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GAUGING σ-MODEL ISOMETRIES IN (2,0)-SUPERSPACE

by

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ABSTRACT

Considering (2,0)-supersymmetric non-linear σ-models defined over Kählerian coset manifolds, we discuss the gauging of the isotropy and isometry groups in (2,0)-superspace and present the action coupling these σ-models to the (2,0)-Yang-Mills supermultiplets.

Key-words: (2,0)-σ-Models; Isometries; Yang-Mills.

Supersymmetric (p,q)-like σ-models defined in two space-time dimensions [1,2] may have an important application to conformal field theory. Actually, under specific conditions on the target space geometry, a two-dimensional non-linear σ-model will define a conformally invariant field theory: supersymmetric non-linear σ-models defined on Ricci-flat Kähler spaces or hyperKähler manifolds [1,3,4] constitute an important class of conformal field theories. In particular, (1,0) and (2,0) heterotic σ-models with conformal invariance have been extensively discussed in connection with the problem of superstring classical configurations [1,5,6].

Bearing therefore in mind the intimate connection σ-models have with conformal field theories, we would like to exploit certain aspects of the former expecting to provide new examples of the latter. For that purpose, we draw our attention to the issue of gauging σ-model isometries [7-12] and carry out the consequent coupling of σ-models to Yang-Mills supermultiplets. We contemplate a (2,0)-supersymmetric σ-model described directly in (2,0)-superspace [13-16] and discuss the gauging of the target space isotropy and isometry groups, while working in terms of (2,0) matter and Yang-Mills supermultiplets [15,16]. The resulting gauge model might be expected to give an example of a new conformal field theory.

We begin by fixing our superspace algebra and notational conventions. Next, we give a brief discussion on the (2,0)-superfields we shall be dealing with, including the formulation of gauge theories and σ-models in (2,0)-superspace. Finally, we discuss and present the details of our analysis of the formulation of a gauge-invariant non-linear σ-model with global (2,0) supersymmetry.

We consider our superspace parametrised by the coordinates $(x^{++},x^{--};\ \theta_-,\bar{\theta}_-)$, where $\theta_-,\bar{\theta}_-$ are complex Grassmannian variables, which transform under the 2-dimensional Lorentz group as right-handed Weyl spinors:

$$\theta'_-=e^{-\alpha/2}\theta_- \quad , \qquad (1a) \qquad\qquad \bar{\theta}'_-=e^{-\alpha/2}\bar{\theta}_- \quad , \qquad (1b)$$

where α is the real parameter of SO(1,1). The light-cone coordinates are defined by

$$x^{++} = \sqrt{\frac{1}{2}}\,(x^0 + x^1)\ ,\ (2a) \qquad x^{--} = \sqrt{\frac{1}{2}}\,(x^0 - x^1)\ . \qquad (2b)$$

The supersymmetry covariant derivatives are taken as:

$$D_+ \equiv \frac{\partial}{\partial\theta_-} + i\bar{\theta}_-\partial_{++}\ ,\ (3a) \qquad \bar{D}_+ \equiv \frac{\partial}{\partial\bar{\theta}} + i\theta_-\partial_{++}\ , \qquad (3b)$$

where

$$\partial_{++} \equiv \frac{\partial}{\partial x^{++}}\ ; \qquad \partial_{--} \equiv \frac{\partial}{\partial x^{--}}\ .$$

They fulfil the algebra

$$\{D_+,D_+\} = 0 \quad (4a) \quad ; \qquad \{D_+,\bar{D}_+\} = 2i\partial_{++}\ . \qquad (4b)$$

To obtain the pure matter supermultiplets, we need to constrain scalar and spinor superfields according to

$$\bar{D}_+\Phi = \bar{D}_+\Psi_- = 0\ , \qquad (5)$$

so that the θ-expansions of these "chiral" superfields read:

$$\Phi = \phi(x) + \theta_-\eta_+(x) + i\theta_-\bar{\theta}_-\partial_{++}\phi(x)\ , \qquad (6)$$

$$\Psi_- = \psi_-(x) + \theta_-F(x) + i\theta_-\bar{\theta}_-\partial_{++}\psi_-(x)\ . \qquad (7)$$

ϕ and F are complex scalar fields, whereas η_+ and ψ_- are left- and right-handed Weyl spinors, respectively.

A manifest (2,0)-supersymmetric kinetic action for the matter superfields above can be written as [13]

$$S = -\frac{i}{2}\int d^2x\, d\theta_+ d\bar{\theta}_+ \left[\bar{\Phi}\partial_{--}\Phi - (\partial_{--}\bar{\Phi})\Phi\right] + \frac{1}{2}\int d^2x\, d\theta_+ d\bar{\theta}_+ \bar{\Psi}_-\Psi_-\ , \qquad (8)$$

which in component fields take the form[#]

$$S_M = \int d^2x \left[\bar{\phi}\,\Box\,\phi - i\bar{\eta}_+\partial_{--}\eta_+ - i\bar{\psi}_-\partial_{++}\psi_- + \frac{1}{2}\bar{F}F\right] \quad . \qquad (9)$$

To introduce the (2,0)-Yang-Mills supermultiplets, we define the gauge covariant derivatives as below:

$$\nabla_+ \equiv D_+ - ig\Gamma_+ \quad , \quad (10a) \qquad \nabla_{++} \equiv \partial_{++} - ig\Gamma_{++} \quad , \quad (10b)$$

$$\bar{\nabla}_+ \equiv \bar{D}_+ + ig\bar{\Gamma}_+ \quad , \quad (10c) \qquad \nabla_{--} \equiv \partial_{--} - ig\Gamma_{--} \quad , \quad (10d)$$

where $\Gamma_A \equiv \Gamma_A^a\, T_a$, and the T_a's are the anti-Hermitian generators of the gauge group G. The parameter g plays the rôle of a mass-dimensional coupling constant.

Now, we set the algebra of the gauge-covariant derivatives in order to impose constraints and obtain an irreducible representation describing a model whose component fields exhibit suitable physical properties. Following Brooks et al. [15], the constraints are:

$$\{\nabla_+,\bar{\nabla}_+\} = 2i\nabla_{++} \quad , \quad (11.a) \qquad [\nabla_+,\nabla_{--}] = -igW_- \quad , \quad (11.d)$$

$$\{\nabla_+,\nabla_+\} = 0 \quad , \quad (11.b) \qquad [\nabla_{++},\nabla_{--}] = -igZ \quad . \quad (11.e)$$

$$[\nabla_+,\nabla_{++}] = 0 \quad , \quad (11.c)$$

The first constraint is the well-known conventional one. It yields:

$$\Gamma_{++} = -\frac{i}{2}\left[D_+\bar{\Gamma}_+ - \bar{D}_+\Gamma_+ - ig\left\{\Gamma_+\,,\bar{\Gamma}_+\right\}\right] \quad . \qquad (12)$$

The constraint (11.b) eliminates a spin-3/2 component field. With the help of the Bianchi identities, it is shown [15] that there

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[#] Our conventions are $\eta_{\mu\nu}$=diag(-1,1) and $\epsilon_{01} = -\epsilon^{01} = +1$.

are only two independent field-strength superfields, namely $\bar{W}_-$ and W_- . On the other hand, (11.c) (or eq. (12)) implies that Γ_{++} must be Hermitian. An additional requirement is that also Γ_{--} be Hermitian, in order that the theory display only one gauge field [16].

The coupling between the (2,0) matter and gauge supermultiplets is obtained upon the covariantisation of the action (8), which becomes:

$$S_{Matter-gauge} = -\frac{i}{2}\int d^2x\, d\theta_+ d\bar{\theta}_+ \left[\bar{\Phi}\, e^{aV}\nabla_{--}\Phi - (\nabla_{--}\bar{\Phi})\, e^{aV}\Phi\right] + \frac{1}{2}\int d^2x\, d\theta_+ d\bar{\theta}_+ \bar{\Psi}_- e^{aV}\Psi_- \quad , \qquad (13)$$

where V is a Lie-algebra-valued real scalar superfield and a is a mass-dimensional coupling constant.

Considering that the matter superfields transform like

$$\Phi' = e^{i\Lambda}\Phi \quad , \quad (14a) \qquad\qquad \Psi'_- = e^{i\Lambda}\Psi_- \quad , \qquad (14b)$$

where $\Lambda \equiv \Lambda^a T_a$ is a chiral scalar superfield satisfying the same chirality constraint of Φ and Ψ_-, the gauge invariance of the action (13) requires that V transforms according to

$$e^{aV'} = e^{i\bar{\Lambda}}\, e^{aV} e^{-i\Lambda} \quad . \qquad (15)$$

It is interesting to point out that the prepotential V acts in such a way to change a Λ- to a $\bar{\Lambda}$-representation of G [17]:

$$(e^{aV}\Phi)' = e^{i\bar{\Lambda}}\,(e^{aV}\Phi) \;, \quad (16) \qquad (\bar{\Phi}e^{aV})' = (\bar{\Phi}e^{aV})e^{-i\Lambda} \quad . \qquad (17)$$

The connection between the prepotential V and the gauge-potential superfields Γ_+ and $\bar{\Gamma}_+$ becomes clear if we consider that we can define

$$\nabla_+ \equiv e^{-aV} D_+ e^{aV} \quad , \quad (18) \qquad\qquad \bar{\nabla}_+ \equiv e^{-aV}\, \bar{D}_+ e^{aV} \quad . \qquad (19)$$

Consequently,

$$\Gamma_{+} = \frac{i}{g} e^{-gV}(D_{+}e^{gV}) \quad , \tag{20}$$

and, according to ref.[15], this relationship enforces the presence of a single gauge potential in the theory.

From the Hermitian nature of the connection Γ_{--}, its gauge transformation must read as below:

$$\delta\Gamma_{--} = \frac{1}{g}\, \partial_{--}(\Lambda + \bar{\Lambda}) + i[\Lambda + \bar{\Lambda}, \Gamma_{--}] \quad . \tag{21}$$

The Yang-Mills action for the (2,0) gauge supermultiplet reads

$$S_{g} = \frac{1}{2}\int d^{2}x\, d\theta_{+}d\bar{\theta}_{+}\, \mathrm{Tr}\, \bar{W}_{-}W_{-} \quad . \tag{22}$$

Now, it is interesting to notice that, in a complete analogy with the case of N=1 - D=4 supersymmetry, we can write down the (2,0) analogue of the Fayet-Iliopoulos term:

$$S_{FI} = \int d^{2}x\, d\theta_{+}d\bar{\theta}_{+}\xi\Gamma_{--} \quad . \tag{23}$$

Its Abelian invariance is evident from the transformation law (21) and the constraints on Λ and $\bar{\Lambda}$ [18].

The raise of interest on the study of supersymmetric non-linear σ-models stems from the fact that they describe the background field configurations for superstrings. Actually, seeking compactifications of the Green-Schwarz [19] and heterotic [5] superstrings that leads to the presence of a 4-dimensional supersymmetry corresponds to searching for locally supersymmetric σ-models that, for flat two-dimensional supergravity backgrounds, exhibit N=2 (or (2,2)) and (2,0) supersymmetries, respectively [3].

Following the results of Zumino [20] and Hull and Witten [1], it is known that the requirement of an extended supersymmetry for a two-dimensional non-linear σ-model dictates geometric restrictions on the σ-model target space. In particular, (2,0) and (2,2) supersymmetric models without Wess-Zumino term [21] require the σ-model target manifold to be Kählerian [22].

Considering the case of (2,0)-supersymmetry, we take the chiral and anti-chiral superfields, $\Phi^i(x;\theta,\bar\theta)$ and $\bar\Phi^i(x;\theta,\bar\theta)\equiv\Phi^{\bar i}$ respectively ($i=1,2,\ldots,n$), and regard them as the coordinates of some n-dimensional Hermitian manifold. In this case, the σ-model action written in (2,0) superspace reads [13,14]:

$$S = -\frac{1}{2}\int d^2x\, d\theta_+ d\bar\theta_+ \left[K_i(\Phi,\bar\Phi)\partial_{--}\Phi^i - K_{\bar i}(\Phi,\bar\Phi)\partial_{--}\Phi^{\bar i}\right] , \qquad (24)$$

where the target space vector $K_i(\Phi,\bar\Phi)$, in the absence of the Wess-Zumino term (torsion-free case), can be written as the gradient of a real scalar (Kähler) potential, $K(\Phi,\bar\Phi)$:

$$K_i = \frac{\partial}{\partial\Phi^i} K(\Phi,\bar\Phi) \equiv \partial_i K(\Phi,\bar\Phi) , \quad (25a) \qquad K_{\bar i} = \frac{\partial}{\partial\bar\Phi^i} K(\Phi,\bar\Phi) \equiv \partial_{\bar i} K(\Phi,\bar\Phi) . \quad (25b)$$

Making use of the $\theta,\bar\theta$-expansions for the superfields Φ^i and $\bar\Phi^i$, one finds that the component-field version of the action (24) reads:

$$S_\sigma = \frac{1}{2}\int d^2x \left[g_{i\bar j}\partial_{--}\phi^{\bar j}\partial_{++}\phi^i + \frac{1}{2} b_{ij}\epsilon^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j + g_{i\bar j}\eta^i D_{--}\eta^{\bar j} + \right.$$

$$\left. + \text{c.c} \right] , \qquad (26)$$

where we recognize the ordinary bosonic σ-model action accompanied by its (2,0)-supersymmetric partners, and

$$D_{--}\eta^{\bar j} = \partial_{--}\eta^{\bar j} + \Gamma^{\bar j}_{\bar k \bar l}\, \eta^{\bar k}\, \partial_{--}\phi^{\bar l} , \qquad (27)$$

$$g_{i\bar j} \equiv K_{i,\bar j} + K_{\bar j,i} , \qquad (28)$$

$$b_{ij} \equiv K_{i,j} - K_{j,i} . \qquad (29)$$

$g_{i\bar j}$ stands for the metric, b_{ij} is the torsion tensor and Γ^k_{ij} is the affine connection of the target space.

We shall contemplate here the torsion-free case, and the

Kählerian target spaces will be taken as symmetric manifolds of the form G/H. The generators of the isometry group G are denoted by Q_α (α=1,2,...,dim G), whereas those of the isotropy group H are specified by $Q_{\bar{\alpha}}$ ($\bar{\alpha}$=1,2,...,dim H). We must also have that

$$\alpha - \bar{\alpha} = 2n \qquad , \tag{30}$$

for we have assumed G/H to be labelled by n coordinates, Φ^i, and their corresponding complex conjugates.

The infinitesimal transformations of the isotropy group are linearly realised and act by matrix multiplication, just as on flat manifolds:

$$\delta\Phi^i = i\lambda^{\alpha} (Q_{\bar{\alpha}})^i{}_j \, \Phi^j \qquad , \tag{31}$$

where $(Q_{\bar{\alpha}})^i{}_j$ denote the matrix elements of the Hermitian generators of the subgroup H in some n-dimensional representation and $\lambda^{\bar{\alpha}}$ are global (x-and Φ-independent) parameters. As for the isometry group, its infinitesimal action on G/H can be written as

$$\delta\Phi^i = \lambda^{\alpha} \, k^i_{\alpha}(\Phi) \tag{32a}$$

and

$$\delta\bar{\Phi}_i = \lambda^{\alpha} \, \bar{k}_{\alpha i}(\bar{\Phi}) \qquad , \tag{32b}$$

where k^i_α and $\bar{k}_{\alpha i}$ are Killing vectors of the target manifold. Exponentiating (32a) and (32b) yields finite isometry group transformations that we write as

$$\Phi^i \xmapsto{\;G\;} \Phi'^i = \exp(L_{\lambda.k}) \, \Phi^i \qquad , \tag{33a}$$

$$\bar{\Phi}_i \xmapsto{\;G\;} \bar{\Phi}'_i = \exp(L_{\lambda.\bar{k}}) \, \bar{\Phi}_i \qquad , \tag{33b}$$

with

$$L_{\lambda.k} \, \Phi^i \equiv \left[\lambda^{\alpha} k^j_{\alpha} \frac{\partial}{\partial\Phi^j} \, , \Phi^i \right] = \delta\Phi^i \qquad . \tag{34}$$

Before discussing the invariance of the action (24) under isotropy and isometry transformations, we can readily check, upon use of the chiral and anti-chiral constraints, that it is invariant under the so-called Kähler gauge transformations:

$$K(\Phi,\bar{\Phi}) \longmapsto K'(\Phi,\bar{\Phi}) = K(\Phi,\bar{\Phi}) + \eta(\Phi) + \bar{\eta}(\bar{\Phi}) \quad , \qquad (35)$$

where $\eta(\Phi)$ is a holomorphic function of the coordinates Φ^i.

Now, an isotropy or isometry transformation induces a change δK on the Kähler potential:

$$\delta K = \partial_i K \; \delta\Phi^i + \partial_{\bar{i}} K \; \delta\Phi^{\bar{i}} \quad . \qquad (36)$$

In the isotropic case, it can be shown, with help of the so-called Kähler gauges, that K can always be chosen in such a way that

$$\delta K = 0 \quad , \qquad (37)$$

that is, the Kähler scalar potential can always be taken H-invariant. This ensures the invariance of the action (24) under the isotropy group. The same, however, is not generally true for the isometry group. Considering the G-transformation given by (32a) and (32b), the variation δK reads:

$$\delta K = \lambda^\alpha \Big[(\partial_i K) \; k^i_\alpha + (\partial^{\bar{i}} K) \; k_{\alpha \bar{i}} \Big] = \lambda^\alpha \Big[\eta_\alpha(\Phi) + \bar{\eta}_\alpha(\bar{\Phi}) \Big] \quad , \qquad (38)$$

where the holomorphic and anti-holomorphic functions η_α and $\bar{\eta}_\alpha$ are to be determined up to a purely imaginary quantity as below:

$$(\partial_i K) \; k^i_\alpha \equiv \eta_\alpha + i M_\alpha(\Phi;\bar{\Phi}) \qquad (39a)$$

and

$$(\partial^{\bar{i}} K) \; k_{\alpha\bar{i}} \equiv \bar{\eta}_\alpha - i M_\alpha(\Phi;\bar{\Phi}) \quad . \qquad (39b)$$

The functions M_α are real scalars whose existence is crucial for the gauging of the isometry group [7,9], as we shall see in the sequel. Therefore, from (38) and by virtue of the constraints on Φ^i and $\bar{\Phi}^i$, it immediately follows that the superspace action (24) is invariant under global isometries.

Now, a relevant issue in the framework of

(2,0)-supersymmetric σ-models would be the gauging of the isometry group of the target space [7-12]. This amounts to studying the minimal coupling of the (2,0)-σ-model to the (2,0)-Yang-Mills supermultiplets. The eventual importance of such a study appears in connection with the dynamics of world-sheet gauge fields, which according to the works of ref. [23], might play a significant rôle in the understanding of the breaking of the large string gauge groups to lower-dimensional simple groups. Another possible relevance for pursuing such an investigation is in connection with 2-dimensional conformal field theories. We already know that 2-dimensional σ-models define conformal theories provided that some constraints are imposed on the geometry of the target space. Now, the coupling of these models to the Yang-Mills sector might yield new conformal theories of interest.

To carry out the analysis in superspace, one has to draw the attention to the fact that, rather than the metric, there appears in the superaction the Kähler scalar $K(\Phi,\bar{\Phi})$, which potentially exhibits extra symmetries leaving the metric invariant. This fact, as we shall explain below, requires the need for simultaneously gauging the Kähler invariance of eq.(35) [9]. However, the four- and two-dimensional cases differ in that the latter exhibits the Kähler function K subject to a space-time derivative. Moreover, the (2,0) case has an extra supersymmetry at play, and this is responsible for the different form of the action respect to the N=1 - D=4 and (1,0) - D=2 cases.

The gauging of the isotropy group (or any of its subgroups), whose transformations now read

$$\Phi^i \longmapsto \Phi'^i = e^{i\Lambda}\,\Phi^i \quad , \tag{40}$$

with $\Lambda \equiv \bar{\Lambda}^{\alpha}(x;\theta,\bar{\theta})\,Q_{\bar{\alpha}}$ and $\bar{D}_+ \Lambda^{\bar{\alpha}} = 0$, can be immediately performed if we replace $\bar{\Phi}_i$ by the combination

$$\tilde{\Phi}_i \equiv \bar{\Phi}_j\,(e^V)^j{}_i \quad , \tag{41}$$

since this transforms as below:

$$\tilde{\Phi}_i \longmapsto \tilde{\Phi}'_i = \tilde{\Phi}_i\,e^{-i\Lambda} \quad . \tag{42}$$

It is worthwhile to mention that V is Lie algebra- valued, $V \equiv V^{\bar\alpha}(x;\theta,\bar\theta)\, Q_{\bar\alpha}$, and the generators of the subgroups we are gauging are written in an n-dimensional representation.

The replacement (41) guarantees that the scalar function $K(\Phi,\tilde\Phi)$ is locally invariant once $K(\Phi,\bar\Phi)$ has been chosen to be globally invariant. This is always the case whenever the group in consideration is at most the isotropy group. So, all we are left with is the standard gauge-covariantisation of the derivative $\partial_{--}\Phi^i$:

$$\partial_{--}\Phi^i \longmapsto \nabla_{--}\Phi^i \equiv \partial_{--}\Phi^i - ig\Gamma_{--}^{\bar\alpha}\,(Q_{\bar\alpha})^i{}_j\,\Phi^j \quad . \qquad (43)$$

Therefore, we finally get that the action

$$S = -\frac{i}{2}\int d^2x\, d\theta_+ d\bar\theta_+\left[\partial_i K(\Phi,\tilde\Phi)\nabla_{--}\Phi^i - \partial_{\bar i}K(\Phi,\tilde\Phi)\nabla_{--}\Phi^{\bar i}\right] \qquad (44)$$

is invariant under the local Yang-Mills transformations generated by any subgroup of the isotropy group H. Notice that the replacement (41) of $\bar\Phi$ by $\tilde\Phi$ takes place only in the Kähler function, but not in the derivatives $\partial_{\bar i}$ and $\partial_{--}\bar\Phi^i$.

The next and final step would be the gauging of the full isometry group G, whose transformations now take the form

$$\Phi^i \longmapsto \Phi'^i = \exp(\, L_{\Lambda.k}\,)\, \Phi^i \quad , \qquad (45a)$$

$$\bar\Phi_i \longmapsto \bar\Phi'_i = \exp(\, L_{\bar\Lambda.\bar k}\,)\, \bar\Phi_i \quad , \qquad (45b)$$

where

$$L_{\Lambda.k}\,\cdot \equiv \left[\Lambda^\alpha\, k^i{}_\alpha(\Phi)\, \frac{\partial}{\partial\Phi^i}\, ,\, \cdot\,\right] \quad , \qquad (45c)$$

$$L_{\bar\Lambda.\bar k}\,\cdot \equiv \left[\bar\Lambda^\alpha\, \bar k_{\alpha i}(\bar\Phi)\, \frac{\partial}{\partial\bar\Phi_i}\, ,\, \cdot\,\right] \quad , \qquad (45d)$$

with

$$\bar D_+\Lambda^\alpha(x;\theta,\bar\theta) = 0 \quad , \qquad (46a) \qquad\qquad D_+\bar\Lambda^\alpha(x;\theta,\bar\theta) = 0 \quad . \qquad (46b)$$

In order to covariantise the potential K and express all the gauge variations exclusively in terms of the superfield parameter $\Lambda^{\alpha}(x;\theta,\bar{\theta})$, in such a way to mimic the case of global transformations, we propose the replacement $\bar{\Phi} \longmapsto \tilde{\Phi}$ by means of the superfield V according to

$$\tilde{\Phi}_i \equiv \exp(L_{iV.\bar{k}})\,\bar{\Phi}_i = \exp(iL_{V.\bar{k}})\,\bar{\Phi}_i \quad , \qquad (47)$$

where the gauge transformation of V is fixed through the equation below:

$$e^{iL_{V'.\bar{k}}} = e^{L_{\Lambda.\bar{k}}}\, e^{iL_{V.\bar{k}}}\, e^{-L_{\bar{\Lambda}.\bar{k}}} \qquad (48a)$$

and

$$L_{iV'.\bar{k}}\,\cdot \equiv \left[iV'^{\alpha}\bar{k}_{\alpha i}\frac{\partial}{\partial\bar{\Phi}_i}\,,\,\cdot\,\right] \quad . \qquad (48b)$$

Therefore, $\tilde{\Phi}$ transforms as

$$\tilde{\Phi}_i \longmapsto \tilde{\Phi}_i' = \exp(\,L_{\Lambda.\bar{k}}\,)\,\tilde{\Phi}_i \quad , \qquad (49a)$$

which infinitesimally reads

$$\delta\tilde{\Phi}_i = \Lambda^{\alpha}(x;\theta,\bar{\theta})\,\bar{k}_{\alpha i} \quad . \qquad (49b)$$

However, unlike the gauging of the isotropy group, the prescription of replacing $\bar{\Phi}$ by $\tilde{\Phi}$ does not yield a gauge-invariant scalar $K(\Phi,\tilde{\Phi})$, for the symmetry group is no longer linearly realised. Indeed, an infinitesimal isometry transformation induces on $K(\Phi,\tilde{\Phi})$ the variation

$$\delta K(\Phi,\tilde{\Phi}) = \Lambda^{\alpha}\,(\eta_{\alpha} + \tilde{\eta}_{\alpha}) \quad , \qquad (50)$$

where

$$\tilde{\eta}_{\alpha} = (\partial^{\tilde{i}}K)\,\bar{k}_{\alpha i}(\tilde{\Phi}) + iM_{\alpha}(\Phi,\tilde{\Phi}) \quad , \qquad (51)$$

and

$$\partial^{\tilde{i}} \equiv \frac{\partial}{\partial \tilde{\Phi}_i} \tag{52}$$

The isometry variation δK computed above reads just like a (local) Kähler transformation and this is a direct consequence of the existence of the real scalars $M_\alpha(\Phi,\tilde{\Phi})$, as discussed in refs. [7] and [9].

The result (50) immediately suggests the introduction of a pair of chiral and anti-chiral auxiliary superfields, $\zeta(\Phi)$ and $\bar{\zeta}(\bar{\Phi})$, whose Yang-Mills transformations are able to compensate the isometry variation of K. This can be realised by introducing a Lagrangian given by

$$\mathcal{L}_\zeta = \partial_i K(\Phi,\tilde{\Phi})\ \nabla_{--}\Phi^i - \partial_{\tilde{i}}\ K(\Phi,\tilde{\Phi})\ \nabla_{--}\tilde{\Phi}^i - \partial_i\zeta(\Phi)\ \nabla_{--}\Phi^i + \\ + \partial_{\tilde{i}}\ \tilde{\zeta}(\tilde{\Phi})\ \nabla_{--}\tilde{\Phi}^i \quad , \tag{53}$$

which can still be rewritten as

$$\mathcal{L}_\zeta = \partial_i\Big[\ K(\Phi,\tilde{\Phi}) - \zeta(\Phi) - \tilde{\zeta}(\tilde{\Phi})\ \Big]\ \nabla_{--}\Phi^i + \\ - \partial_{\tilde{i}}\Big[\ K(\Phi,\tilde{\Phi}) - \zeta(\Phi) - \tilde{\zeta}(\tilde{\Phi})\ \Big]\ \nabla_{--}\tilde{\Phi}^i \quad , \tag{54}$$

where the covariant derivatives $\nabla_{--}\Phi^i$ and $\nabla_{--}\tilde{\Phi}^i$ are defined in analogy to what is done for the bosonic σ-model:

$$\nabla_{--}\Phi^i \equiv \partial_{--}\Phi^i - g\Gamma^\alpha_{--}\ k^i_\alpha \tag{55a}$$

and

$$\nabla_{--}\tilde{\Phi}_i \equiv \partial_{--}\tilde{\Phi}_i - g\Gamma^\alpha_{--}\ \bar{k}_{\alpha i} \quad . \tag{55b}$$

Now, if the holomorphic and anti-holomorphic auxiliary scalar superfields are so chosen that

$$(\partial_i\zeta)\ k^i_\alpha = \eta_\alpha(\Phi) \tag{56a}$$

and

$$(\partial^{\tilde{i}}\tilde{\zeta})\ \bar{k}_{\alpha i} = \tilde{\eta}_{\alpha}(\tilde{\Phi}) \quad , \tag{56b}$$

the combination [$K(\Phi,\tilde{\Phi}) - \zeta(\Phi) - \tilde{\zeta}(\tilde{\Phi})$] becomes an invariant and the Lagrangian $\mathcal{L}_{\zeta}$ of eq.(54) is symmetric under local isometry transformations.

The striking fact concerning the locally invariant Lagrangian obtained above regards the need for the introduction of the auxiliary superfields $\zeta(\Phi)$ and $\bar{\zeta}(\bar{\Phi})$. This is not surprising, since a similar mechanism occurs in the gauging of N=1-supersymmetric σ-models in 4 dimensions. However, contrary to what happens in the case of (2,0)-supersymmetric σ-models, the superfields ζ and $\bar{\zeta}$ drop out from the N=1-D=4 action. The basic reason for the persistence of the auxiliary superfields ζ and $\bar{\zeta}$ here follows from the presence of the space-time derivatives $\partial_{--}\Phi^{i}$ and $\partial_{--}\bar{\Phi}^{i}$ in the globally symmetric action. Though the target space geometry in the cases N=1-D=4 and (2,0)-D=2 are the same, the action of the former involves only the Kähler potential whereas the action of the latter displays explicit space-time derivatives. Therefore, besides the scalar function discussed in refs.[7] and [9], the auxiliary superfields ζ and $\bar{\zeta}$ (fixed non-uniquely from their gauge transformations) remain in the (2,0)-D=2 action and no chirality arguments may throw them away as it happens in N=1-D=4. Nevertheless, since the rôle of ζ and $\bar{\zeta}$ is to add holomorphic and anti-holomorphic pieces to the scalar potential K, the metric of the target space does not feel the presence of them both and we claim that actions with different choices of ζ and $\bar{\zeta}$ (this is just a superspace artifact) correspond to the same component-field σ-model action and describe equivalent conformal field theories with non-compact gauge group.

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