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AND AXIAL MESONS

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QUARK MODEL COUPLINGS AND STRONG DECAYS OF TENSOR  
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ABSTRACT

The quark model couplings of tensor, axial and scalar mesons to PP and PV meson states have been evaluated under the assumption that the former have the  $Q\bar{Q}$  structures  $3_{P_{0,1,2}}$  and  $1_{P_1}$ . The results for the strong decay widths for the known modes are in rather good agreement with observations.

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The spectacular success of the quark model in the realm of elementary particle physics during the last couple of years <sup>1</sup> tempts one to look for its predictions in less conventional areas through some bold assumptions. One of the interesting areas of study lies in the strong decay modes of hadrons into hadrons and pseudoscalar mesons (P) via Yukawa-type  $\bar{Q} Q P$  couplings folded into the quark structure of the initial and final hadrons. In such a model, the final meson must be regarded as "elementary", and since no simple dynamical explanation seems to be available <sup>2</sup> for such an assumption, it must be judged almost entirely by the results. Such a model was recently used by MITRA and ROSS <sup>3</sup> to calculate the decay widths of the 10 baryons as well as the negative parity resonances under the assumption of  $SU(6) \times O(3)$  structures for these particles <sup>4, 5</sup>. The model reproduced not only the well-known  $SU(6)$  results for the 56 decays, but several of the established modes of the negative parity baryons as well.

We have felt sufficiently encouraged by these results to venture a corresponding evaluation for the couplings and decays of tensor and axial vector mesons to the meson states PP or PV. A similar evaluation for VPP coupling was done by BECCHI and MORPURGO <sup>6</sup> for the important case of  $\rho \rightarrow 2\pi$  decay and recently extended for other modes by COOK <sup>7</sup>, with very good results. The mathematical objections against such an asymmetrical treatment to the two final state pions was already noted by the authors of Ref. (6), yet their deduction of the  $\rho\pi\pi$  coupling constant in terms of  $NN\pi$  coupling, in good accord with experiment, encourages the hope

that one could still obtain "correct" results in a quark model, even though the dynamics is not understood. Taking such a pragmatic attitude, we have obtained coupling constant relations for MPV and MPP couplings, where M is  $0^+$ ,  $1^+$ ,  $2^+$ , and also the decay widths for several modes of experimental interest. Our results are in rather good accord with experiment.

While the  $2^+$  mesons are firmly established, certain  $1^+$  mesons (especially A, (1080), B(1210) and  $K_A(1320)$ ) received strong support at the last Berkely Conference <sup>1, 8</sup>, and  $0^+$  mesons are still at a conjectural stage. However, in a quark model all these cases must be considered together since their  $Q\bar{Q}$  structures, viz.,  ${}^3P_{0,1,2}$  and  $1P_1$ , all have a common orbital wave function. This is the essential due to the possibility of obtaining a rich variety of correlations among the MPP and MPV coupling constant, since both  $V({}^3S_1)$  and  $P({}^1S_0)$  have again a common orbital structure <sup>9</sup>).

To indicate the essential features of the calculations we assume an SU(3) invariant  $Q\bar{Q}P$  coupling <sup>3</sup>

$$c \sum_{i=1}^2 \sum_{\alpha=1}^8 \bar{q} \lambda_{\alpha}^{(i)} P_{\alpha} \sigma^{(i)} \underline{q} Q \quad (1)$$

where  $\underline{q}$  is the meson momentum,  $\sigma^{(1)}$ ,  $\sigma^{(2)}$  are the spin operators for  $Q$  and  $\bar{Q}$ ,  $\lambda_{\alpha}^{(i)}$  the corresponding Gell-Mann matrices <sup>10</sup>, and  $c$  is an overall constant in the strict SU(3) limit. Violation of SU(3), which could be especially important in the pion versus kaon modes, must be taken in phase space, and perhaps also in the coupling constant <sup>11</sup>.

With the assumption of the quark structures indicated for the various mesons, it is easily seen that only s and d waves are possible for the outgoing meson states. For each mode of transition, the overlap integral may be taken as a free parameter. Both integrals are proportional to  $q^2$  and give the same structure for the form factor for large  $q^2$ . Thus we have two sets of coupling constants, (i) the scalar couplings  $g_s(\text{MPV})$ ,  $g_s(\text{MPP})$ , which in the SU(3) limit bear geometrical ratios to one another, and (ii) the tensor coupling constants  $g_T(\text{MPV})$ ,  $g_T(\text{MPP})$  which are again related by geometrical factors. We list below morely the extra SU(3) relations between these coupling constants for the important meson states. For this purpose, we have used the "ideal mixing angle"  $\tan \beta = 1/\sqrt{2}$  for  $\phi - \omega$  mixing, and the angle

$$\tan \beta = \frac{1}{2\sqrt{2}}, \quad (2)$$

for  $f_0, f_0'$  mixing, the latter designed to give a zero width for the decay  $f_0(1250) \rightarrow K\bar{K}$ , in conformity with experiment<sup>12</sup>.

$$\begin{aligned} \frac{1}{2} g_s(A_1 \rho \pi) &= g_s(B\omega\pi) = -\sqrt{2} g_s(K_A^* K\omega) = -g_s(K_A^* K\phi) \\ &= \frac{1}{6} \sqrt{2} g_s(\sigma_1 \pi\pi) = \frac{1}{3} g_s(\sigma_8 \pi\pi) \end{aligned} \quad (3)$$

$$g_s(B\phi\pi) = g_T(f_0\eta\eta) = g_T(f_0'\eta\eta) = g_T(f_0 K\bar{K}) = 0 \quad (4)$$

$$\begin{aligned}
\frac{3}{10} \sqrt{5} g_T(A_1 \rho \pi) &= -\frac{3}{10} \sqrt{5} g_T(B \omega \pi) = \frac{1}{2} g_T(A_2 \rho \pi) \\
&= -\sqrt{2} g_T(K_V^* K \omega) = -g_T(K_V^* K \phi) \quad (5) \\
&= \frac{1}{4} \sqrt{2} g_T(f_0 \pi \pi) = \frac{1}{2} g_T(f_0' \pi \pi) = \frac{1}{2} \sqrt{3} g_T(A_2 \eta \pi) \\
&= \frac{1}{6} \sqrt{6} g_T(f_0' K \bar{K}) = -\frac{1}{3} \sqrt{3} g_T(f_0' K^* \bar{K})
\end{aligned}$$

An interesting feature which immediately emerges from this model is that the decay of  $1^+$  mesons in particular,  $A_1$  and  $B$ , into two pseudoscalars is strictly forbidden, in good accord with observation. However, the decays of tensor and scalar mesons into such states should be appreciable.

Finally we give the results for some decay widths of experimental interest. According to this model, these widths can be parametrized in the form

$$g_{IY}^2 \left[ \lambda_S S_{LSJ}^2 + \lambda_T T_{LSJ}^2 \right] q^5 F^2(q) \quad (6)$$

where  $g_{IY}$  is the unitary spin factor in the meson couplings for each model of decay,  $S_{LSJ}$ ,  $T_{LSJ}$  are the reduced matrix elements resulting from the spin and angular integrations, and  $\lambda_S$ ,  $\lambda_T$  represent the squares of the overall scalar and tensor coupling strengths respectively, with the modification indicated <sup>11</sup> for pion versus kaon modes. The form factor  $F(q)$ , representing the centrifugal effect has been parametrized as

$$F(q) = \exp(-\frac{1}{2}\alpha q^2) \quad (7)$$

About the best results in this model are obtained with

$$\sqrt{\alpha} \approx (250 \text{ MeV})^{-1}, \quad \lambda_S \approx 2 \lambda_T, \quad (8)$$

and Table I gives the calculated widths for the physically interesting cases.

Taking account of the large uncertainties in several of the experimental widths, the general trend seems to be rather well reproduced. Thus the main decay modes are quite faithfully represented, though for some of the smaller modes there are some discrepancies. We hope more extensive measurements will clear up the picture.

As a few final remarks, this model is capable of yielding decay widths for transitions in which one of the  $2^+$ ,  $1^+$  or  $0^+$  mesons is involved in the final state in association with a pion or kaon. As a matter of fact, there seem to have been observed certain decays of these heavy mesons with precisely such features, e.g.,  $D(1285) \rightarrow \pi_V(1003)\pi$ ,  $E(1420) \rightarrow \pi_V(1003)\pi$  and  $K_A(1800) \rightarrow K_V(1420)\pi$ . It would be interesting to compute such modes as and when more data of this kind are available.

Finally, the model gives an interesting prediction for  $\sigma \rightarrow 2\pi$  decay. For an  $SU(3)$  singlet of 400 MeV it gives a decay width of 63 MeV, while the assumption of octet structure for this particle gives only half this value (31.5 MeV).

TABLE I

Decay widths of principal modes of  $2^+$  and  $1^+$  mesons

Transition	$\Gamma_{\text{calc}}$ (MeV)	$\Gamma_{\text{expt}}$ (MeV) *
$A_1(1080) \rightarrow \rho + \pi$	130 (input)	$130 \pm 40$
$K_A^*(1320) \rightarrow K^* \pi$ $\rightarrow K\rho$ $\rightarrow K\omega$	102 6.5 0.8	} large overlap } < 10% } $80 \pm 20$
$B(1210) \rightarrow \omega\pi$ $\rightarrow \phi\pi$	110 0	$119 \pm 24$ 0
$A_2(1300) \rightarrow \rho\pi$ $\rightarrow K\bar{K}$ $\rightarrow \eta\pi$	108 14.9 22.1	$75 \pm 8$ $4 \pm 1.2$ $3 \pm 2.5$
$K_V^*(1420) \rightarrow K^* \pi$ $\rightarrow K\rho$ $\rightarrow K\omega$ $\rightarrow K\pi$ $\rightarrow K\eta$	41.0 9.4 2.6 22.7 19.0	$33 \pm 7$ $8 \pm 4$ $1 \pm 2$ $46 \pm 6$ $2 \pm 3$
$f_0(1250) \rightarrow \pi\pi$ $\rightarrow K\bar{K}$ $\rightarrow \eta\eta$	60.0 0 0	} large } 2% } $117 \pm 15$ unseen }
$f_0'(1500) \rightarrow \pi\pi$ $\rightarrow K\bar{K}$ $\rightarrow \eta\eta$ $\rightarrow K\bar{K}^* + K\bar{K}^*$	4.48 19.2 0 31.4	} < 14% } > 60% } $86 \pm 23$ unseen } < 40% }

\* Estimated from the data cards (November 1966) of Rbsenfeld et al.



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9. Ignoring a possible  ${}^3D_1$  component in V.
10. With appropriate sign changes for  $\lambda_2, \lambda_3, \lambda_5, \lambda_7, \lambda_8$  in the  $\bar{Q}$ -representation.
11. A simple modification, found in Ref. 7, it to assume that the coupling constant is roughly proportional to the inverse square root of the meson mass, so that the Kaon modes are reduced by a factor  $(m/m_K)$  relative to the pion modes, in the decay rates.

12. Here  $\sigma_1$  and  $\sigma_8$  refer to the  $3_{P_0}$  states of  $T = 0$  under the assumptions of SU(3) singlet and octet respectively.