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CRITICALITY OF THE DISCRETE N-VECTOR FERROMAGNET IN PLANAR SELF-DUAL LATTICES

by

C. TSALLIS*, A.M. MARIZ[+&], A. STELLA[§] and L.R. da SILVA[+]

*Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brazil

[+]Departamento de Física
Universidade Federal do Rio Grande do Norte
59000 - Natal, RN - Brazil

[§]Dipartimento di Fisica "Galileo Galilei"
Università degli Studi di Padova
Via F. Marzolo 8, 35131 Padova, Italy

[&]Service de Physique Théorique - CEN Saclay
91190 Gif-sur-Yvette, France

ABSTRACT

We study an extended version of the discrete N-vector (or cubic) ferromagnetic model within a real space renormalization group approach which preserves the two-spin correlation function. The N-evolution (for *real* values of N) of the Wheatstone-bridge hierarchical lattice phase diagram, which presents paramagnetic, intermediate (nematic-like) and ferromagnetic phases, as well as of the thermal (ν) and crossover (ϕ) critical exponents, is presented. The self-avoiding walk problem is recovered in the $N \to 0$ limit, and the so called "corner-rule" is reobtained in a larger context. The Ising, N- and 2N-state Potts ferromagnets are recovered as particular cases. An interchange of stability occurs at $N=N^* \simeq 6.9$ in such a way that the 2N-state Potts special point (where all three existing phases join) is a multicritical one if $N < N^*$ but only a critical one if $N > N^*$ (consistently $\phi(N^*)=0$). For the cubic model, $\nu(N)$ presents a maximum at $N=N_{max} \simeq 1.5$. The results are exact, for all N, for the Wheatstone-bridge hierarchical lattice, and approximate, for $N \leq 2$, for the square lattice. Last but not least, we discuss the connection between the present approach and the phenomenological renormalization group.

Key-words: N-vector model; Planar self-dual; Phase transition; Hierarchical lattice.

1 INTRODUCTION

In recent years several real space renormalization group (RG) methods have been developed, whose transformations describe with reasonable approximation spin models on Bravais lattices, and become exact for the same systems (if classical) on hierarchical structures. The Migdal-Kadanoff approximation as well as the methods developed in Refs. [1-4] are examples of this kind of approaches. Even if sometimes the approximations involved are not able to reproduce important qualitative features of models on Bravais lattices (like the first-order transitions of a Potts ferromagnet for sufficiently high number of states[5]), other results can even turn out to be exact, especially when the choice of the basic RG clusters respects some important symmetries of the infinite system (like self-duality for the square lattice). Moreover, as discussed in Section 4 of the present paper, if applied to big clusters, the methods of Refs. [1-4] can be shown to have the same potentialities as a phenomenological renormalization approach [6], in which the interfacial tension between different domains in a block is used, in place of the correlation length, as the basic scaling quantity. As illustrations of these potentialities see Refs. [7,8].

In this context, particularly appealing is the possibility of treating, within relatively simple renormalization schemes, whole classes of models, like the q-state Potts model for arbitrary q, or the Z(N) model for arbitrary N. Whereas for the former the renormalization transformation considered does not require a parameter space with dimension increasing with q, for the most general Z(N) model such dimension grows linearly with N, making the RG soon untractable. An important consequence of these facts is that, whereas for the Potts model it is possible to have the results for arbitrary *real* values of q (and consequently the important $q \to 1$ and $q \to 0$ limits, respectively the bond percolation and resistor problems, are easily accessible), to obtain for the Z(N) model results which are analytical in N is a non trivial task.

The main purpose of this paper is to present results, for arbitrary *real* values of N, for a particular realization of the Z(2N) model, the so called *discrete N-vector* (or *N-component*) *model* or even *cubic model*. This problem is tractable because, as we shall see, it presents the considerable advantage of requiring, in order to remain closed under renormalization, a parameter space which is, for any N, at most bidimensional. The cubic model has already been focused within various theoretical frameworks, such as Mean Field Approximation [9], Niemejer and van Leeuwen RG [10], Migdal RG [11], variational and dedecoration RG's [12], Monte Carlo-like approach [13], conformal invariance [14] and Monte Carlo RG [15]. Possible physical motivations (e.g., rare-earth compounds) are discussed in Refs. [9,11,12]. Here we study the cubic model within a RG approach which preserves appropriate two-spin correlation functions. All the results are exact for the Wheatstone-bridge hierarchical lattice; they are either exact (e.g., parts of the phase diagram for $N \leq 2$) or approximate (e.g., the critical exponents ν and ϕ) for the square lattice.

In Section 2 we introduce the model and the formalism; in Section 3 we present the general results as well as those corresponding to the $N \to 0$ limit (self-avoiding walk); in Section 4 we make the connection between the present approach and the phenomenological RG; finally we conclude in Section 5.

2 MODEL AND FORMALISM

The cubic model elementary interaction between spins i and j is described by the following dimensionless Hamiltonian:

$$\beta \mathcal{H}_{ij} = -NK\vec{S}_i . \vec{S}_j \qquad (1)$$

where $\beta \equiv 1/k_B T$ and where the spin $\vec{S}_i$ at any given site is a N-component unitary vector which can point only along the 2N positive or negative orthogonal coordinate directions, i.e., $\vec{S}_i=(\pm 1,0,0,\ldots,0)$ or $(0,\pm 1,0,\ldots,0)$ or $\ldots(0,0,0,\ldots,\pm 1)$. This interaction is a discrete version of the classical N-vector model. In what follows we shall consider a generalized form of it, namely given by

$$\beta \mathcal{H}_{ij} = -NK\ \vec{S}_i . \vec{S}_j - N^2 L\ (\vec{S}_i . \vec{S}_j)^2 \qquad (2)$$

which will prove to be closed under RG.

Hilhorst [10] has verified that model (1) reproduces, in the $N \to 0$ limit for L=0, the grand-canonical statistics of a self-avoiding walk (SAW) with step fugacity K. This result also holds for model (2) and extends to discrete spins the de Gennes' result [16] for continuous spins; it was in fact exploited for the early RG analysis of the SAW mentioned above [10]. For the particular case N=1, model (2) reduces to the spin 1/2 Ising model for all values of L. For N=2 we recover the Z(4) model (see, for example, Ref. [4] and references therein). If $NL = K$, model (2) recovers the 2N-state Potts model with dimensionless coupling constant 2NK. If K=0, model (2) recovers the N-state Potts model with dimensionless coupling constant N^2L. For finite K and $NL/|K| \to \infty$ we recover, for all values of N, the spin 1/2 Ising model with dimensionless coupling constant NK. Indeed, the second term of Hamiltonian (2) becomes dominant, and therefore only parallel and antiparallel spin configurations are possible at any finite temperature. To summarize all these particular situations, let us say, by using the notation (N_α, N_β)-model introduced by Domany and Riedel [11], that Hamiltonian (2) corresponds to the (N,2)-model.

Hamiltonian (2) is in general associated with a three-level system. For instance, if we assume $K > 0$ and $L > 0$, we have a fundamental level whose energy is $-N(K+NL)$ and whose degeneracy is 2N; the energy of the first excited level is 0 and its degeneracy is $4N(N-1)$; finally, the energy of the second excited level is $N(K-NL)$ and its degeneracy is 2N.

If we consider now a two-rooted graph made by a series array of two bonds with coupling constants $(K^{(1)}, L^{(1)})$ and $(K^{(2)}, L^{(2)})$ respectively, its Hamiltonian will be given by

$$\beta\mathcal{H}_{123} = -NK^{(1)}\vec{S}_1.\vec{S}_3 - N^2L^{(1)}(\vec{S}_1.\vec{S}_3)^2 - NK^{(2)}\vec{S}_3.\vec{S}_2 - N^2L^{(2)}(\vec{S}_3.\vec{S}_2)^2 \tag{3}$$

where $\vec{S}_1$ and $\vec{S}_2$ are the terminal spins and $\vec{S}_3$ the internal one. For all statistical equilibrium properties which do not directly involve $\vec{S}_3$, $\beta\mathcal{H}_{123}$ can be replaced by

$$\beta\mathcal{H}'_{12} = -NK^{(s)}\vec{S}_1.\vec{S}_2 - N^2L^{(s)}(\vec{S}_1.\vec{S}_2)^2 - K'_o \tag{4}$$

where we impose

$$e^{-\beta\mathcal{H}'_{12}} = \operatorname*{Tr}_3 e^{-\beta\mathcal{H}_{123}} \tag{5}$$

with $K^{(s)}, L^{(s)}$ and K_o' to be determined. The results (except for K_o', which is not important in the present context) can be written as follows:

$$t_r^{(s)} = t_r^{(1)}\, t_r^{(2)} \qquad (r=1,2) \tag{6}$$

where the *vector thermal transmissivity* (t_1,t_2) (see [2,4,12]) is related to (K,L) through the definitions

$$t_1 \equiv \frac{1-e^{-2NK}}{1+2(N-1)\,e^{-N(K+NL)} + e^{-2NK}} \tag{7.a}$$

and

$$t_2 \equiv \frac{1-2\,e^{-N(K+NL)} + e^{-2NK}}{1+2(N-1)\,e^{-N(K+NL)} + e^{-2NK}} \tag{7.b}$$

For the 2N-state Potts model (K=NL) we have $t_1=t_2$, for the N-state Potts model (K=0) we have $t_1=0$, and for the Ising model $(NL/|K| \to \infty)$ we have $t_2=1$. In all these cases we recover the definition of thermal transmissivity introduced in [2]. For N=2, (t_1,t_2) reproduce the vector transmissivity of the Z(4) model as defined in [4]. It is finally worthy to mention that the cubic model (L=0) corresponds to the equation $(N-2)t_2^2 + 2t_2 = Nt_1^2$.

Equations (7) yield, through inversion,

$$e^{-N(K+NL)} = \frac{1-t_2}{1+Nt_1+(N-1)t_2} \tag{8.a}$$

and

$$e^{-2NK} = \frac{1-N\,t_1 + (N-1)\,t_2}{1+N\,t_1 + (N-1)\,t_2} \tag{8.b}$$

We note that for N=2 and only then, the functional forms of the transformation $(t_1,t_2) \rightleftarrows (e^{-N(K+NL)}, e^{-2NK})$ are one and the same. In other words, if we define $(t_1,t_2) = F_N(e^{-N(K+NL)}, e^{-2NK})$, in general $F_N^{-1} \neq F_N$, but $F_2^{-1} = F_2$. This fact will make, as we shall see further on, the N=2 model to be a special one.

Let us now consider a parallel (instead of series) array of two bonds with coupling constants $(K^{(1)},L^{(1)})$ and $(K^{(2)},L^{(2)})$. The equivalent coupling constants $(K^{(p)},L^{(p)})$ will be now given by

$$K^{(p)} = K^{(1)} + K^{(2)} \tag{9.a}$$

and

$$L^{(p)} = L^{(1)} + L^{(2)} \tag{9.b}$$

or equivalently

$$t_1^{(p)} = \frac{t_1^{(1)} + t_1^{(2)} + (N-1)\,t_2^{(1)}\,t_1^{(2)} + (N-1)\,t_1^{(1)}\,t_2^{(2)}}{1 + N\,t_1^{(1)}\,t_1^{(2)} + (N-1)\,t_2^{(1)}\,t_2^{(2)}} \tag{10.a}$$

$$t_2^{(p)} = \frac{t_2^{(1)} + t_2^{(2)} + N\,t_1^{(1)}\,t_1^{(2)} + (N-2)\,t_2^{(1)}\,t_2^{(2)}}{1 + N\,t_1^{(1)}\,t_1^{(2)} + (N-1)\,t_2^{(1)}\,t_2^{(2)}} \tag{10.b}$$

These equations can also be written as follows:

$$(t_r^{(p)})^D = (t_r^{(1)})^D\,(t_r^{(2)})^D \qquad (r=1,2)$$

with

$$t_1^D \equiv \frac{1-N t_1 + (N-1) t_2}{1+N t_1 + (N-1) t_2} \qquad (11.9)$$

$$t_2^D \equiv \frac{1-t_2}{1+N t_1 + (N-1) t_2} \qquad (11.b)$$

For a full discussion of this kind of "dual" variables see [17].

Now that we have introduced the variables t_1 and t_2 (very convenient at the present time for representing the RG flow diagrams, and possibly in future for formulating a Break-collapse method [2,4,17,18]), let us focus the ferromagnetic model in square lattice. The Hamiltonian will be given by

$$\beta\mathcal{H} = -NK \sum_{<i,j>} (\vec{S}_i.\vec{S}_j) - N^2 L \sum_{<i,j>} (\vec{S}_i.\vec{S}_j)^2 \qquad (12)$$

where the sums run over all pairs of first-neighbouring sites, $K > 0$ and $L \geq -K/N$.

In a way similar to what happens for the Potts ferromagnet, the L=0 transition is expected to become first order on a Bravais lattice for high enough N. Mean field predicts first order for $N > 3$ [9]. Real space renormalization group calculations in two dimensions indicated first order for $N > N_c = 2$ [15]. At the present moment we will leave out of the discussion the aspects connected with the first order transition, and focus more on the peculiar features of the phase diagram of hierarchical lattices, which can be obtained exactly without the introduction of vacancies [12]. The hierarchical lattice we consider here in particular is that corresponding to the Wheatstone bridge cluster of Fig. 1. This cluster, due to its self-duality, warranties coincidence of the critical couplings with those of the infinite square lattice, in all cases in which the model becomes self-dual.

To construct our RG we impose

$$e^{-\beta \mathcal{H}'_{12}} = \mathrm{Tr}_{3,4}\; e^{-\beta \mathcal{H}_{1234}} \tag{13}$$

where $\mathcal{H}'_{12}$ and $\mathcal{H}_{1234}$ respectively are the Hamiltonians associated with the small and large graphs of Fig. 1 ($\mathcal{H}'_{12}$ in particular is explicitly written in Eq. (4) with (K',L') substituting $(K^{(s)},L^{(s)})$). Equation (13) yields

$$K' = \frac{1}{2N} \ln \frac{G_1}{G_2} \tag{14}$$

and

$$L' = \frac{1}{2N^2} \ln \frac{G_1 G_2}{G_3^2} \tag{15}$$

with

$$G_1 \equiv e^{5N^2L}(e^{5NK} + e^{-3NK} + 2e^{-NK}) + 2(N-1)[2e^{2N^2L}$$
$$(e^{2NK} + e^{-2NK}) + e^{N^2L}(e^{NK} + e^{-NK}) + 2N-4] \tag{16}$$

$$G_2 \equiv 2e^{5N^2L}(e^{NK} + e^{-NK}) + 2(N-1)[4e^{2N^2L}$$
$$+ e^{N^2L}(e^{NK} + e^{-NK}) + 2N-4] \tag{17}$$

$$G_3 \equiv 2\Big\{e^{3N^2L}(e^{3NK} + 3e^{-NK}) + e^{2N^2L}(e^{2NK} + 2 + e^{-2NK})$$
$$+ (N-2)[5e^{N^2L}(e^{NK} + e^{-NK}) + 2N-6]\Big\} \tag{18}$$

Equations (14) and (15) provide the RG recurrence relations we were looking for. For fixed N, the RG flow in the (K,L) space (or equivalently in the (t_1,t_2) space) will determine the phase diagram as well as the universality classes. The numerical values of thermal and crossover exponents (ν and ϕ respectively) can be obtained through the calculation of the Jacobian matrix $\partial(K',L')/\partial(K,L)$ on the various semi-stable or fully unstable fixed points. More specifically, if we denote λ_1 and λ_2 the eigenvalues of the matrix we have:

(i) $\lambda_1 > 1 > \lambda_2$ for critical (semi-stable) fixed points, and

$$\nu = \frac{\ell n\, B}{\ell n\, \lambda_1} \tag{19}$$

where B is the linear expansion factor (B=2 for Fig. 1).

(ii) $\lambda_1 > 1$ and $\lambda_2 \gtrsim 1$ for multicritical (fully unstable) fixed points,

$$\nu_s = \frac{\ell n\, B}{\ell n\, \lambda_s} \qquad (s=1,2) \tag{20}$$

and

$$\phi = \frac{\ell n\, \lambda_2}{\ell n\, \lambda_1} \tag{21}$$

where λ_2 denotes that eigenvalue which, while varying N, tends to unity whereas λ_1 remains greater than unity.

3 GENERAL RESULTS

The phase diagrams for typical values of N are presented in Figs. 2(a) (in the (t_1,t_2) variables) and 2(b) (in the (1/K,NL/K) variables). For a given value of N, the phase diagram presents three phases, namely the *paramagnetic* (P; characterized by the fully stable fixed point $t_1=t_2=0$), the *ferromagnetic* (F; characterized by the fully stable fixed point $t_1=t_2=1$) and the *intermediate* (I; characterized by the fully stable fixed point $(t_1,t_2)=(0,1)$) ones. The existence of three distinct phases is well known for N=2 (Z(4) model). This structure analytically remains so for *all* values of N, including for $N \leq 1$ where it should be considered as a mathematical artifact. Indeed, for N=1 (Ising model), the P-I critical frontier should be considered as a spurious one, since for this model only two distinct phases exist, namely the ferromagnetic phase (F) and the paramagnetic one (P and I); as expected, the physically meaningfull critical temperature for N=1, does not depend on NL/K ("vertical" line in Fig. 2(a), and "horizontal" line in Fig. 2(b)).

The critical frontier corresponding to a given value of N contains *four* special points, namely three semi-stable fixed points (*critical* points) and a fully unstable one (*multicritical* point). Two of the three critical points are the Ising one ($(t_1,t_2) = (\sqrt{2}-1,1)$) and the N-state Potts one ($(t_1,t_2)=(0,1/(\sqrt{N}+1))$). The third and fourth special points are the 2N-state Potts one ($t_1=t_2=1/(\sqrt{2N}+1)$) and the *extended cubic* one ($(t_1,t_2)=(t_1^c,t_2^c)$ where the associated transmissivities and coupling constants are given in Figs. 3(a) and 3(b) respectively). For $N < N^* \simeq 6.9$ the 2N-state Potts model corresponds to the multicritical one and the extended cubic model corresponds to the critical one; the situation is reversed for $N>N^*$. At $N=N^*$ a special multicritical point emerges as the 2N-state Potts and the extended cubic fixed points collapse; at this value of N the two models exchange stability.

The thermal critical exponent ν_T as well as the crossover exponent ϕ are shown in Figs. 4(a) and 4(b) for the 2N-state Potts and the extended cubic models respectively. In particular, in Fig. 4(a) we recover well known values of ν_T for the Wheatstone-bridge hierarchical lattice Potts model, namely $\nu_T \simeq 1.43$ for the bond percolation model (N=1/2), and $\nu_T \simeq 1.15$ for the Ising model (N=1); it is also worthy to mention that ϕ=1 for N=1/2. Also, in the $N \to \infty$ limit we obtain $\nu_T = \ell n\, 2/\ell n\, 5 \simeq 0.43$, in accordance with the conjecture [19,20] that ν_T should give $1/d_f$ where d_f is the intrinsic fractal dimensionality. Finally, our numerical results suggest that, in the limit $N \to \infty$, the exponents ν_T associated with the 2N-Potts and extended cubic models coincide.

A limit of special interest is the $N \to 0$ one as it corresponds to the self-avoiding walk problem (SAW). In the Fig. 3(b) we see that $K_c = (\sqrt{3}-1)/2 \simeq 0.366$ which corresponds to the exact critical fugacity for the Wheatstone bridge hierarchical lattice (for the square lattice we have $K_c \simeq 0.3790$ [21]. The corresponding value for ν_T is given by $\nu_T = \ell n\, 2/\ell n(4-\sqrt{3}) \simeq 0.85$ (see Fig. 4(b)) to be compared with the value 3/4 [21]. In fact, the present RG precisely recovers (and consequently further supports), in the $N \to 0$ limit, the "corner rule" [22]. Indeed, this rule provides the RG recursive relation $K' = 2K^2 + 2K^3$, whose critical fixed point and thermal exponent precisely are $K_c = (\sqrt{3}-1)/2$ and $\nu_T = \ell n\, 2/\ell n(4-\sqrt{3})$.

4 CONNECTION WITH THE PHENOMENOLOGICAL RG APPROACH

As stressed in the previous sections, the renormalization procedure applied in this work [1-4] is exact for a hierarchical lattice, while it is expected to be a more or less good approximation for systems on a Bravais lattice. In this section we intend to better clarify the nature of this approximation by making explicit the connection between the present approach and the phenomenological RG [6] (see also Ref. [23]).

To avoid unnecessary complications, let us focus on the particular case of the d=2 Ising model (N=1). We can omit vector notations and represent simply by $S_i = \pm 1$ the spin at site i.

Successive clusters of the Wheatstone-bridge family are reported in Fig. 5 (the b=1 and b=2 clusters are shown in Fig. 1). On each of these clusters (with b(b-1) internal spins), the summation procedure leading to the renormalized coupling constant K' can be interpreted as the calculation of an *interface* free energy for blocks of the type indicated in Fig. 6. The spins on the upper and bottom horizontal sides of the block are left out of the summation. Indeed, if we indicate by {S} the configurations of the internal spins of the cluster (i.e., other than S_1 and S_2) we have:

$$e^{K'S_1S_2+g} = \mathop{\mathrm{Tr}}_{\{S\}} e^{-\beta\mathcal{H}(\{S\};S_1,S_2)} \equiv Z_{S_1S_2}(K) \qquad (22)$$

where g is an appropriate spin-independent term. From Eq. (22) we obtain

$$K' \equiv K'(K,b) = \frac{1}{2}[\ln Z_{++} - \ln Z_{+-}] \qquad (23)$$

This means that K' is nothing but the dimensionless excess free energy produced by fixing the horizontal sides to (+) and (-), compared to the case in which both sides are fixed, say, to (+). By definition of the (dimensionless) surface tension σ, we thus have

$$K'(K,b) = (b-1)\sigma(K,b) \tag{24}$$

where $\sigma(K,b)$ is expected to become independent of b in the $b \to \infty$ limit (thermodynamic limit).

From finite size scaling [24] we expect, for $K \sim K_c$ and $b \to \infty$,

$$\sigma(K,b) \simeq b^{-1}\,\sigma_0(b/\xi_\infty(K)) \sim 1/\xi(K,b) \tag{25}$$

where $\xi_\infty(K)$ is the correlation length of the infinite system, σ_0 is a scaling function with $\sigma_0(0) \neq 0$, and $\xi(K,b)$ is the correlation length in the finite block.

If we now define, as often done [1,2,7], a renormalized coupling constant K_{ren} corresponding to a linear rescaling factor b/b' ($b'<b$), through the following cell to cell recurrence relation

$$K'(K_{ren},b') = K'(K,b) \quad , \tag{26}$$

it follows, from Eqs. (24) and (25) and for large b and b', that

$$\xi(K_{ren},b') = \frac{b'}{b}\,\xi(K,b) \tag{27}$$

This is nothing but the definition of renormalized coupling constant in a phenomenological approach [6]. It is clear that various choices can be done for the cells to be used. In particular, the standard choice in the phenomenological approach is finite x infinite strips, whereas here we are using finite x finite self-dual clusters. In view of the nice convergence of results generally obtained with phenomenological renormalization methods, the preceeding arguments justify the usual strategy of improvement of the results herein obtained (as well as in similar treatments) as that of considering cell to cell transformations $K \to K_{ren}$, like in Eq. (26), with both b and b' becoming increasingly large (as usually done in the phenomenological RG).

The above derivation can of course be easily generalized to the case of dimensionality $d \neq 2$, and to models different from the Ising one.

Summarising, we see that the procedure we have used here should not be interpreted as another type of decimation RG approximation. Indeed, although we impose the correlation function to be preserved, we do so between the roots of the graphs, which corresponds to imposing the surface free energy to be preserved in the Bravais blocks, whereas in the decimation procedures what is imposed is the preservation of the correlation function between two sites of the Bravais lattice. This makes a substantial difference since the decimation procedures, unless conveniently handled, bring along intrinsic difficulties related to the spin rescaling. These difficulties do not exist in the present approach.

The present analysis makes clear that the well known limitations of the Migdal-Kadanoff-like approaches are not due to the fact that correlation functions are preserved, but rather to the fact that diamond (or tress) choices for the graphs lead, even for large clusters, to topologies which are not at all those of the Bravais lattices which are suposed to be approached.

5 CONCLUSION

We have considered the criticality of the discrete N-vector ferromagnet in planar self-dual lattices. The approach is a real space renormalization group one which exactly preserves correlation functions between the roots of conveniently chosen two-rooted graphs. The renormalization leaves invariant not the standard discrete N-vector model (cubic model) but a generalized version of it. The results are exact for the associated hierarchical lattices, and good estimates for the square lattice ($N \leq 2$). The phase diagram (including multicritical points) associated with fixed N, as well as the thermal and crossover exponents, are calculated. At a value of N (noted N*) an exchange of stability is observed between the Potts and cubic models ($N^* \simeq 6.9$ for the Wheatstone-bridge hierarchical lattice). In the $N \to 0$ limit we recover the self-avoiding walk, and give support to the "corner rule" since long used in this problem.

In addition to the above results, we have exhibited the connection between the present (correlation-function preserving) renormalization procedure and the phenomenological renormalization group. This connection makes clear that these two commonly used renormalization procedures share essentially the same advantages and limitations.

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CAPTION FOR FIGURES

Fig. 1 - Iteration associated with the Wheatstone-bridge RG (● 0 respectively denote the internal and terminal sites of the graph).

Fig. 2 - (a) Phase diagram in the (t_1,t_2) space for typical values of N: P, F and I respectively denote the paramagnetic, ferromagnetic and intermediate phases. The arrows indicate the RG flow; ■ and ● respectively indicate stable and unstable fixed points. The line $t_1=t_2$ corresponds to the 2N-state Potts model. (b) Phase diagram in the (1/K, 1+LN/K) space for typical values of N.

Fig. 3 - N-dependence of the location of the extended cubic fixed point: (a) (t_1,t_2) variables; (b) (K, 1+NL/K) variables.

Fig. 4 - N-dependences of the thermal critical exponent ν_T and the crossover exponent $\emptyset$: (a) 2N-state Potts model; (b) extended cubic model.

Fig. 5 - b=3 and b=4 generating graphs of the Wheatstone-bridge family of hierarchical lattices.

Fig. 6 - b=3 and b=4 blocks of spins respectively corresponding to those of Fig. 5.

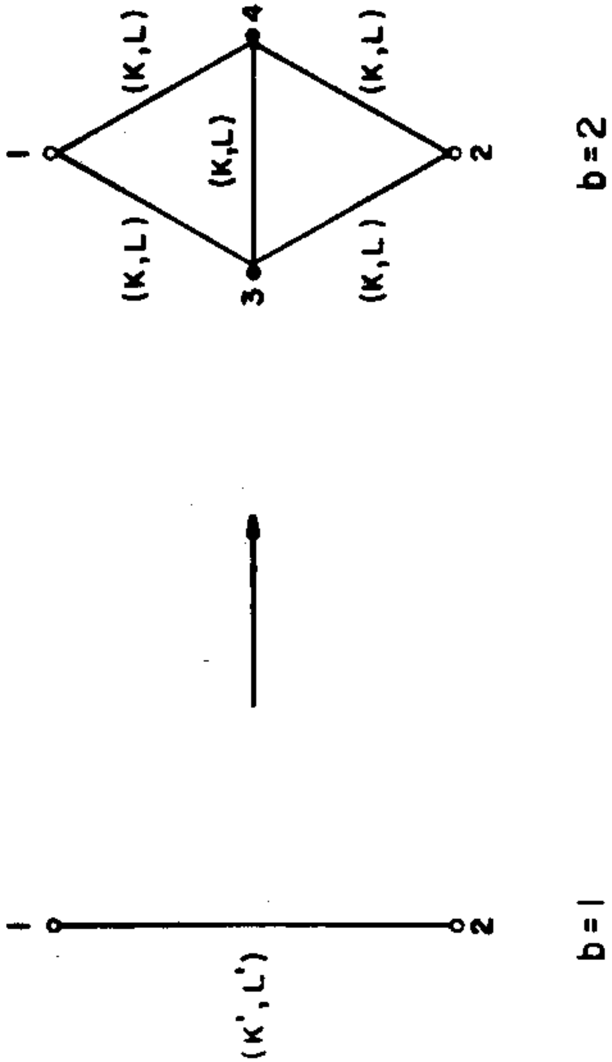


FIG. 1

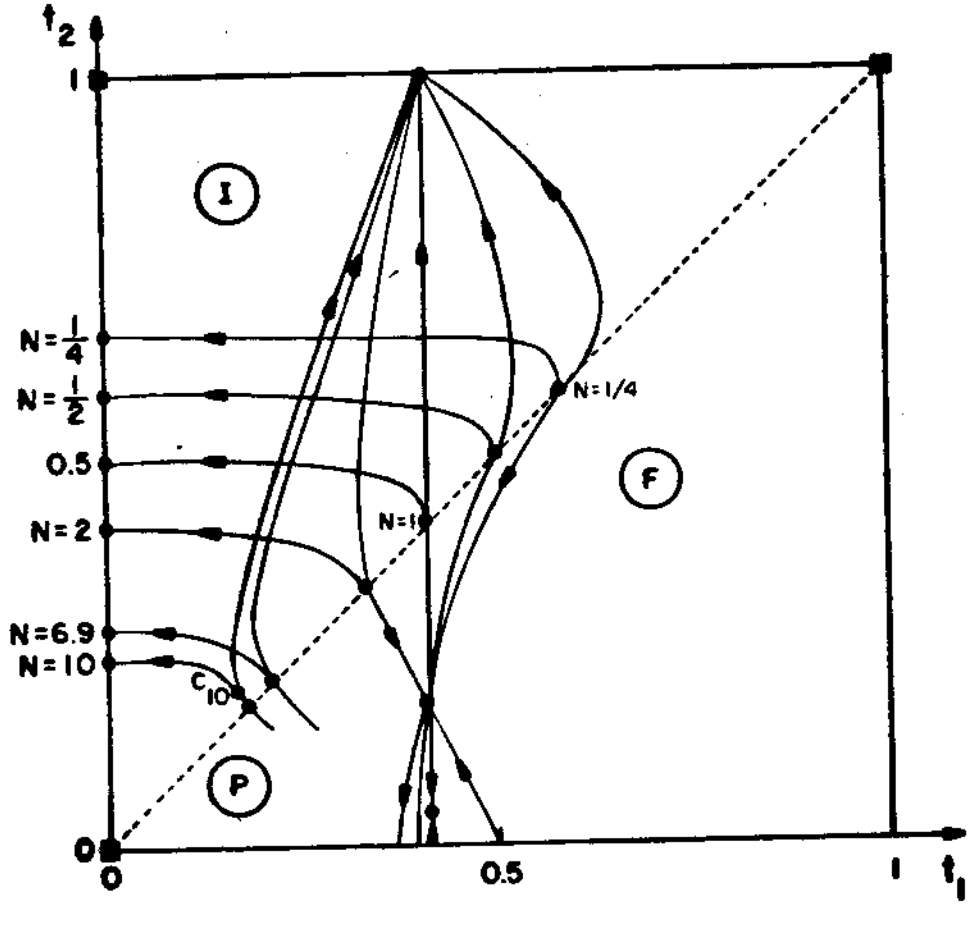


FIG. 2 (a)

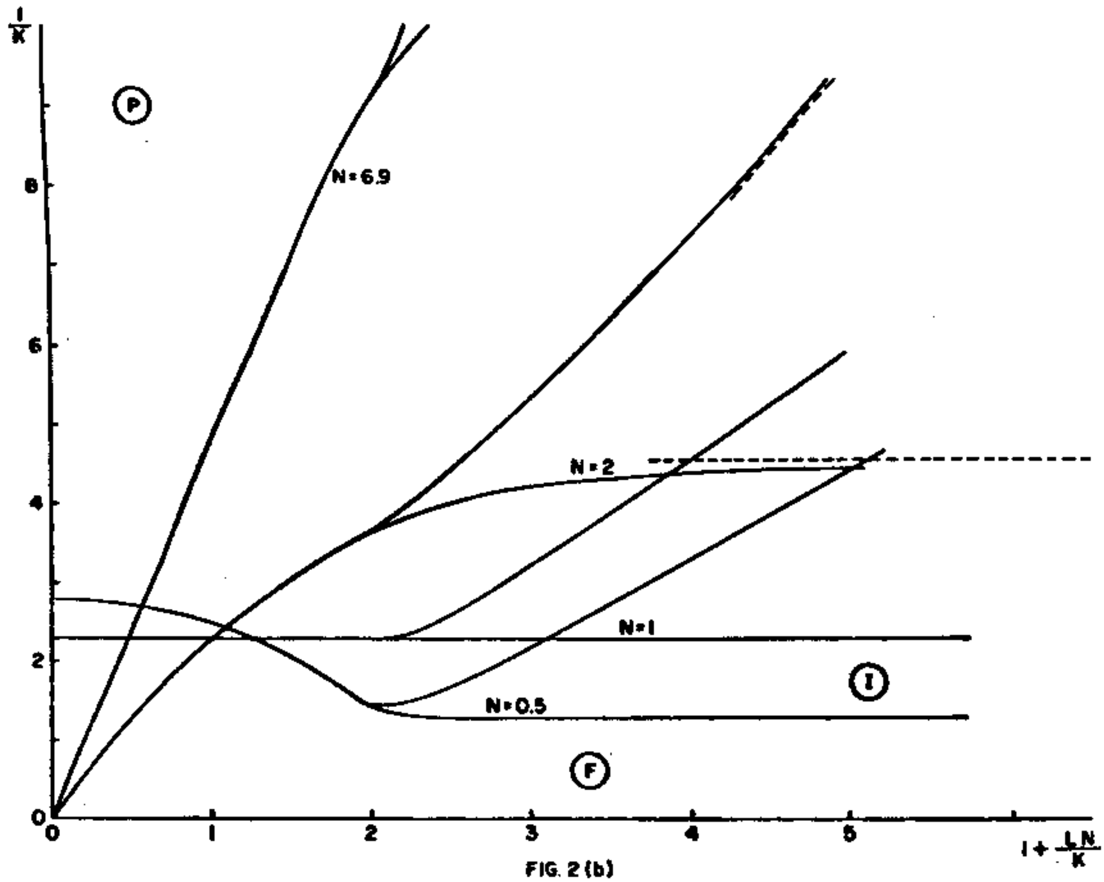


FIG. 2 (b)

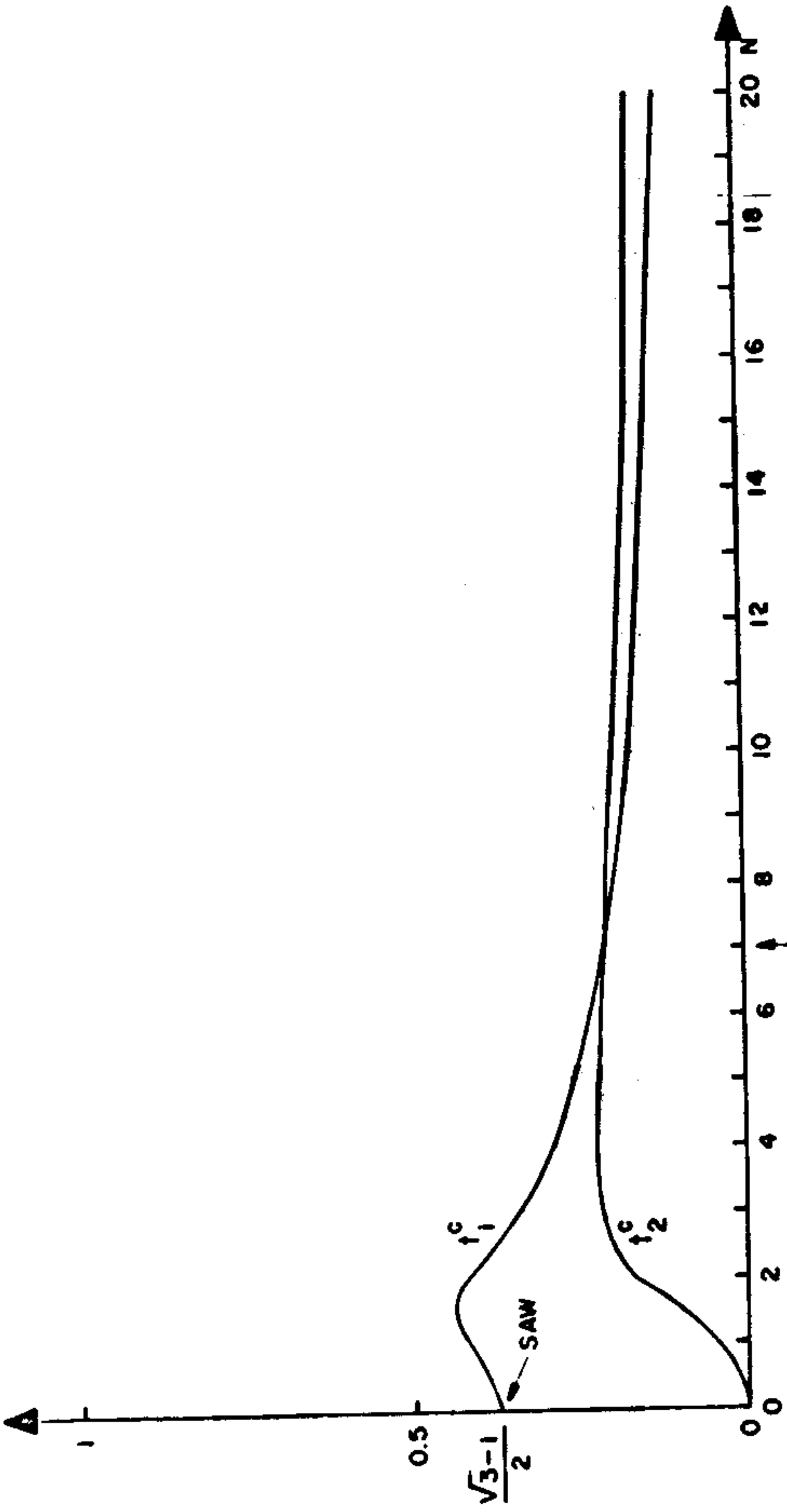


FIG. 3(a)

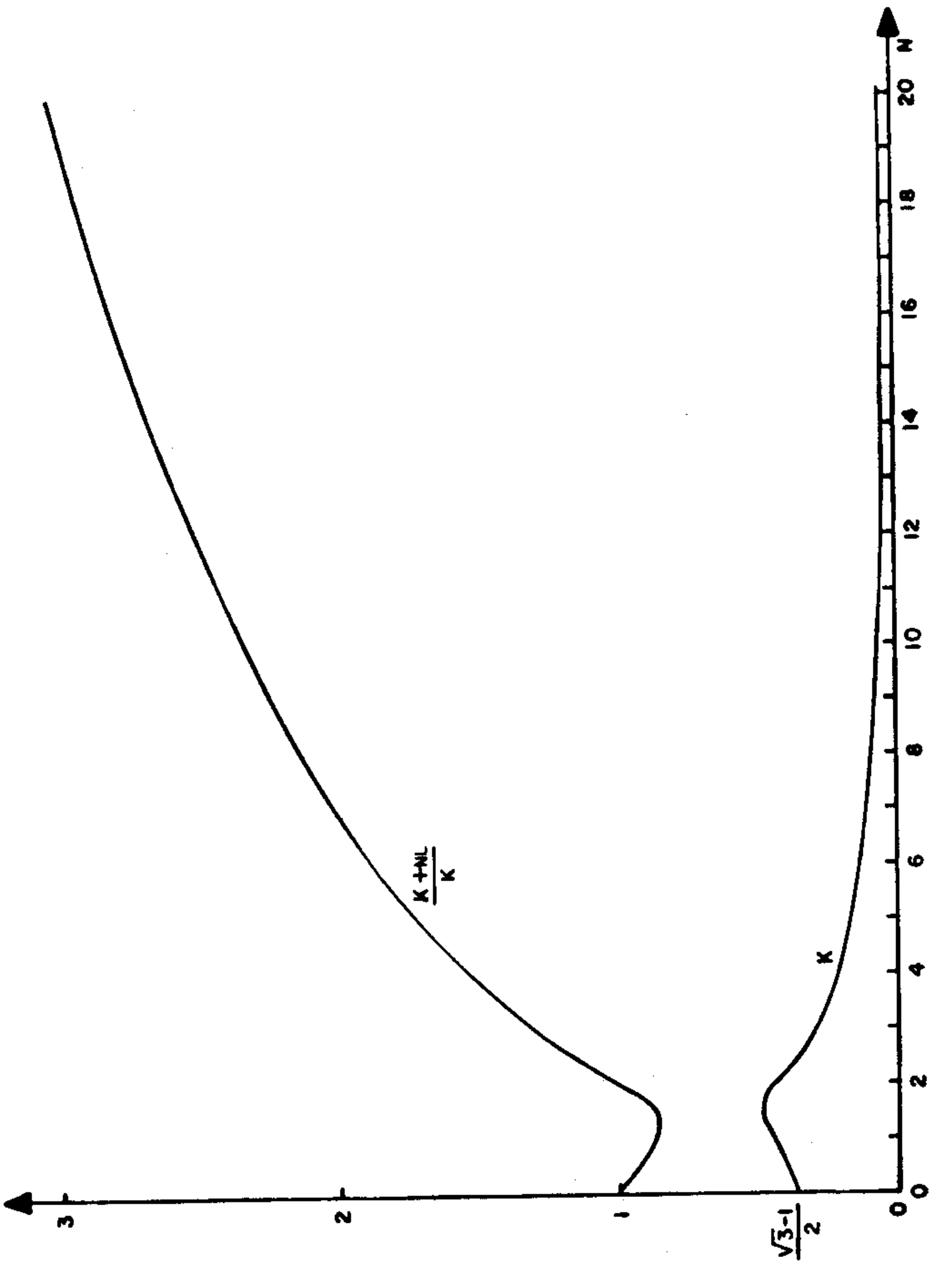


FIG. 3(b)

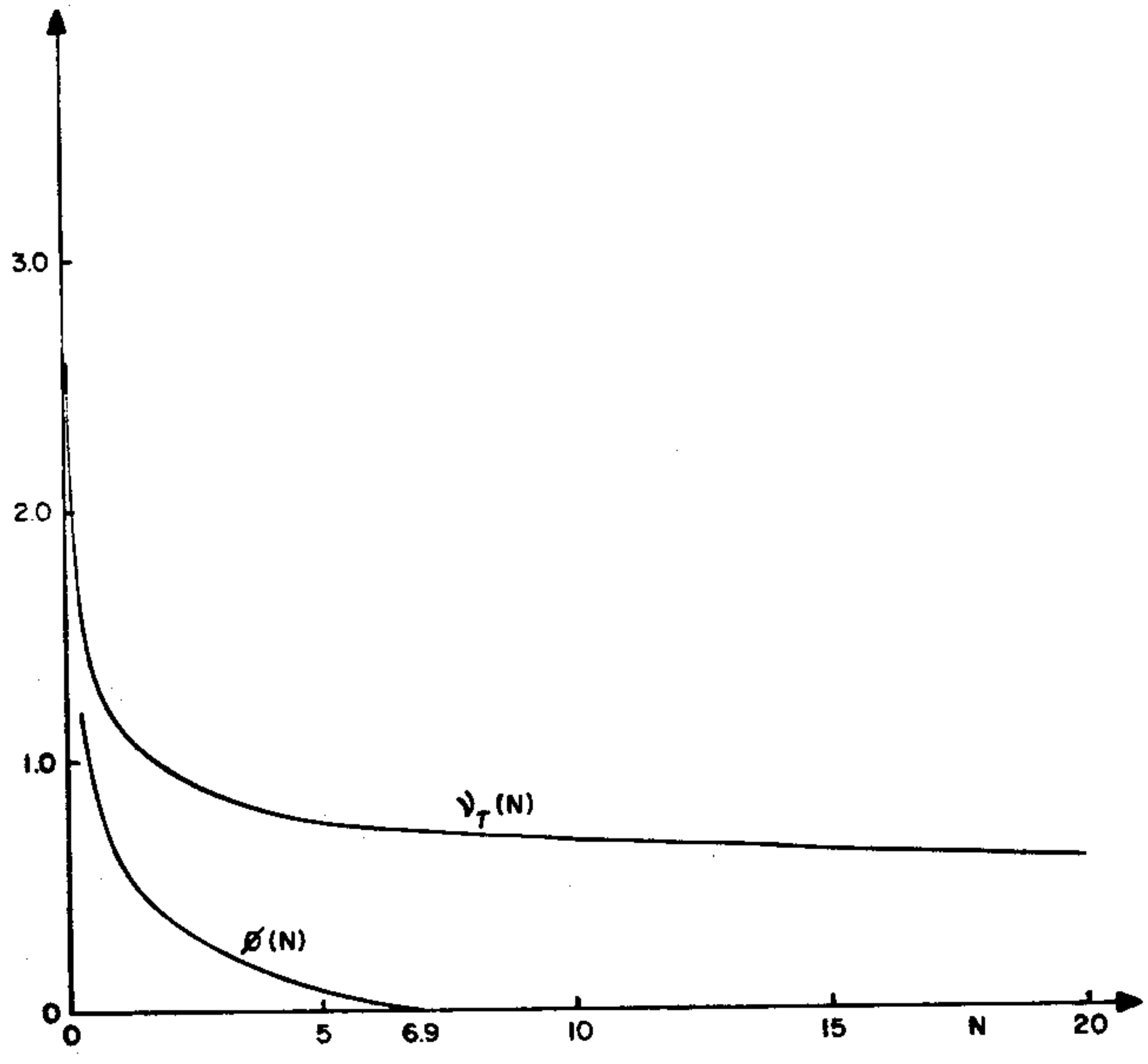


FIG. 4(a)

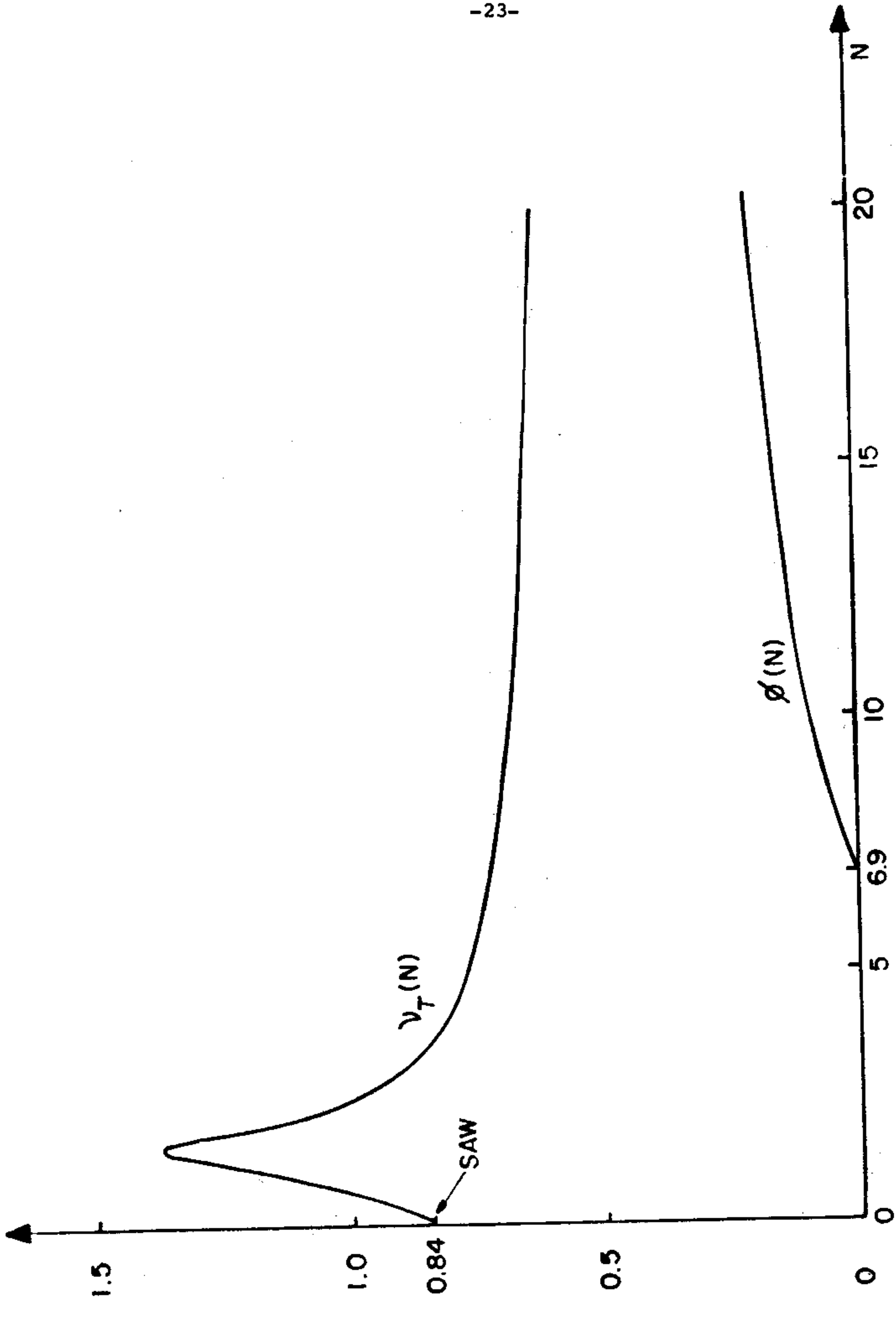


FIG. 4 (b)

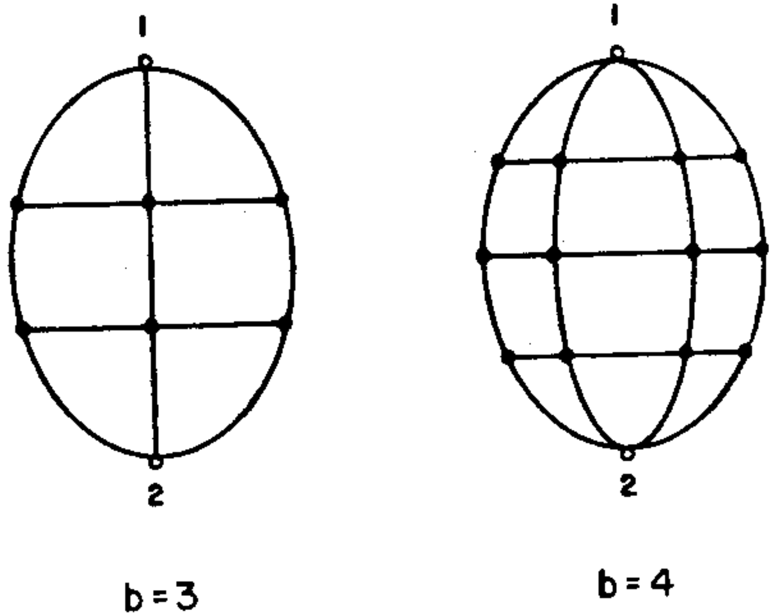
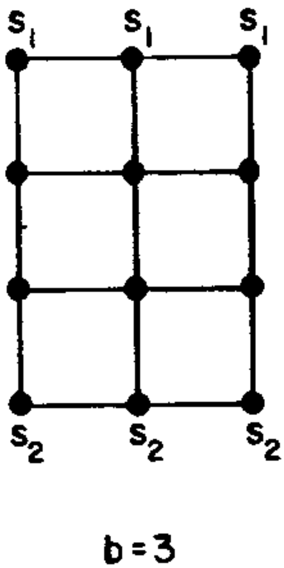


FIG. 5



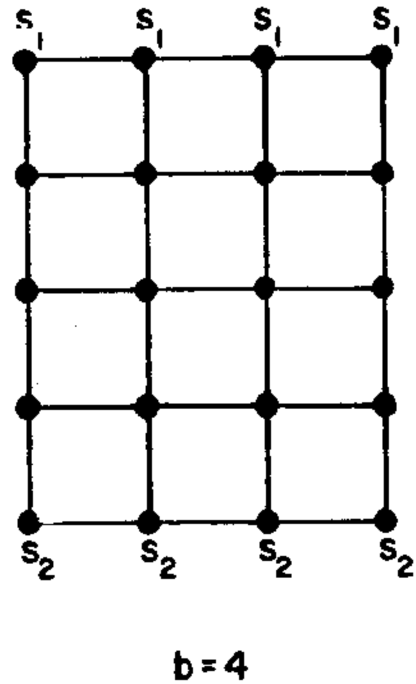


FIG. 6

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