline{\psi}\gamma^{\mu}(i\partial_{\mu} - ig_0\gamma_5\partial_{\mu}\phi)\psi + (1/2)(\partial_{\mu}\phi)^2 - (m_o^2/2)\phi^2 + \overline{\psi}\theta + \overline{\theta}\psi + K\phi. \tag{9}$$

We can see that the only differences between (8b) and (9) are the corresponding free bosonic Lagrangian densities.

Using the rule (7), we have for the photon propagator:

$$D^{\mu\sigma}(x-y) = (1/i^2)\ \delta^2 Z/\delta J_{\mu}(x)\delta J_{\sigma}(y)\Big|_{J=0} =$$

$$= (g/e)^2\epsilon^{\mu\nu}\epsilon^{\sigma\rho}\partial_{\nu}\partial_{\rho}(1/i^2)\delta^2\overline{Z}/\delta K(x)\delta K(y)\Big|_{K=0} \tag{10a}$$

and so

$$D^{\mu\sigma}(x-y) = (g/e)^2\epsilon^{\mu\nu}\epsilon^{\sigma\rho}\partial_{\nu}\partial_{\rho}D(x-y). \tag{10b}$$

In momentum space, the free propagator $D^0_{\kappa}(k)$ for the boson χ in $\mathcal{L}_{\kappa}$ is given by

$$D^0_\kappa(k) = (e/g)^2/k^4. \tag{11}$$

Substituting (11) in (10b) and using formula (6), we obtain the free photon propagator,

$$D^0_{\mu\sigma}(k) = (1/k^2)\left(-g_{\mu\sigma} + k_\mu k_\sigma/k^2\right), \tag{12}$$

which corresponds to the expected result in the Landau gauge.

Now, we will use (10b) to find the relation between boson self-energies of the SM and AM, as noticed by Rothe and Stamatescu[3], although there it appears without demonstration. The complete propagators for these models are

$D_{\mu\sigma}(x-y) =$ 

$$= D^0_{\mu\sigma} / (1 - \Pi^{\rho\nu} D^0_{\rho\mu}), \tag{13a}$$

for the field A_μ in the Schwinger model, and

$$D(x-y) = D^0/(1 - \Pi D^0), \tag{13b}$$

for χ in $\mathcal{L}_\kappa$, equation (8b). Using (10a), and remembering that this relation is valid also for free propagators, and making straightforward calculations, we obtain:

$$\Pi D^0 D^0_{\mu\sigma} = (g/e)^2 \epsilon^{\mu\nu} \epsilon^{\sigma\rho} \partial_\nu \partial_\rho \Pi^{\mu\sigma} D^0 D^0_{\mu\sigma}, \tag{14}$$

so that, in momentum space we have

$$\Pi(k) = (g/e)^2 \epsilon^{\mu\nu} \epsilon^{\sigma\rho} k_\nu k_\rho \Pi^{\mu\sigma}, \tag{15}$$

which, up to the multiplicative factor $(g/e)^2$, is equal to that which appears in the original work in the AM[3]. Besides we observe that this relation is valid only when we are working in the Landau gauge ($\alpha = 0$). It is interesting to note that now we can obtain similar relations for other Green's functions as, for example, the four photon scattering amplitude. Finally, we observe that the above demonstration, made for the field χ, can be easily repeated for the field ϕ of the AM, because it does not depend on the explicit form of the propagator D(x-y) used.

III) The Anomalies:

We observe that the Lagrangian density (8b) differs from that of the AM (9) only through the form of the free boson propagator, and that in two dimensions the anomaly appears in the polarization tensor, that depends only on the fermionic loop. It is not difficult to conclude that these models have the same anomaly. If this is true, we can obtain the anomaly of the SM from that of the AM. The anomaly of the AM is expressed as[12]

$$\langle \partial_\mu j^{\mu,5}_{AM}(x) \rangle = (ag/\pi)\Box\phi, \tag{16}$$

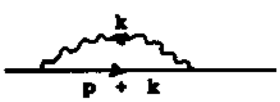
where a is an arbitrary regularization parameter, using now (4), we have that

$$e\epsilon^{\rho\mu}\partial_\rho A_\mu = g\Box\chi \tag{17}$$

so that, if we identify the field ϕ of the AM with the field χ of the SM, we get

$$\langle \partial_\mu j^{\mu,5}_{AM}(x) \rangle = (ae/2\pi)\epsilon^{\rho\mu}F_{\rho\mu} = \langle \partial_\mu j^{\mu,5}_{SM}(x) \rangle \tag{18}$$

So, as we have proved that the two models have the same anomaly, we are stimulated to ask if the Feynman graphs of the two models are also related . Let us start calculating the electron self-energy to 1-loop

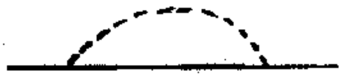


$$= e^2\int (d^2k/4\pi^2)\,\gamma^\mu \frac{1}{\gamma^\sigma(p_\sigma + k_\sigma)}\,\gamma^\nu D^0_{\mu\nu}, \tag{19}$$

where $D^0_{\mu\nu}$ is the photon propagator at zero order, that is given in (12). Substituting (12) in (19), we obtain that

$$\equiv \sum_{SM}(p) = e^2\int (d^2k/4\pi^2)\left(\frac{1}{\gamma^\sigma(p_\sigma + k_\sigma)}\right)\left(\frac{1}{k^2}\right); \tag{20}$$

for the case of the massless AM, we have also:



$$= - g^2 \int (d^2k/4\pi^2) \left(\frac{1}{\gamma^{\sigma}(p_{\sigma} + k_{\sigma})}\right)\left(\frac{1}{k^2}\right) \equiv \Sigma_{AM}(p),$$

so that

$$\Sigma_{SM}(p) = - (e/g)^2 \Sigma_{AM}(p). \tag{21}$$

It is also not difficult to show that for the 1-particle irreducible graphic of the 4-point Green function:

$$\equiv \Gamma(p_1,p_2),$$

the relation is given by:

$$\Gamma_{SM}(p_1,p_2) = + (e/g)^4 \Gamma_{AM}(p_1,p_2). \tag{22}$$

In general, the 1PI graphics of n-point Green's functions will be related, at least at 1-loop level, by:

$$\Gamma^{(n)}_{SM} = (-e/g)^n \Gamma^{(n)}_{AM}. \tag{23}$$

For graphics with photon external legs, there will exist similar relations to that in equation (10a), by repeated use of the rule (10b).

The above relations were obtained using the propagators at zero order. If we use the full photon propagator (and we know that the photon will acquire mass) we can see that the equivalence will be established between the SM and the AM with mass.

In this case, it is easy to see that the complete propagators of the photon and the pseudoscalar field are given respectively by

$$D^c_{\mu\nu}(k) = (k^2 - m_e^2)^{-1}\left(-g_{\mu\nu} + k_\mu k_\nu/k^2\right), \text{ with } m_e^2 = e^2/\pi \ , \qquad (24a)$$

and

$$D^c(k) = Z_\phi (k^2 - m_\phi^2)^{-1}, \qquad (24b)$$

with $Z_\phi = (1 - \lambda g^2/\pi)^{-1}$ and $m_\phi^2 = m_0^2 Z_\phi$. Now, we can recalculate the electron self-energy, obtaining:

$$\sum_{SM}(p) = - Z_\phi^{-1}(e/g)^2 \sum_{AM}(p). \qquad (25)$$

If the masses are identified by making use of the arbitrary parameter a in the AM, we come to

$$m_e^2 = m_\phi^2 \ \Rightarrow \ a = \pi(1 - m_0^2/e^2)/g^2. \qquad (26)$$

The other n-point 1PI vertex functions can be similarly obtained.

IV) Nonperturbative approach:

In this section we intend to show the relation between these models directly, by solving exactly the SM in an arbitrary gauge of Lorentz type, and then show that in a particular gauge the correlation functions can be related. For this we will use chiral rotation as a decoupling transformation[15,16]. So we start from the Lagrangian density

$$\mathcal{L} = \bar{\psi}\gamma_\mu(i\partial_\mu - eA_\mu)\psi - (1/4)F_{\mu\nu}F_{\mu\nu} - (1/2\alpha)(\partial_\mu A_\mu)^2 + J_\mu A_\mu + \bar{\psi}\theta + \bar{\theta}\psi, \tag{27}$$

and using the identities (3) and (4) we get

$$\mathcal{L} = \bar{\psi}\gamma_\mu(i\partial_\mu - g\partial_\mu\eta - ig\gamma_5\partial_\mu\chi)\psi + (g^2/2e^2)\chi\,\Box^2\chi - (1/2\alpha e^2)\eta\,\Box^2\eta +$$
$$+ (g/e)J_\mu\Big[\partial_\mu\eta + \epsilon_{\mu\nu}\partial_\nu\chi\Big]. \tag{28}$$

Making now the transformation in the Euclidean generating functional:

$$\psi = \exp\Big(-ig\eta + g\gamma_5\chi\Big)\psi' \equiv U_5\psi' \tag{29a}$$

$$\bar{\psi} = \bar{\psi}'\exp\Big(ig\eta + g\gamma_5\chi\Big) \equiv \bar{\psi}'\bar{U}_5, \tag{29b}$$

we obtain

$$Z = \int \mathcal{D}\bar{\psi}'\mathcal{D}\psi'\mathcal{D}\eta\mathcal{D}\chi \ J_F \ \exp\left[-\int d^2x \ \mathcal{L}'\right], \tag{30}$$

where

$$\mathcal{L}' = \bar{\psi}'(i\gamma_\mu\partial_\mu)\psi' + (g^2/2e^2)\chi \ \Box^2\chi - (g^2/2\alpha e^2)\eta \ \Box^2\eta +$$

$$+ (g/e)J_\mu\left[\partial_\mu\eta + \epsilon_{\mu\nu}\partial_\nu\chi\right] + \bar{\psi}'\bar{U}_5\theta + \bar{\theta}U_5\psi'. \tag{31}$$

As the Jacobian from the fermionic chiral transformation is not trivial, it is necessary to compute it. In order to make its calculation we will follow Banerjee[7] and Roskies and Schaposnik[15]. This consists essentially to use Fujikawa's procedure[17], so that the infinitesimal fermionic Jacobian comes from:

$$\mathcal{D}\bar{\psi}\mathcal{D}\psi = J_F \ \mathcal{D}\bar{\psi}'\mathcal{D}\psi'$$

with

$$J_F = \exp - \int d^2x \sum g\chi(x)\phi_m^+(x)\gamma_5\phi_m(x) \ , \tag{32}$$

where ϕ_m is a complete set of normalized eigenvectors of a given operator, with which we can expand the fermion fields. As is well known this is an ill-defined quantity, so we regularize it in order to supress large eigenvalues. With this in mind we replace the singular part of the Jacobian by

$$\lim_{M\to} \sum_{m} \phi_m^{+}(x)\gamma_5 \exp\left(-\lambda_m^2/M^2\right)\phi_m(x) \quad , \tag{33}$$

with λ_m being the eigenvalues of the regulator in the basis ϕ_m. In general we can choose the regularization operator as

$$D = i\gamma_\mu\partial_\mu - ga\gamma_\mu A_\mu \quad , \tag{34}$$

and a is the regularization arbitrary parmeter. Using it we can rewrite (36) as

$$\lim_{M\to} \sum_{m} \phi_m^{+}(x)\gamma_5 \exp\left(-D^2/M^2\right)\phi_m(x) \quad , \tag{35}$$

and after some manipulations[17], we obtain

$$J_F = \exp - \int d^2x\, \frac{ag^2}{2\pi}\chi \,\Box\, \chi \quad , \tag{36}$$

where was necessary to use the method of reference 15 in order to compute the Jacobian for the finite transformation. Now we can write the effective Lagrangian density:

$$\mathcal{L}' = \bar{\psi}'(i\gamma_\mu\partial_\mu)\psi' + (g^2/2e^2)\chi\left[\Box^2 + m_e^2\Box\right]\chi - (g^2/2\alpha e^2)\eta\,\Box^2\eta +$$

$$+ (g/e)J_\mu\left[\partial_\mu\eta + \epsilon_{\mu\nu}\partial_\nu\chi\right] + \bar{\psi}'\bar{U}_s\theta + \bar{\theta}U_s\psi' \quad , \qquad (37)$$

with $m_e^2 = ae^2/\pi$. At this point we can introduce a new field in order to get rid of the higher derivatives. For this we perform the transformation

$$\chi = (\ e/\sqrt{\Box}\ )\varphi \quad , \qquad (38)$$

so that

$$\mathcal{L}_{eff} = \bar{\psi}'(i\gamma_\mu\partial_\mu)\psi' + (g^2/2)\varphi\left[\Box + m_e^2\right]\varphi - (g^2/2\alpha e^2)\eta\,\Box^2\eta +$$

$$+ (g/e)J_\mu\left[\partial_\mu\eta + e\epsilon_{\mu\nu}(\ \partial_\nu/\sqrt{\Box}\ )\varphi\right] + \bar{\psi}'\bar{U}_s\theta + \bar{\theta}U_s\psi' \quad . \qquad (39)$$

It is easy to see now that, up to the source terms and a gauge fixing, we can identify this effective Lagrangian density with that of the axial model, provided that we impose the evenness of the masses through the regularization arbitrary parameter, as is stated in equation (26). Now one can calculate the complete Green functions from the generating functional of the above Lagrangian density.

The bosonic propagator is easily computed if we follow a similar way to that used in equation (10), so we get for the full photon propagator:

$$D_{\mu\nu}(k) = (g/e)^2\Big[k_\mu k_\nu D_\eta(k) + e^2\epsilon_{\mu\lambda}\epsilon_{\nu\sigma}\Big(k_\lambda k_\sigma/k^2\Big)D_\varphi(k)\Big] , \qquad (40)$$

where the propagators of the fields η and φ are given respectively by

$$D_\eta(k) = -(g/e)^2\Big(\alpha/k^4\Big) , D_\varphi(k) = (1/g^2)\Big(1/(k^2 - m_e^2)\Big) , \qquad (41)$$

and consequently we obtain

$$D_{\mu\nu}(k) = \Big(1/(k^2 - m_e^2)\Big)\Big\{- g_{\mu\nu} + \Big[1 - \alpha\Big(1 - (m_e^2/k^2)\Big)\Big]\Big(k_\mu k_\nu/k^2\Big)\Big\}. \qquad (42)$$

From the above Green function we easily see that for the Landau gauge ($\alpha = 0$), the propagator (12) is reobtained with the addition of dinamically generated mass, as we have asserted before. Similarly we could work with the fermionic correlation functions.

V)Point-Splitting:

Here we discuss the equivalence of the anomaly of these models via the point-splitting method. For this we remember firstly that the gauge current and the chiral one, in Euclidean two-dimensional space, are related through

$$j_5^\mu(x) = i\epsilon^{\mu\nu}j^\nu(x), \qquad (43)$$

where we defined $j^{\mu}(x) \equiv \bar{\psi}(x)\gamma^{\mu}\psi(x)$ and $j_5^{\mu}(x) \equiv \bar{\psi}(x)\gamma^{\mu}\gamma_5\psi(x)$. So, we only need to calculate the vectorial current and, from it, obtain the chiral one.

In the point-splitting method we can define the vectorial current as[18]:

$$j^{\mu}(x) \equiv (1/2)\left[\lim_{\varepsilon\to 0+} j^{\mu}(x,\varepsilon,a) + \lim_{\varepsilon\to 0-} j^{\mu}(x,\varepsilon,a)\right], \tag{44a}$$

and

$$j^{\mu}(x,\varepsilon,a) \equiv \bar{\psi}(x-\varepsilon/2)\gamma^{\mu}\psi(x+\varepsilon/2)\exp\left\{-ie(2a+1)\int_{x-\varepsilon/2}^{x+\varepsilon/2} dz^{\nu}A^{\nu}(z)\right\}, \tag{44b}$$

where a is the regularization parameter. In principle the above current will be gauge-invariant only in the particular case when $a = -1$; below we will see that the introduction of a new gauge-dependent external field, the Wess-Zumino one, restores the gauge invariance for any value of a.

The averaged vectorial current is then written as

$$\langle j_{\mu}(x,\varepsilon,a)\rangle = \mathrm{Tr}\left\{\gamma_{\mu}\, G_{+}(x+\varepsilon/2, x-\varepsilon/2, A_{\mu})\exp\left[-ie(2a+1)\int_{x-\varepsilon/2}^{x+\varepsilon/2} dz^{\nu}A^{\nu}(z)\right\}, \tag{45}$$

where $G_{+}(x+\varepsilon/2, x-\varepsilon/2, A_{\mu})$ is the fermionic Green function in presence of the external field $A_{\mu}(x)$, and is given by

$$G_+(x + \varepsilon/2, x - \varepsilon/2, A_\mu) = \exp\left(-ie\left[\chi(x + \varepsilon/2) - \chi(x - \varepsilon/2)\right]\right) G_+(\varepsilon), \tag{46a}$$

and $G_+(\varepsilon)$ is given by

$$G_+(\varepsilon) = (-i/2\pi)\frac{\gamma^\mu \varepsilon^\mu}{\varepsilon^2}. \tag{46b}$$

Expanding the expression in powers of ε, we obtain that

$$\langle j_\mu(x,\varepsilon,a)\rangle = (-i/2\pi)\mathrm{Tr}\left\{\gamma_\mu\left(1 + ie\gamma_5(\partial_\sigma\chi)\varepsilon_\sigma\right)\frac{\gamma^\nu\varepsilon^\nu}{\varepsilon^2}\left(1 + -ie(2a + 1)\varepsilon^\rho A^\rho\right) + O(\varepsilon^2)\right\}. \tag{47}$$

Taking the symmetric limit defined in (44a), and after straigthforward calculations, we see that the current defined in (44b) is given by

$$\langle j^\mu(x)\rangle = (e/\pi)\left[\partial_\mu\partial_\nu/\Box - a\delta_{\mu\nu}\right]A_\nu(x). \tag{48}$$

Now, with this result at hand we can compute the divergence of the currents:

$$\langle\partial_\mu j_\mu(x)\rangle = (e/\pi)(1 + a)\partial_\mu A_\mu(x), \tag{49a}$$

$$\langle\partial_\mu j_{\mu,5}(x)\rangle = -(iea/\pi)\epsilon_{\mu\nu}\partial_\mu A_\nu(x). \tag{49b}$$

For $a = -1$, there is no gauge anomaly, all anomaly is concentrated in the chiral current.

With the above results we discuss the equivalence with the AM. For this we use the decomposition of the gauge field in equation (4), and obtain:

$$\langle \partial_\mu j^\mu_{SM}(x) \rangle = (g/\pi)(1 + a)\Box\eta, \tag{50a}$$

and

$$\langle \partial_\mu j^{\mu,5}_{SM}(x) \rangle = -(iag/\pi)\Box\chi. \tag{50b}$$

The divergency of the chiral current is exactly that for the AM, as can be seen in the work of Banerjee[7] and, for $a = 1$, in the original work of Rothe and Stamatescu[3]. Besides for a special gauge choice ($\eta = 0$), as is made in reference 14, the vector current vanishes as in the case of the AM. This result corroborates and clarify the observation in Banerjee's work, that there is a "striking resemblance between the two theories".

However it could be argued that the arbitrary parameter appearing in the anomaly of the AM, would not appear in SM because of gauge invariance. This argument is not valid because a Wess-Zumino field can be added, without changing physics, and the gauge invariance restored for any value of a [13].

In the point-splitting method this can be made by a suitable redefinition of the vectorial current,

$$j^{\mu}(x,\varepsilon,a) = \bar{\psi}(x - \varepsilon/2)\gamma^{\mu}\psi(x + \varepsilon/2)\exp\left\{ie\left[-(2a + 1)\int_{x-\varepsilon/2}^{x+\varepsilon/2} dz^{\nu}A^{\nu}(z) + \right.\right.$$

$$\left.\left. - 2(a + 1)\int_{x-\varepsilon/2}^{x+\varepsilon/2} dz^{\nu}\partial^{\nu}\theta(z)\right]\right\}, \quad (51)$$

where the fields $A_{\mu}(x)$ and $\theta(x)$, change by a gauge transformation as:

$$A_{\mu}(x) \rightarrow A_{\mu}(x) + \partial_{\mu}\alpha(x),$$

$$\theta(x) \rightarrow \theta(x) - \alpha(x). \quad (52)$$

With this definition we see that the gauge invariance is restored for the vectorial current.

VI) The effective action:

In this section, we calculate the photon effective action for the SM in the trivial topological sector, showing that it can keep its mass not fixed (through an arbitrary parameter) and simultaneously preserving gauge invariance. Usually it is argued that this symmetry can be preserved only for the mass obtained originally by Schwinger[1] ($m_e{}^2 = e^2/\pi$). Recently however, Harada and Tsutsui[13] showed that, if we perform the integration in the gauge volume, this gauge symmetry is kept for an arbitrary mass

($m_e^2 = ae^2/\pi$). Here, we intend to reobtain the result of Harada and Tsutsui[13] without performing such integration.

Starting from the Lagrangian density (5) without the sources, performing a gauge rotation, doing an integration by parts and neglecting surface terms, we obtain that

$$\mathcal{L} = \bar{\psi}\gamma^{\mu}(i\partial_{\mu})\psi + ig\chi\langle\partial_{\mu}j_{\mu,5}\rangle + (1/2e^2)\chi\Box\Box\chi, \tag{53}$$

Now, we can invert the transformation (4) for the field χ, so

$$\chi = (e/2g)\epsilon_{\mu\nu}\left(\frac{F^{\mu\nu}}{\Box}\right), \tag{54}$$

and after the substitution of the divergence of the chiral current calculated above, we have for the effective action:

$$S_{eff} = (-1/4)\int d^2x\left[F_{\mu\nu}\Box^{-1}\left(\Box + m_e^2\right)F^{\mu\nu}\right], \tag{55}$$

where $m_e^2 = ae^2/\pi$. This expression is identical to that obtained by Harada and Tsutsui for this model, but has appeared in a completely different way, without using any gauge group integration at any stage. This is an intriguing result, because it is generally assumed that the SM is gauge invariant only when the parameter a is made equal to one, except if one introduces the Wess-Zumino term via gauge volume integration. Besides it is not difficult to verify that the above procedure does not lead to an analogous result for the chiral Schwinger model (CSM)[4]. In fact we think that this occurs because when people are dealing with such models, the gauge-fixing term that comes from the Faddeev-Popov trick is not considered, and the absence of this term corresponds, in practice, to a particular choice of the gauge-fixing parameter α. Had we used this term, the longitudinal part of the gauge field would appear in the SM and so, the effective Lagrangian density would be not gauge invariant. On the other hand, this part of the gauge field appears quite naturally in the CSM, and this prevents the gauge invariance of the effective Lagrangian density.

In our opinion, this makes necessary the consideration of the gauge-fixing term from the begining in the Lagrangian density. Consequently we would have a modified Wess-Zumino term. This conjecture is presently under investigation.

VII) Conclusions:

We have proved that the SM and AM are equivalent, and this was made through the rules necessary to go from one model to the other and that their effective Lagrangians are identical, provided that the masses of the models are identified. Similarly, other models, as the CSM[4] and the generalized SM[19] , may have their "partner models" with Green functions related. In this sense they would be redundant models.

Besides, as a byproduct of our calculations, we rederived the effective Lagrangian of the SM obtained in reference 11 without any gauge volume integration, and discussed the possible explanation of this intriguing result.

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