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ON THE CONTRIBUTION OF A HEAVY NEUTRAL PARTICLE TO
W-BOSON PAIR PRODUCTION IN ELECTRON-POSITRON
ANNIHILATION

by

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ABSTRACT

We study the effect of a heavy neutral particle in the process $e^+e^- \rightarrow W^+W^-$. From the unitarity of the cross section we have limits on the mass and couplings of the N . Considering the contribution of this heavy particle to the anomalous magnetic moment of the electron we infer a limit to the mixing angle between ordinary and excited (heavy) matter.

Key-words: $e^+e^- \rightarrow W^+W^-$; Heavy neutrinos; Mass limits.

The available experimental data confirms quite remarkably the predictions of the standard model of the electroweak interactions based on the $SU(3) \times SU(2) \times U(1)$ group [1] (the Glashow-Salam-Weinberg model). But, as has been exhaustively discussed, the number of parameters and unexplained structures indicates that it is not a truly fundamental theory.

All the candidates to be an extension of the GSW must necessarily reproduce the standard model at low energies. This feature is usually satisfied by imposing that the new particles and/or properties will only show themselves at an energy scale soon accessible in the big machines: LEP II, SLAC, TEVATRON, SSC, ...

The phenomenological agreement at low energies is not the only requirement we can impose. It is very important to have a theory that is self-consistent, renormalizable and anomaly free in order to perform reliable estimates.

The advent of the new colliders will also permit to test two remaining problems of the standard model: the existence of the Higgs particle and the Yang-Mills structure of the self-couplings of the electroweak vector-bosons.

The W boson pair production in e^+e^- annihilation is a process whose acceptable behavior at high energies (σ_{obs}^{-1} ns) is given by the gauge theory cancellations between the direct diagrams (γ and Z exchange) and the cross-channel diagram (ν diagram). To emphasize we repeat that this gauge cancellation is vital for the renormalizability of the theory.

The effect of a new neutral boson, hereafter named Z_2 , in the process $e^+e^- \rightarrow W^+W^-$ has been studied by many authors[2]. Our purpose here in this paper is to investigate the contribution of a new heavy neutral lepton, N , in the W boson pair production besides the Z_2 role. The relevant diagrams are depicted in figure 1.

A heavy neutral lepton is proposed in almost every extension of the standard model: superstring inspired[3], mirror fermions[4], and composite models[5], for example.

We will take a general N-electron coupling and analyze the possible bounds on it as well as on the N mass.

The general interaction is described by the lagrangian:

$$\begin{aligned} \mathcal{L} = & -e \bar{e} \gamma^\mu e A_\mu + \sum_{a=1,2} \bar{e} \gamma^\mu (g_V^{(a)} - g_A^{(a)} \gamma^5) e Z_\mu + G_{VA} \bar{e} \gamma^\mu (1 - \gamma^5) \nu W_\mu + \\ & + G_{VA} \bar{e} \gamma^\mu (a + b \gamma^5) N W_\mu + \text{h.c.} + \mathcal{L}_{\text{boson}} \end{aligned} \quad (1)$$

where $g_V(a)$ and $g_A(a)$ depends on the particular model we use, and G_{VA} is the usual standard model $e\nu$ coupling. Z_1 is the known neutral boson.

We consider a general mixing between Z_1 and Z_2 so that the mass eigenstates are related to the physical ones (Z, Z') by:

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_M & \sin \theta_M \\ -\sin \theta_M & \cos \theta_M \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix} \quad (2)$$

and

$$\begin{aligned} \mathcal{L}_{\text{boson}} = & -ie [(A^\nu W^\mu - A^\mu W^\nu) \partial_\mu W_\nu^\dagger + \\ & + (A^\nu W^{\mu\dagger} - A^{\mu\dagger} W^\nu) \partial_\mu W_\nu + (\partial^\mu A^\nu - \partial^\nu A^\mu) W_\mu W_\nu^\dagger] \\ & - ie \cos \theta_M \cotan \theta_W [A \leftrightarrow Z_1] + ie \sin \theta_M \cotan \theta_W [A \leftrightarrow Z_2] \end{aligned} \quad (3)$$

A straightforward calculation gives:

$$\frac{d\sigma}{dt} = \frac{2\pi\alpha^2}{s^2} \sum_{ij} B_{ij} \quad (4)$$

where the B_{ij} are:

$$\begin{aligned}
 B_{\gamma\gamma} &= A(s, t, u) & D_{\nu\nu} &= \frac{8G_{VA}^4}{e^4} E(s, t, u) \\
 B_{z_1 z_1} &= \cos^2 \theta_M \left(\frac{e_2}{e^2} \right)^2 \left[(g_V^{(1)})^2 + (g_A^{(1)})^2 \right] \left(\frac{s}{s - M_{z_1}^2} \right)^2 A(s, t, u) \\
 B_{z_2 z_2} &= \sin^2 \theta_M \left(\frac{e_2}{e^2} \right)^2 \left[(g_V^{(2)})^2 + (g_A^{(2)})^2 \right] \frac{s^2}{(s - M_{z_2}^2)^2 + M_{z_2}^2 \Gamma_{z_2}^2} A(s, t, u) \\
 B_{\gamma\nu} &= - \frac{4 G_{VA}^2}{e^2} I(s, t, u) \\
 B_{\gamma z_1} &= - 2 \cos \theta_M \left(\frac{e_2}{e^2} \right) g_V^{(1)} \frac{s}{s - M_{z_1}^2} A(s, t, u) \\
 B_{\gamma z_2} &= 2 \sin \theta_M \left(\frac{e_2}{e^2} \right) g_V^{(2)} \frac{s (s - M_{z_2}^2)}{(s - M_{z_2}^2)^2 + M_{z_2}^2 \Gamma_{z_2}^2} A(s, t, u) \\
 B_{z_1 \nu} &= 4 \frac{G_{VA}^2}{e^2} \frac{e_2}{e^2} (g_V^{(1)} + g_A^{(1)}) \frac{s}{s - M_{z_1}^2} \cos \theta_M I(s, t, u) & (5) \\
 B_{z_2 \nu} &= - 4 \frac{G_{VA}^2}{e^2} \frac{e_2}{e^2} \sin \theta_M (g_V^{(2)} + g_A^{(2)}) \frac{s (s - M_{z_2}^2)}{(s - M_{z_2}^2)^2 + M_{z_2}^2 \Gamma_{z_2}^2} I(s, t, u) \\
 B_{z_1 z_2} &= - 2 \left(\frac{e_2}{e^2} \right)^2 \sin \theta_M \cos \theta_M (g_V^{(1)} g_V^{(2)} + g_A^{(1)} g_A^{(2)}) \frac{s^2 (s - M_{z_2}^2)}{(s - M_{z_1}^2) [(s - M_{z_2}^2)^2 + M_{z_2}^2 \Gamma_{z_2}^2]} A(s, t, u) \\
 B_{NN} &= \frac{G_{VA}^4}{e^4} \left[(a^4 + b^4 + 6a^2 b^2) E_2(s, t, u) + M_N^2 (a^2 - b^2)^2 E_N(s, t, u) \right] \\
 B_{\gamma N} &= - 2 (a^2 + b^2) \frac{G_{VA}^2}{e^2} I_1(s, t, u) \\
 B_{N\nu} &= 2 (a - b)^2 \frac{G_{VA}^4}{e^4} E_1(s, t, u) \\
 B_{N z_1} &= 2 \frac{G_{VA}^2}{e^2} \cos \theta_M \left[\frac{e_2}{e^2} (g_V^{(1)} (a^2 + b^2) - 2ab g_A^{(1)}) \right] \frac{s}{s - M_{z_1}^2} I_1(s, t, u) \\
 B_{N z_2} &= - 2 \frac{G_{VA}^2}{e^2} \sin \theta_M \left[\frac{e_2}{e^2} (g_V^{(2)} (a^2 + b^2) - 2ab g_A^{(2)}) \right] \frac{s (s - M_{z_2}^2)}{(s - M_{z_2}^2)^2 + M_{z_2}^2 \Gamma_{z_2}^2} I_1(s, t, u)
 \end{aligned}$$

here $ez = e \cot \theta_w$.

The functions $A(s,t,u)$, $E(s,t,u)$ and $I(s,t,u)$ are given in the classical work of Brown and Mikaelian^[6]. The others are:

$$\begin{aligned} E_1(s,t,u) &= \left(\frac{t}{t-M_w^2} \right) E(s,t,u) \\ I_1(s,t,u) &= \left(\frac{t}{t-M_w^2} \right) I(s,t,u) \\ E_2(s,t,u) &= \left(\frac{t}{t-M_w^2} \right)^2 E(s,t,u) \end{aligned} \quad (6)$$

$$E_h(s,t,u) = \left(\frac{t}{t-M_w^2} \right)^2 \left[\frac{M_w^2}{t^2} \left(\frac{ut}{M_w^4} - 1 \right) + \frac{s}{M_w^4} \left(\frac{1}{4} + \frac{M_w^4}{t^2} \right) \right]$$

The total cross section is obtained by integrating (4) from t_{\min} to t_{\max} :

$$t_{\min} = M_w^2 - \frac{s}{2} (1 + \beta) \quad (7)$$

$$t_{\max} = M_w^2 - \frac{s}{2} (1 - \beta)$$

with

$$\beta = \sqrt{1 - \frac{4M_w^2}{s}} \quad (8)$$

When we let $s \rightarrow \infty$ there remains in the total cross section a term that is proportional to s and another that is constant in s . To guarantee unitarity we impose that the coefficient of the linear term, L , as well as the constant, C , must vanish. Explicitly:

$$\begin{aligned} L &= \left[-1 + \left(\frac{e_2}{e^2} \right) (g_V^{(1)} \cos \theta_M - g_V^{(2)} \sin \theta_M) \right]^2 + \left[\frac{e_2}{e^2} (g_A^{(1)} \cos \theta_M - g_A^{(2)} \sin \theta_M) \right]^2 + \\ &+ \frac{8G_{VA}^4}{e^4} + \frac{4G_{VA}^2}{e^2} \left\{ \frac{e_2}{e^2} \left[(g_V^{(1)} + g_A^{(1)}) \cos \theta_M - (g_V^{(2)} + g_A^{(2)}) \sin \theta_M \right] - 1 \right\} + \\ &+ \frac{G_{VA}^4}{e^4} \left[a^4 + b^4 + 6a^2b^2 + 2(a-b)^2 \right] + \frac{2G_{VA}^2}{e^2} \left\{ \frac{e_2}{e^2} \left[(a^2 + b^2) (g_V^{(1)} \cos \theta_M - g_V^{(2)} \sin \theta_M) - \right. \right. \\ &\left. \left. - 2ab (g_A^{(1)} \cos \theta_M - g_A^{(2)} \sin \theta_M) \right] - (a^2 + b^2) \right\} \equiv 0 \end{aligned} \quad (9)$$

$$\begin{aligned}
C = & \frac{2}{3} \left\{ \left[-1 + \frac{e_2}{e^2} (g_V^{(1)} \cos \theta_M - g_V^{(2)} \sin \theta_M) \right]^2 + \left[\frac{e_2}{e^2} (g_A^{(1)} \cos \theta_M - g_A^{(2)} \sin \theta_M) \right]^2 \right\} + \\
& + \frac{20}{3} \frac{G_{VA}^4}{e^4} + \frac{36 G_{VA}^2}{e^2} \left\{ \frac{e_2}{e^2} \left[(g_V^{(1)} + g_V^{(2)}) \cos \theta_M - (g_V^{(1)} - g_V^{(2)}) \sin \theta_M \right] - 1 \right\} + \\
& + \frac{G_{VA}^4}{e^4} \left[(a^4 + b^4 + 6 a^2 b^2) \left(\frac{M_W^4 + 18 M_W^2 M_Z^2 - 5 M_Z^4}{M_Z^2} \right) + 6 M_Z^2 (a^2 - b^2)^2 + \right. \\
& \left. + 2(a-b)^2 (20 M_W^2 - 3 M_Z^2) \right] + 2 \frac{G_{VA}^2}{e^2} \left\{ \frac{e_2}{e^2} \left[(a^2 + b^2) (g_V^{(1)} \cos \theta_M - \right. \right. \\
& \left. \left. - g_V^{(2)} \sin \theta_M) - 2ab (g_A^{(1)} \cos \theta_M - g_A^{(2)} \sin \theta_M) \right] - (a^2 + b^2) (18 M_W^2 - 3 M_Z^2) \right\} \equiv 0 \quad (10)
\end{aligned}$$

Comparing the sum of the terms without a , b , or M_W (the first four in L , p.e.) we immediately recognize the same expressions present in the standard model if we just redefine the coupling of the Z 's with the fermions. We mean:

$$\begin{aligned}
g_V^{(1)} \cos \theta_M - g_V^{(2)} \sin \theta_M &= g_V^{SM} \\
g_A^{(1)} \cos \theta_M - g_A^{(2)} \sin \theta_M &= g_A^{SM}
\end{aligned} \quad (11)$$

This is what happens in all breaking patterns of $E_6^{[7]}$:

$$\begin{aligned}
g_V^{(1)} &= \frac{1}{2} g_Z \left(-\frac{1}{2} + 2x \right) \cos \theta_M + \frac{1}{2} g_{Y'} \sin \theta_M \left(\frac{1}{2} Y'_{L,t} + \frac{1}{2} Y'_{R,t} \right) \\
g_A^{(1)} &= \frac{1}{2} g_Z \left(-\frac{1}{2} \right) \cos \theta_M + \frac{1}{2} g_{Y'} \sin \theta_M \left(\frac{1}{2} Y'_{L,t} - \frac{1}{2} Y'_{R,t} \right) \\
g_V^{(2)} &= \frac{1}{2} g_Z \left(-\frac{1}{2} + 2x \right) \sin \theta_M + \frac{1}{2} g_{Y'} \cos \theta_M \left(\frac{1}{2} Y'_{L,t} + \frac{1}{2} Y'_{L,r} \right) \\
g_A^{(2)} &= \frac{1}{2} g_Z \left(\frac{1}{2} \right) \sin \theta_M + \frac{1}{2} g_{Y'} \cos \theta_M \left(\frac{1}{2} Y'_{L,t} - \frac{1}{2} Y'_{L,r} \right)
\end{aligned} \quad (12)$$

$$(x = \sin^2 \theta_W)$$

where $g_Z = e/(\sin \theta_W \cos \theta_W)$ and $Y'_{L,R}$ depends on the symmetry breaking. From equations (12) and (11) we see that the linear terms in σ are independent of the particular choice for the $U_{Y'}(1)$ (x, ψ, η) and reduce to the standard model.

So we have:

$$L = \frac{a^4 + b^4 + 6a^2b^2}{2} - (a-b)^2 \equiv 0 \Rightarrow a^4 + b^4 + 6a^2b^2 = 2(a-b)^2 \quad (13)$$

Taking this result in (10) and defining $y = \left(\frac{M_W}{M_N}\right)^2$

$$y^2 + 2y - 2 + 3(a+b)^2 = 0 \quad (14)$$

or, better

$$y = -1 + \sqrt{3[1 - (a+b)^2]} \quad (15)$$

y must be real and positive. This implies

$$(a+b)^2 < \frac{2}{3} \quad (16)$$

From (15) we can also see that

$$M_N > M_W \quad (17)$$

Now we will discuss some couplings:

I) Pure vector, $b=0$. From (13) $a=i\sqrt{2}$. But this case is ruled out by (16).

II) Pure axial, $a=0$. It cannot happen by the same argument of I).

III) Pure V+A, $a=b$. The bound (13) implies on $a=0$, $b=0$.

IV) Pure V-A, $a=-b$. From (13) $a=i$. This means a full strength coupling in the eN interaction! And, from

(15) $M_N \approx M_W$.

We will then suppose that there is a general mixing between the neutral (ν, N) and the charged particles (e, E), where E is a heavy charged lepton.

$\phi_{L,R}$ is the mixing angle between the neutral particles and $\phi_{L,R}$ the one in the charged fermionic sector^[8]. A simple manipulation shows that:

$$L^{cc} = G_{VA} \left[\bar{N} \gamma^\mu (1 - \gamma^5) e \sin(\phi_L - \phi_e) + \bar{N} \gamma^\mu (1 + \gamma^5) \cos \phi_R \sin \phi_R + \right. \\ \left. + \bar{\nu} \gamma^\mu (1 - \gamma^5) e \cos(\phi_L - \phi_e) + \bar{\nu} \gamma^\mu (1 + \gamma^5) e \sin \phi_R \sin \phi_R \right] W_\mu \quad (18)$$

So,

$$a = \sin(\phi_L - \phi_e) + \cos \phi_R \sin \phi_R \quad (19)$$

$$b = \cos \phi_R \sin \phi_R - \sin(\phi_L - \phi_e)$$

If the mixing is small and $\cos \phi_R \approx 1$ the condition (13) is satisfied by

$$\sin^2 \phi_R \approx \sin(\phi_L - \phi_e) \quad (20)$$

From (16)

$$|\sin \phi_R| < \frac{1}{\sqrt{6}} \quad (21)$$

Now (15) is written as

$$y = -1 + \sqrt{3[1 - 4 \sin^2 \Phi_R]} \quad (22)$$

Varying $\sin \Phi_R$ in the boundary (21) we can obtain a, b, and M_N (see fig.2).

In order to have other bounds on the parameters we have studied the contribution of the N to the anomalous magnetic moment of the electron^[9], through the triangular diagram (fig.3)

The result is^[10]

$$a_e = - \frac{G_F M_W m_e (a^2 - b^2)}{8\sqrt{2} \pi^2} F(y) \quad (23)$$

with

$$F(y) = \frac{\sqrt{y}}{y-1} \left\{ 4 \left[\frac{1}{2} - c + c^2 \ln \left(\frac{1+c}{c} \right) \right] + \frac{1}{y} \left[\frac{1}{2} + c - c(1+c) \ln \left(\frac{1+c}{c} \right) \right] \right\}$$

and

$$c = \frac{1}{y-1}$$

Assuming that the uncertainty in the experimental determination of a_e is saturated by this contribution the shadowed area in fig.4 is excluded. But as in most of the standard model extensions, we can have additional contributions to a_e and this bound can, in principle, be weaker.

In table I we show the values for a, b, and M_N satisfying all these bounds.

To make some comparisons with the known results we calculated the cross section (fig.5) and the angular distribution to the process $e^+e^- \rightarrow W^+W^-$ (fig.6). We have used $M_W = 82 \text{ GeV}$; $x = \sin^2 \theta_W = 0.223$; $M_{Z2} = 200 \text{ GeV}$; $\Gamma_{Z2} = 2.0 \text{ GeV}$ and $\sin \theta_H = 0.1$. The hypercharges are the same as in Dib + Gilman, (ref.2).

It is very interesting to know what happens in the angular distribution at $\sqrt{s} = 2000 \text{ GeV}$ when we look at the various contributions. We depicted in figure 7 the profile of these contributions.

As can be seen, at this energy, we cannot distinguish the contribution of an extra Z boson (diagrams a,b,c,d of fig.1) from the prediction of the standard model (a,b,d in fig.1). However if we add to the standard model a heavy neutrino (diagrams a,b,d,e in fig.1) there is a significant increase in $d\sigma/d(\cos\theta)$ (represented by the curve with balloons in fig.7). The continuous line in fig.7 represents the contribution of all the diagrams in fig.1.

To summarize our conclusions we can say that:

- a) if there exists a neutrino production via ZNN such that $M_N < M_W$ it will not couple to eW ;
- b) we don't have pure V,A, V+A or V-A couplings;
- c) we can only have a heavy neutral particle if there is mixing between the ordinary (ν, e) and excited matter (N, E) . This mixing should be such that $\sin\phi_R < 0.24$, here computed the constraints imposed by the anomalous magnetic moment of the electron. The corresponding N mass is less than 114GeV if the $(g-2)$ bounds is saturated by the N contribution. As we can have other contributions this bound can be significantly increased.

FIGURE CAPTIONS

fig.1 - Relevant diagrams to $e^+e^- \rightarrow W^+W^-$

fig.2 - The shadowed area is excluded by the limits on the anomalous magnetic moment of the electron.

fig.3 - Triangular diagram contribution to the anomalous magnetic moment of the electron.

fig.4 - Cross section of the process $e^+e^- \rightarrow W^+W^-$.

fig.5 - Angular distribution for two sets of (a, b, M_N) . θ is the angle in the center of mass frame between the e^- and the W^- .

fig.6 - The angular distributions considering the various contributions.

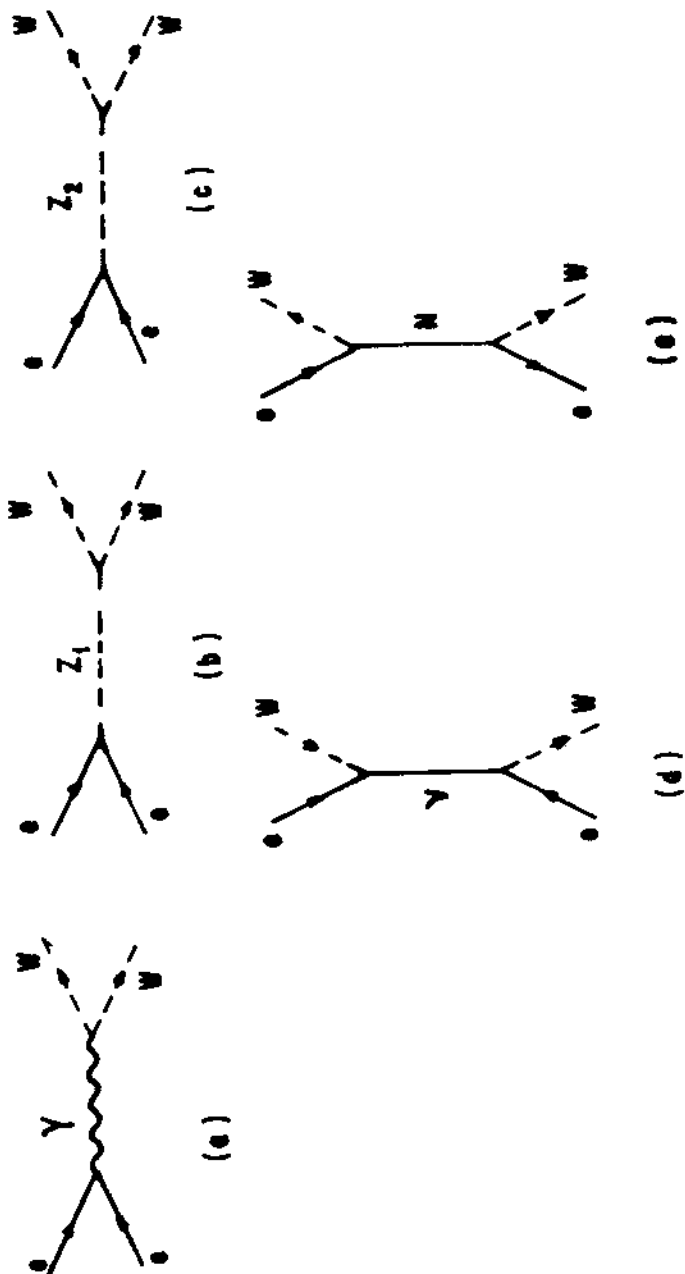


FIG. 1

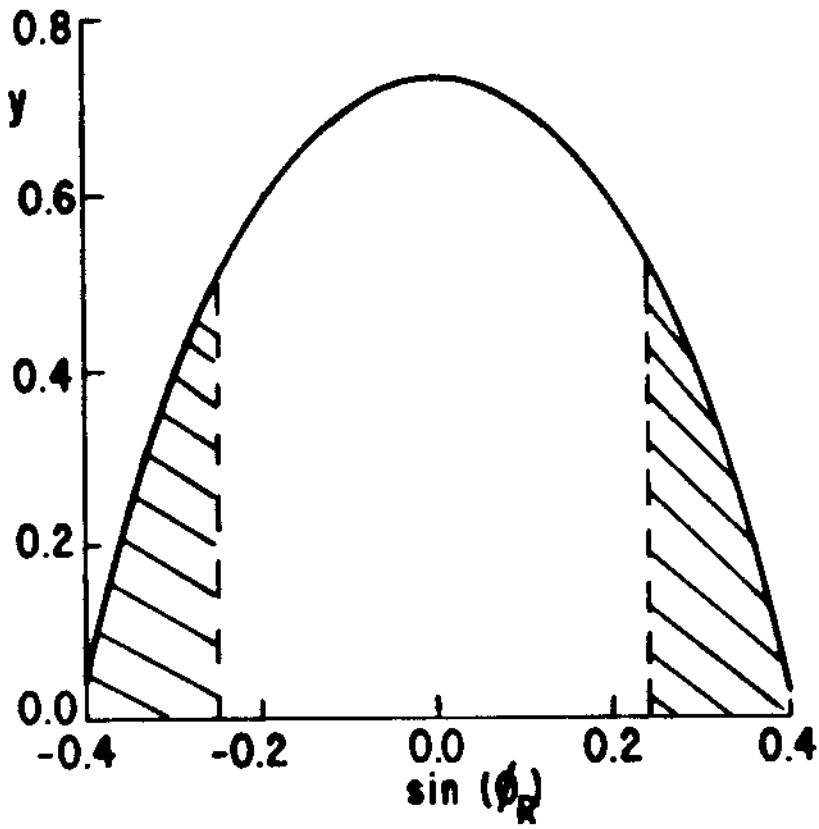


FIG. 2

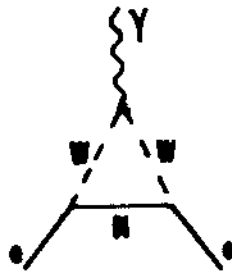


FIG. 3

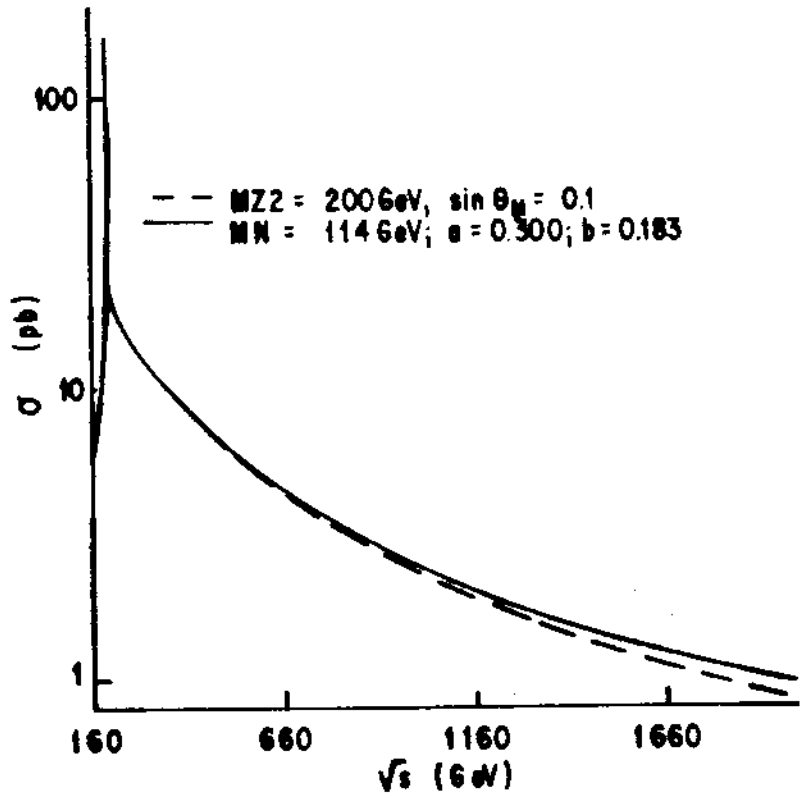


FIG. 4

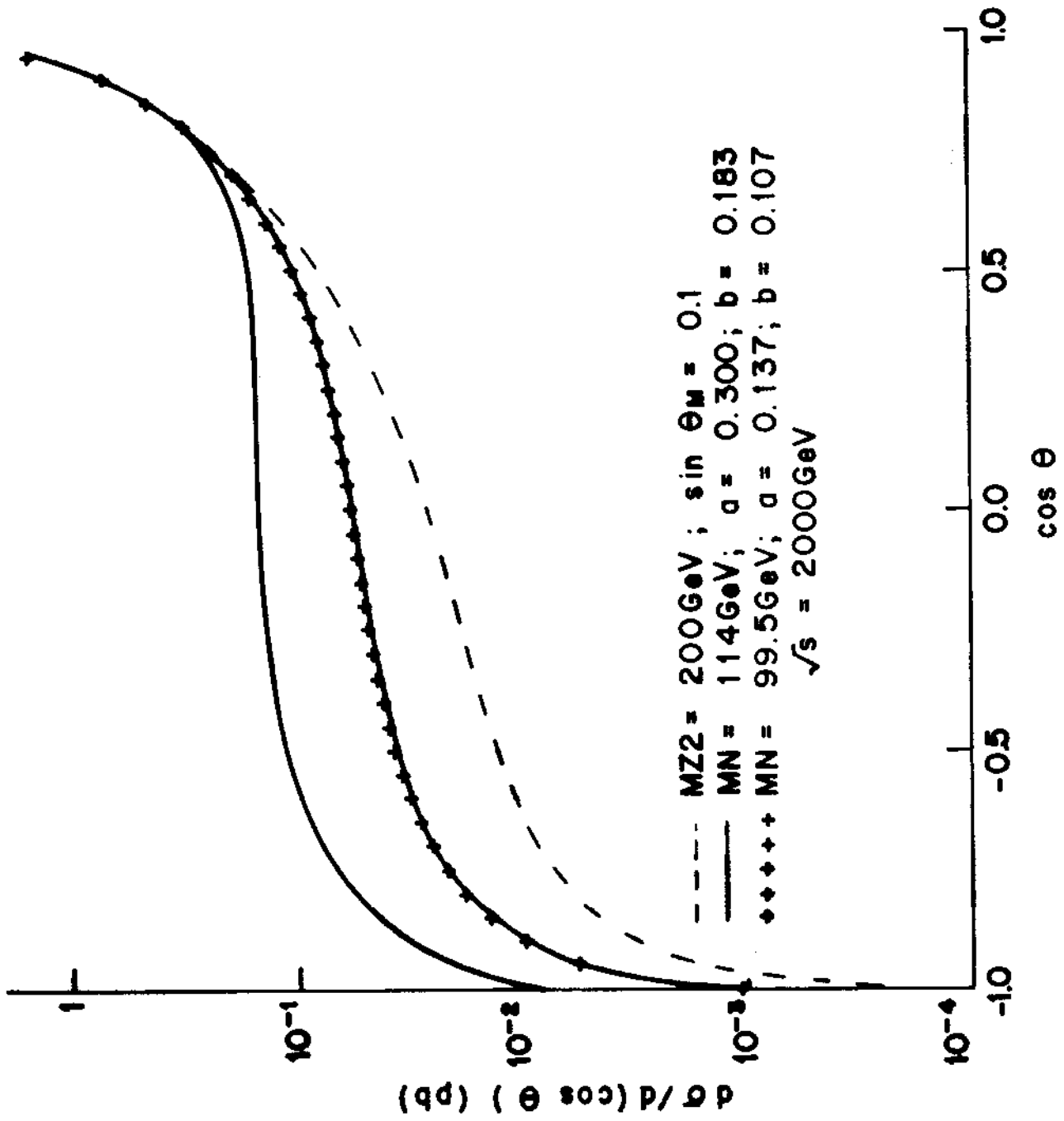


FIG.5

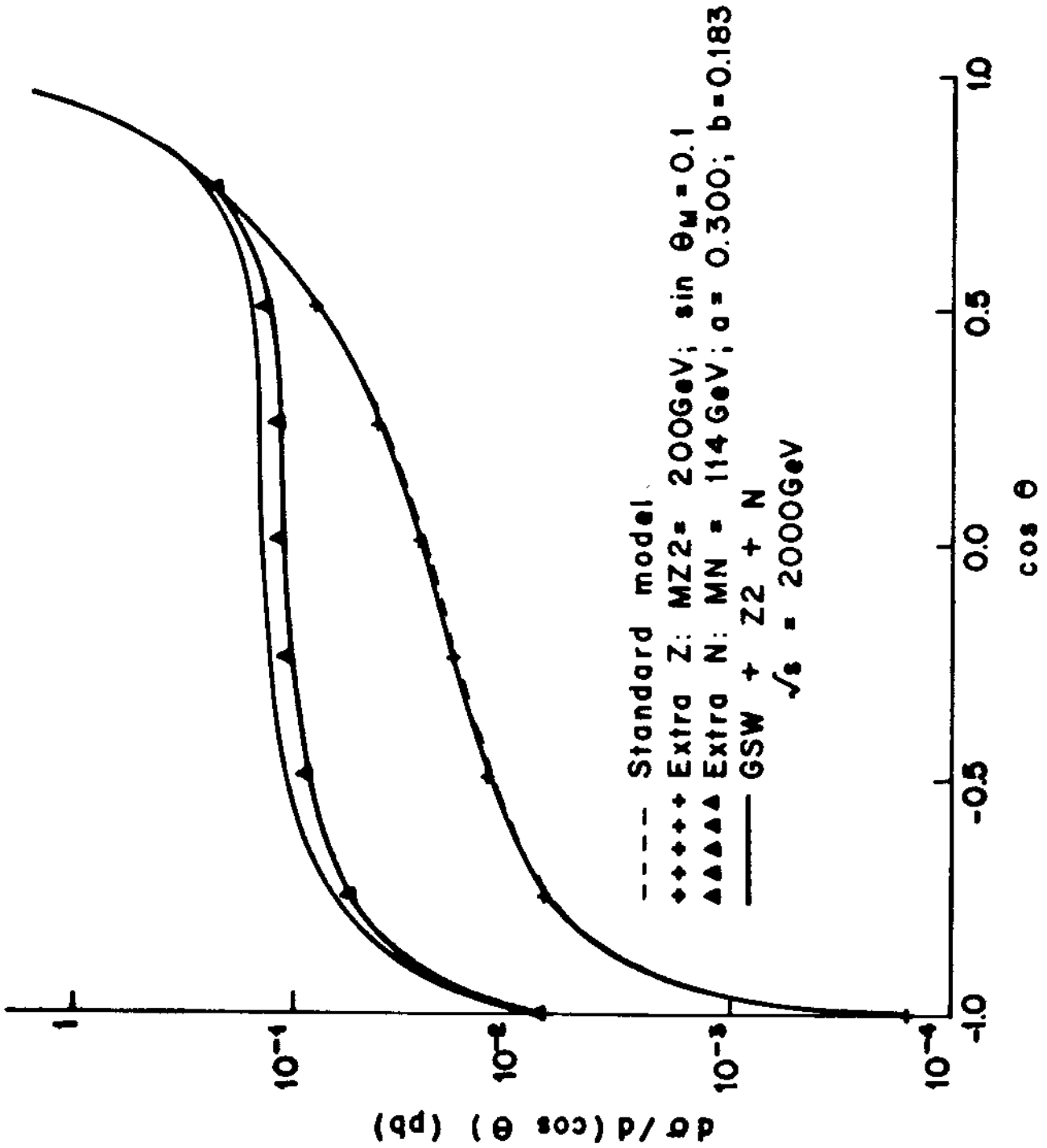


FIG. 6

M_N	$\sin\phi_R$	a	b
115.5	-.248	-.187	-.310
105.5	-.188	-.153	-.224
100.6	-.138	-.119	-.157
96.0	-.028	-.027	-.029
95.9	.012	.012	.012
96.2	.042	.043	.040
98.3	.102	.112	.091
98.8	.112	.124	.099
100.1	.132	.149	.114
105.9	.192	.229	.155
110.3	.222	.271	.173
112.1	.232	.285	.178
114.1	.242	.300	.183

Table 1 - Possible values for M_N , a, b and $\sin\phi_R$ observing the limits imposed by a_e .

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