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STABILITY OF RELATIVISTIC HARTREE STATES

by

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Abstract

It is shown that the Hartree solution of a relativistic hamiltonian of the type considered by Walecka and collaborators is unstable with respect to particle-hole excitations responsible for fluctuations of the nucleon effective mass, unless the negative energy states (Dirac sea) are taken into account and the corresponding renormalization counter terms are properly treated.

It is also shown that static properties as well as low frequency dynamical properties are not too much affected by vacuum polarization effects.

However, renormalization is crucial for a correct description of high energy processes associated with the creation of scalar mesons or of particle-antiparticle pairs, even if the momentum transfer is small. Polarization effects lead to a reduction of the effective mass of the scalar meson in the medium.

Key-words: Relativistic manybody theory; Dirac sea; Stability.

A most successful model of the nucleus regarded as a collection of relativistic nucleons interacting through the exchange of σ and ω mesons has been developed by Walecka and collaborators [1]. In this simple model, the nucleon effective mass has been determined by the mean-field value of the σ field, neglecting the contribution of the negative energy nucleon states, although the replacement of the free nucleon mass M by the nucleon effective mass M* implies an actual involvement of the negative energy states in the underlying mechanism.

It seems very natural to postulate that the ground state of a physical system should have the fundamental property of being stable. By stability we mean that no slight disturbance will decrease the energy of the equilibrium state and the time evolution of the perturbed state only involves real frequencies. We will show in this paper that it is necessary not to disregard the role of the negative energy states in order to guarantee the stability of the system.

Since the ω field does not play a relevant role in the following discussion, it is not explicitly taken into account in the presentation, for simplicity.

The hamiltonian of a system of nucleons interacting with a scalar meson field may be written as

$$H = \int d^3x \left[\psi^\dagger (\vec{\alpha}.\vec{p} + \beta M)\psi - g\,\psi^\dagger \beta \psi \phi + \tfrac{1}{2}(\Pi^2 + \nabla\phi.\nabla\phi + m^2\phi^2)\right]. \quad (1)$$

No counter terms are needed for renormalizing the nucleon and the meson mass, provided the configuration space is spanned by zero momentum mesons and positive energy nucleon states only, so that transitions into negative energy states are artificially forbidden.

Let $|M',\varphi\rangle = |SD(M')\rangle \otimes |\varphi\rangle$ denote the product of a Slater determinant $|SD(M')\rangle$ of positive energy plane wave nucleon states of mass M' and momentum less than p_F multiplied by a coherent state $|\varphi\rangle$ of zero momentum mesons. Note that the mass parameter M' is different from the mass M appearing in the hamiltonian, (eq.(1)). The energy of this nuclear state is

$$E(M',\varphi) = \langle M',\varphi | H | M',\varphi \rangle$$

$$= 4 \sum_{p<p_F} \frac{p^2 + M'(M - g\varphi)}{\sqrt{p^2 + M'^2}} + \frac{1}{2} m^2 \varphi^2 V \,, \tag{2}$$

where V is the normalization volume. It is clear that the Hartree Fock equations for this system are satisfied at a stationarity point W with respect to variations of M' and φ ; i.e.,

$$\frac{\partial E}{\partial M'} = 4(M - g\varphi - M') \sum_{p<p_F} \frac{p^2}{(p^2 + M'^2)^{3/2}} = 0 \,, \tag{3}$$

$$\frac{\partial E}{\partial \varphi} = -4 \sum_{p<p_F} \frac{g M'}{\sqrt{p^2 + M'^2}} + m^2 \varphi V = 0 \,. \tag{4}$$

The energy surface $E(M',\varphi)$ is represented in fig. 1 which shows that the stationarity point W does not provide a minimum of the energy. The fact that, at the stationarity point, the determinant

$$\Delta = \begin{vmatrix} \dfrac{\partial^2 E}{\partial M'^2} & \dfrac{\partial^2 E}{\partial M' \partial \varphi} \\[2ex] \dfrac{\partial^2 E}{\partial M' \partial \varphi} & \dfrac{\partial^2 E}{\partial \varphi^2} \end{vmatrix} \tag{5}$$

$$= -4 \sum_{p<p_F} \frac{p^2}{(p^2 + M'^2)^{3/2}} \left(m^2 V + 4 g^2 \sum_{p<p_F} \frac{p^2}{(p^2 + M'^2)^{3/2}} \right)$$

is negative, proves that this point is a saddle point. This instability is obviously due to keeping the negative energy states empty in the trial Slater determinant $|M',\varphi\rangle$. This means that the negative energy states should be filled up, so, the remedy to the instability problem lies in the replacement of the trial Slater determinant previously considered, by a new Slater determinant of positive energy plane wave states with momentum less than p_F and negative energy plane wave states with momentum less than some cut-off value Λ, which, eventually, will be allowed to increase indefinitely. For simplicity, we use the same symbol $|M',\varphi\rangle$ for the new state. However, the presence of single particle negative energy states requires the renormalization of the hamiltonian through appropriate counter terms, which depend on the value of Λ, so that, following Chin [2], we write

$$\tilde{H} = H + \int d^3x \left(a_\Lambda \phi + \frac{1}{2} b_\Lambda \phi^2 + \frac{1}{3!} c_\Lambda \phi^3 + \frac{1}{4!} d_\Lambda \phi^4 + \frac{1}{2} \lambda_\Lambda \Pi^2 \right). \tag{6}$$

On the other hand, the configuration space should be defined as the space of single particle states of momentum less than Λ, either with positive or negative energy. The coefficients a_Λ, b_Λ, c_Λ, d_Λ and λ_Λ are well defined functions of the cut-off momentum Λ which are determined so as to insure that the physical values of the free nucleon and meson masses, respectively M and m, are reproduced by the model. The energy of the state $|M',\varphi\rangle$ becomes now

$$\begin{aligned} E(M',\varphi) &= \langle M',\varphi|\tilde{H}|M',\varphi\rangle \\ &= 4 \sum_p{}' \frac{p^2 + M'(M - g\varphi)}{\sqrt{p^2 + M'^2}} + V\left(\frac{1}{2} m^2\varphi^2 + a_\Lambda \varphi \right. \\ &\quad \left. + \frac{1}{2} b_\Lambda \varphi^2 + \frac{1}{3!} c_\Lambda \varphi^3 + \frac{1}{4!} d_\Lambda \varphi^4 \right), \end{aligned} \tag{7}$$

where the dash on the summation sign reminds us that

$$\sum_p{}' = \sum_{p<p_F} - \sum_{p<\Lambda} = - \sum_{p_F<p<\Lambda} \tag{8}$$

The stationarity condition is now written as

$$\frac{\partial E}{\partial M'} = 4\,(M - g\varphi - M') \sum_p{}' \frac{p^2}{(p^2+M'^2)^{3/2}} = 0 \ , \tag{9}$$

$$\frac{\partial E}{\partial \varphi} = -\,4 \sum_p{}' \frac{gM'}{\sqrt{p^2+M'^2}} + V\left(m^2\varphi + a_\Lambda + b_\Lambda\varphi + \tfrac{1}{2}c_\Lambda\varphi^2 + \tfrac{1}{3!}d_\Lambda\varphi^3\right) = 0 \ . \tag{10}$$

We require that, at the stationarity point, for the vacuum ($p_F=0$) the nucleon mass M' has the free nucleon value M and the field φ is zero. This requirement yields

$$V a_\Lambda = -4 \sum_{p<\Lambda} \frac{gM}{\sqrt{p^2+M^2}} \ . \tag{11}$$

In order that the derivative $\partial E/\partial\varphi$ remains a well defined function of φ, when the replacement $M' = M - g\varphi$ is made in eq.(10) and Λ is allowed to increase indefinitely, we also require

$$V b_\Lambda = 4g \frac{\partial}{\partial M} \sum_{p<\Lambda} \frac{gM}{\sqrt{p^2+M^2}} \tag{12}$$

$$V c_\Lambda = -\,4g^2 \frac{\partial^2}{\partial M^2} \sum_{p<\Lambda} \frac{gM}{\sqrt{p^2+M^2}} \tag{13}$$

$$V d_\Lambda = 4g^3 \frac{\partial^3}{\partial M^3} \sum_{p<\Lambda} \frac{gM}{\sqrt{p^2+M^2}} \tag{14}$$

From eq.(9),the effective mass of the nucleon in nuclear matter is given by

$$M^* = M - g\varphi_0 \tag{15}$$

where φ_0 is such that

$$-4\sum_p{}' \frac{g(M-g\varphi_0)}{\sqrt{p^2+(M-g\varphi_0)^2}} + V\left(m^2\varphi_0 + a_\Lambda + b_\Lambda\varphi_0 + \right.$$
$$\left. + \frac{1}{2}c_\Lambda\varphi_0^2 + \frac{1}{3!}d_\Lambda\varphi_0^3\right) = 0. \tag{16}$$

When Λ approaches $+\infty$, this equation becomes

$$-\frac{2g}{\pi^2}\int_0^{p_F}\frac{(M-g\varphi_0)p^2}{\sqrt{p^2+(M-g\varphi_0)^2}}dp + \frac{g}{\pi^2}\left\{(M-g\varphi_0)^3\log\frac{M-g\varphi_0}{M}\right.$$
$$\left. + M^2g\varphi_0 - \frac{5}{2}Mg^2\varphi_0^2 + \frac{11}{6}g^3\varphi_0^3\right\} + m^2\varphi_0 = 0. \tag{17}$$

At the stationarity point the determinant

$$\Delta = \begin{vmatrix} \dfrac{\partial^2 E}{\partial M'^2} & \dfrac{\partial^2 E}{\partial M'\partial\varphi} \\ \dfrac{\partial^2 E}{\partial M'\partial\varphi} & \dfrac{\partial^2 E}{\partial\varphi^2} \end{vmatrix}(M^*,\varphi_0)$$
$$= -4\sum_p{}'\frac{p^2}{(p^2+M^{*2})^{3/2}}\left[4g^2\sum_p{}'\frac{p^2}{(p^2+M^{*2})^{3/2}}\right.$$
$$\left. + V\left(m^2 + b_\Lambda + c_\Lambda\varphi_0 + \frac{1}{2}d_\Lambda\varphi_0^2\right)\right] \tag{18}$$

is positive, since both factors in the right hand side are positive. Indeed, as Λ approaches $+\infty$, the quantity

$$S_\Lambda = \frac{4g^2}{V} \sum_p{}' \frac{p^2}{(p^2+M^{*2})^{3/2}} + m^2 + b_\Lambda + c_\Lambda \varphi_0 + \frac{1}{2} d_\Lambda \varphi_0^2 \tag{19}$$

remains well defined and is positive, and

$$S = \lim_{\Lambda\to\infty} S_\Lambda = \frac{4g^2}{2\pi^2} \int_0^{p_F} \frac{p^4}{\sqrt{p^2+(M-g\varphi_0)^2}^{\,2}}\, dp - \frac{4g^2}{(2\pi)^2} \left\{ 3(M-g\varphi_0)^2 \log \frac{M-g\varphi_0}{M} + 3Mg\varphi_0 - \frac{9}{2} g^2 \varphi_0^2 \right\} + m^2 > 0 . \tag{20}$$

In fig. 2, the nuclear energy $E(M', \varphi) - E(M, \varphi=0, p_F=0)$ is plotted as a function of the variables M' and

$$z = \Lambda\, (M' - M + g\varphi) . \tag{21}$$

Expressed in terms of these variables, the energy surface is independent of Λ, provided Λ is large enough. The stationarity point is now a minimum, as is clearly shown in fig. 2.

At this point we would like to briefly remark that the renormalization considered by Chin is the minimal renormalization scheme insuring that the physical consequences of the theory are independent of the cutoff Λ . However, other renormalization schemes are equally possible. It is clear, for instance, that the same effect is accomplished if in eq. (7) the counter terms $a_\Lambda \varphi + \frac{1}{2} b_\Lambda \varphi^2 + \frac{1}{3!} c_\Lambda \varphi^3 + \frac{1}{4!} d_\Lambda \varphi^4$ are replaced by the term

$$-4V^{-1} \sum_{p<\Lambda} \sqrt{p^2+(M-g\varphi)^2}$$

which completely cancels the contribution of negative energy states. Actually, this prescription has the nice feature of providing a justification for Walecka's initial procedure, as far as equilibrium properties are concerned, and, moreover, of stabilizing the Hartree-Fock solution.

In table 1 (columns I and II) we study the influence on several physical quantities of the renormalization procedure accounted for by eq.(7). The values of g_σ and g_ω are fixed by the density and binding energy of nuclear matter. We observe that the effective mass M* and the incompressibility K are slightly improved by the renormalization procedure, since M*=522 MeV is usually considered too low an effective mass and K=546 MeV is also considered to be a too high an incompressibility. This improvement is not sufficient, but the possibility remains of adding extra terms to the hamiltonian. In columns III and IV we study the effect of adding a term of the form $\lambda \int d^3x\, \phi^4$ to the hamiltonian [3]. We consider it an interesting result that the static properties (binding energy,density, nucleon effective mass,incompressibility, etc.) remain the same when this term is added, whether the negative energy states and corresponding counter terms are omitted or not. This result provides a justification for the original Walecka prescription of taking only into account positive energy states. We observe that the values of the effective mass M* and incompressibility K obtained when the term $\lambda \int d^3x\, \phi^4$ with λ=80, is added to the hamiltonian are very reasonable. We recall that the inclusion of the Dirac sea insures the stability of the system.

We return now to the previous hamiltonian (eq.(6)), i.e., we omit the term $\lambda\int d^3x\phi^4$. We wish now to investigate, in the Random Phase Approximation the dynamics of scalar excitations with zero momentum of our system. To this end we consider the following canonical transformation of the zero momentum component of the meson field

$$\phi = \varphi_0 + \tilde{\phi}\ , \tag{22}$$

$$\Pi = \tilde{\Pi}\ , \tag{23}$$

so that

$$[\tilde{\Pi}, \tilde{\phi}] = -i V^{-1}\ . \tag{24}$$

We denote by $c_p^\dagger$ the creation operator for a particle in a state with momentum p (creation operator for a positive energy state) and by $b_p^\dagger$ the creation operator for an antiparticle with momentum p (destruction operator for a negative energy state). Zero momentum excitations may then be described by the hamiltonian

$$\tilde{H} = E(M^*, \varphi_0) + H_{RPA}\ , \tag{25}$$

and

$$H_{RPA} = \sum_{p<p_F} \tilde{\varepsilon}_p (c_p^\dagger c_p + b_p^\dagger b_p) - g \sum_{p_F<p<\Lambda} (\eta_p^* c_p^\dagger b_{-p}^\dagger + \eta_p b_{-p} c_p) + \frac{1}{2} V \Big[(\tilde{\Pi}^2 + m^2\tilde{\phi}^2) + \lambda_\Lambda \tilde{\Pi}^2 + (b_\Lambda + c_\Lambda \varphi_0 + \frac{1}{2} d_\Lambda \varphi_0^2)\tilde{\phi}^2 \Big]\ . \tag{26}$$

Here

$$\tilde{\varepsilon}_p = \sqrt{p^2 + M^{*2}} \tag{27}$$

$$\eta_p = < \chi_s | - \frac{\vec{\sigma}\cdot\vec{p}}{\tilde{\varepsilon}_p} | \chi_s > \tag{28}$$

We regard the operators $c_p^+ \; b_{-p}^+$ and $b_{-p} \; c_p$ as quasi boson operators, so that the RPA hamiltonian is diagonalized by the canonical transformation

$$\theta^+ = \sum_p (x_p \, c_p^+ b_{-p}^+ + y_p \, b_{-p} c_p) + i X \tilde{\Pi} + Y \tilde{\Phi} . \tag{29}$$

We have, therefore

$$[H_{RPA} , \theta^+] = \omega \, \theta^+ , \tag{30}$$

provided ω is such that

$$\frac{\omega^2}{1+\lambda_\Lambda} = - \frac{16 g^2}{V} \sum_p{}' \frac{p^2}{\tilde{\varepsilon}_p (\omega^2 - 4 \tilde{\varepsilon}_p^2)} + m^2 + b_\Lambda + c_\Lambda \varphi_0 + \frac{1}{2} d_\Lambda \varphi_0^2 . \tag{31}$$

The renormalization constant λ_Λ is fixed by the requirement that, for the vacuum ($p_F = 0$) the energy of the lowest (collective) RPA state is given by the physical meson mass, $\omega = m$. We obtain, therefore, the condition

$$\frac{1}{1+\lambda_\Lambda} = 1 + \frac{4 g^2}{V} \sum_{p<\Lambda} \frac{p^2}{\varepsilon_p^3 (m^2 - 4 \varepsilon_p^2)} , \tag{32}$$

where $\mathcal{E}_p=\sqrt{p^2+M^2}$. Moreover, when the Fermi momentum does not vanish, we have the dispersion relation for the eigenfrequencies

$$\omega^2 = F_\Lambda(\omega^2) \tag{33}$$

with

$$\begin{aligned} F_\Lambda(\omega^2) = & -\frac{16g^2}{V}\sum_{p<p_F}\frac{p^2}{\tilde{\mathcal{E}}_p(\omega^2-4\tilde{\mathcal{E}}_p^2)} \\ & -\frac{4g^2\omega^2}{V}\sum_{p<\Lambda}\left(\frac{p^2}{\mathcal{E}_p^3(m^2-4\mathcal{E}_p^2)}-\frac{p^2}{\tilde{\mathcal{E}}_p^3(\omega^2-4\tilde{\mathcal{E}}_p^2)}\right) \\ & -\frac{4g^2}{V}\sum_{p<\Lambda}\frac{p^2}{\tilde{\mathcal{E}}_p^3}+b_\Lambda+c_\Lambda\varphi_0+\frac{1}{2}d_\Lambda\varphi_0^2+m^2. \end{aligned} \tag{34}$$

The function $F_\Lambda(\omega^2)$ is represented in fig.3. We observe that $F_\Lambda(0)=S_\Lambda>0$, (see eq.(20)). The stability condition $S_\Lambda>0$ insures, therefore, that the solutions of the equation $\omega^2=F_\Lambda(\omega^2)$ are positive (ω is real). The collective energy for the mesonic state remains real when $p_F\neq0$ and the coupling with nucleons yields 50 MeV mass defect for the meson in nuclear matter.

We remark that the Walecka model, without explicit treatment of the negative energy states, also admits a zero momentum RPA mode. Since in his model there can be no particle-hole excitation with zero momentum, such an RPA mode is strictly related to mean field fluctuations, which induce fluctuations of the scalar density through the relation $M'=M-g\varphi$ and give rise to an RPA frequency corresponding to

$$\omega^2=m^2+\frac{4g^2}{V}\sum_{p<p_F}\frac{p^2}{\tilde{\mathcal{E}}_p^3}>m^2$$

This RPA frequency increases with p_F while the solution ω of the corresponding dispersion relation in our approach (eq.(33)) shows the opposite behaviour which seems to be required by experiment. Note that the fluctuations in the Walecka model are constrained on the straight line $M^* = M - g\varphi$ in fig. 1.

The non collective solutions should not be ignored, since they may play a role in the high frequency properties of the system. Indeed, given some transition operator D, the RPA provides well defined prescriptions for computing the transition amplitude from the RPA vacuum $|0)$ to the RPA excited state $|\nu)$

$$(\nu|D|0) = \langle M^*, \varphi_0 | [\theta_\nu, D] | M^*, \varphi_0 \rangle \tag{35}$$

from which the transition strength $\omega_\nu |(\nu|D|0)|^2$ is obtained. The sum rule

$$\sum_\nu \omega_\nu |(\nu|D|0)|^2 = \frac{1}{2} \langle M^*, \varphi_0 | [D, [H, D]] | M^*, \varphi_0 \rangle \tag{36}$$

determines how much strength is distributed over non collective states.

The main conclusion of the present work is that the stability of the Hartree solution of a relativistic hamiltonian of the type considered by Walecka and collaborators requires the appropriate treatment of negative energy states (Dirac sea) and the corresponding renormalization counter terms. It is natural to adjust the counter terms so as to insure that in the vacuum the mean field φ is zero, so that the nucleon effective mass coincides with the mass M. One then finds that the static properties of nuclear matter (binding energy, density, incompressibility, etc.) are not drastically affected by renormalization and in some circumstances may even be quite insensitive to renormalization. This fact provides a justification for the initial Walecka prescription of disregarding renormalization altogether.It seems that Walecka's prescription is also appropriate to describe low frequency dynamical processes, connected with low energy particle-hole excitations close to the Fermi surface (giant resonances). However,renormalization is crucial for a correct description of high energy processes associated with the creation of scalar mesons or of particle-antiparticle pairs (RPA correlations),even if the momentum transfer is small. Processes of this type are responsible for the reduction of the value of the effective mass of the meson in the medium.

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Figure captions

Figure 1

Energy surface $E(M', \varphi)$ of a trial mean field nuclear state composed of positive energy single particle states (Fermi sea), in the presence of the ω field, as a function of the nucleon effective mass M' and of the scalar field φ. The coupling constants g_σ and g_ω are fixed so as to reproduce the properties of nuclear matter [1]. The two solid lines crossing at the stationarity point W represent the Hartree eqs.(3) and (4), i.e., $M'=M-g\varphi$ and

$$\varphi = \frac{4 g_\sigma}{m^2 V} \sum_{p<p_F} \frac{M'}{\sqrt{p^2+M'^2}} \; .$$

Figure 2

Energy surface $E(M', \varphi) - E(M, \varphi=0, p_F=0)$ of a trial mean field nuclear state composed of positive and negative energy single particle states (Fermi sea plus Dirac sea), in the presence of the ω field, as a function of the nucleon effective mass M' and of the parameter z. The coupling constants g_σ and g_ω are fixed so as to reproduce the properties of nuclear matter [2].

Figure 3

Graphical determination of the collective excitation energy corresponding to the scalar meson mass in the medium, in the presence of the Dirac sea. The ordinate $F(0)=S_1$ of the intersection of the function $F(\omega^2)$ with the y axis, is positive as required by the stability of the Hartree state composed of positive and negative single particle states. The chosen parameters for this figure are $g_\sigma = 7.21$, $p_F = 280$MeV, so that $m = 500$ MeV. The solution is $\omega = 450$ MeV, corresponding to a 50 MeV mass defficiency.

Table caption

<u>Table 1</u> - The coupling constants g_σ and g_ω , fixed so as to reproduce the binding energy and the density of nuclear matter, the nucleon effective mass M*, the incompressibility K and the stability of relativistic Hartree states are displayed for several models described in the text. The first two lines summarize the definition of the models considered. We use the masses in Mev, 940 for nucleon, 500 for σ and 780 for ω.

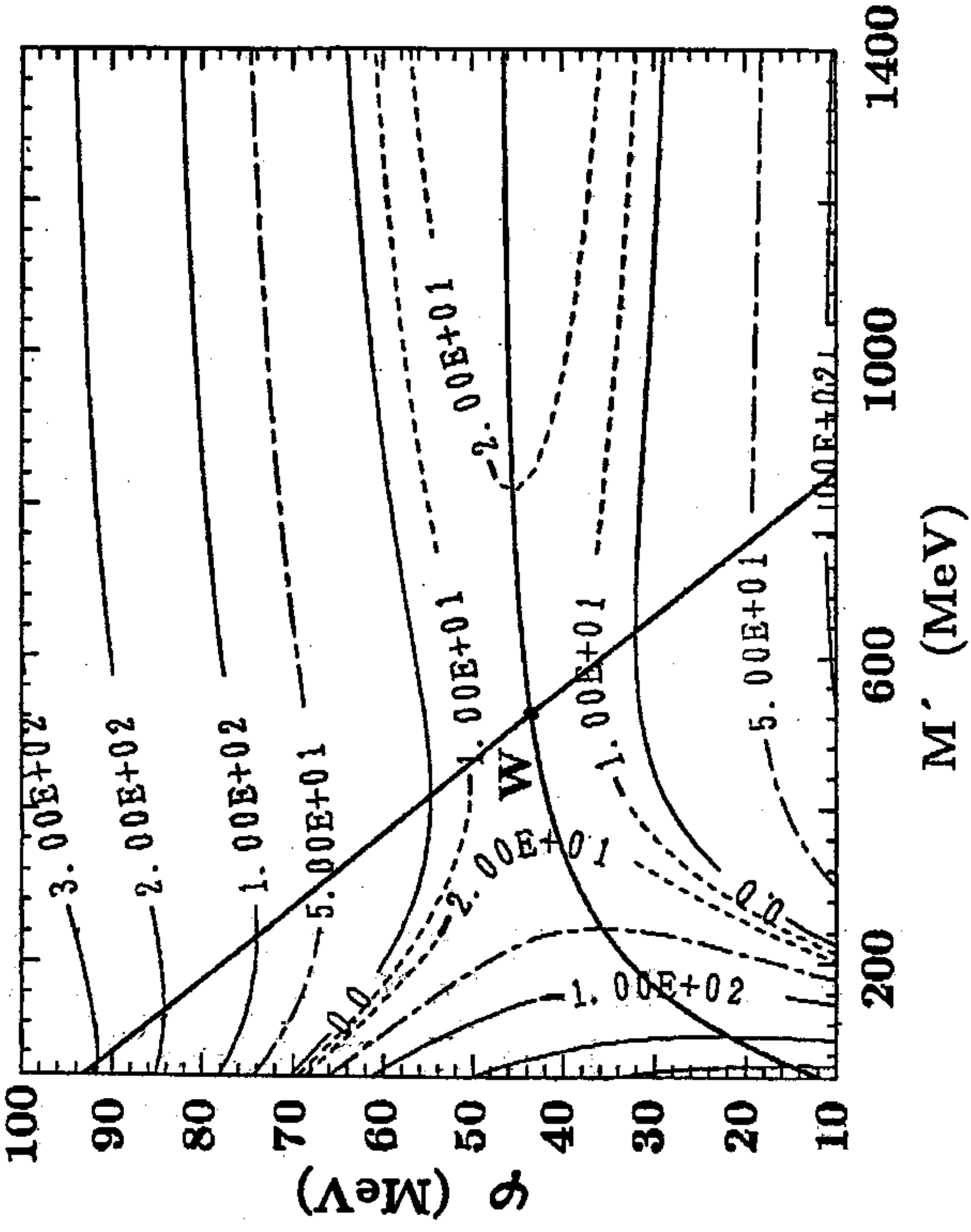
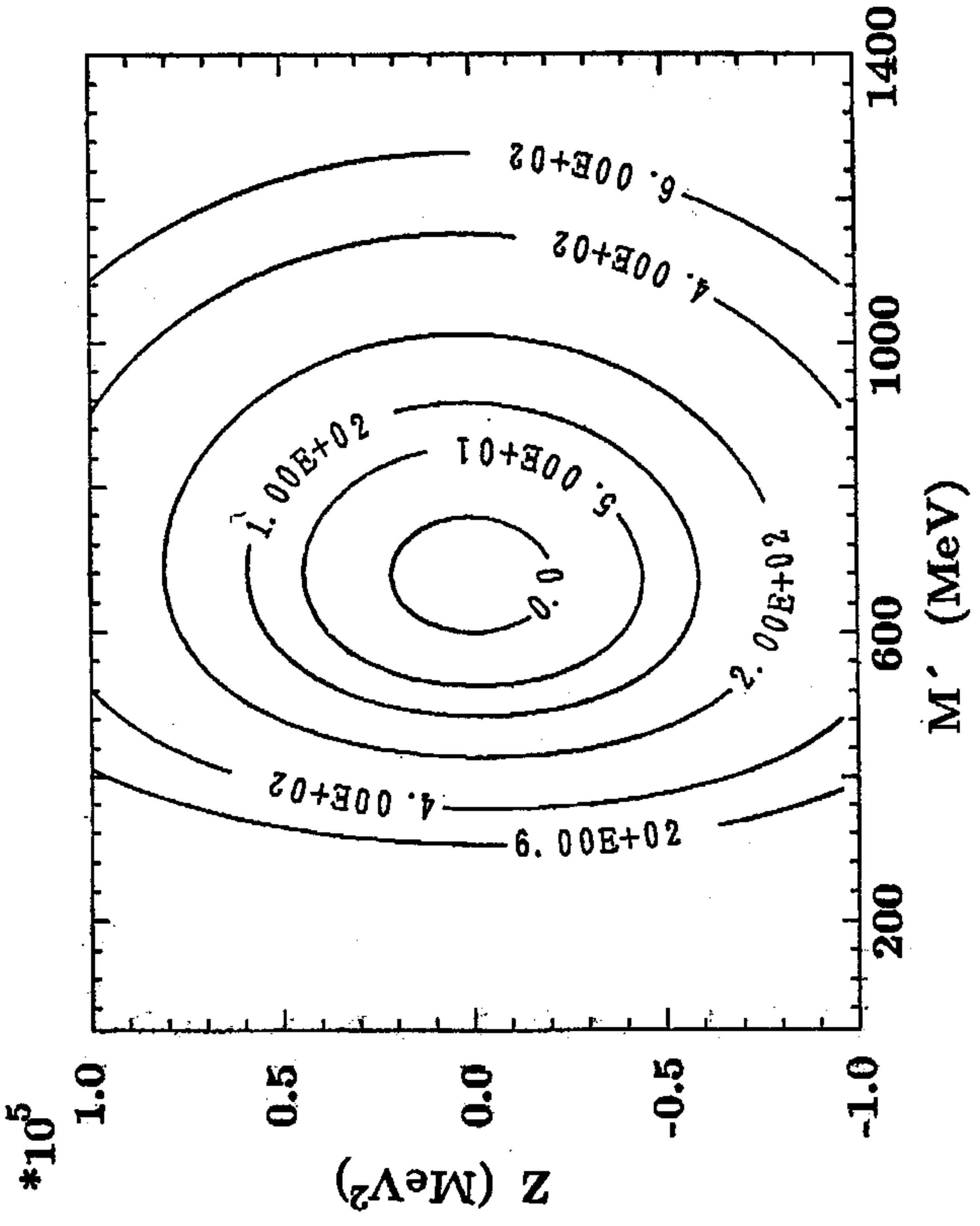

3.00E+02
2.00E+02
1.00E+02
5.00E+01
1.00E+01
2.00E+01
0.0
W
M′ (MeV)
φ (MeV)
200
600
1000
1400
100
90
80
70
60
50
40
30
20
10

Fig. 1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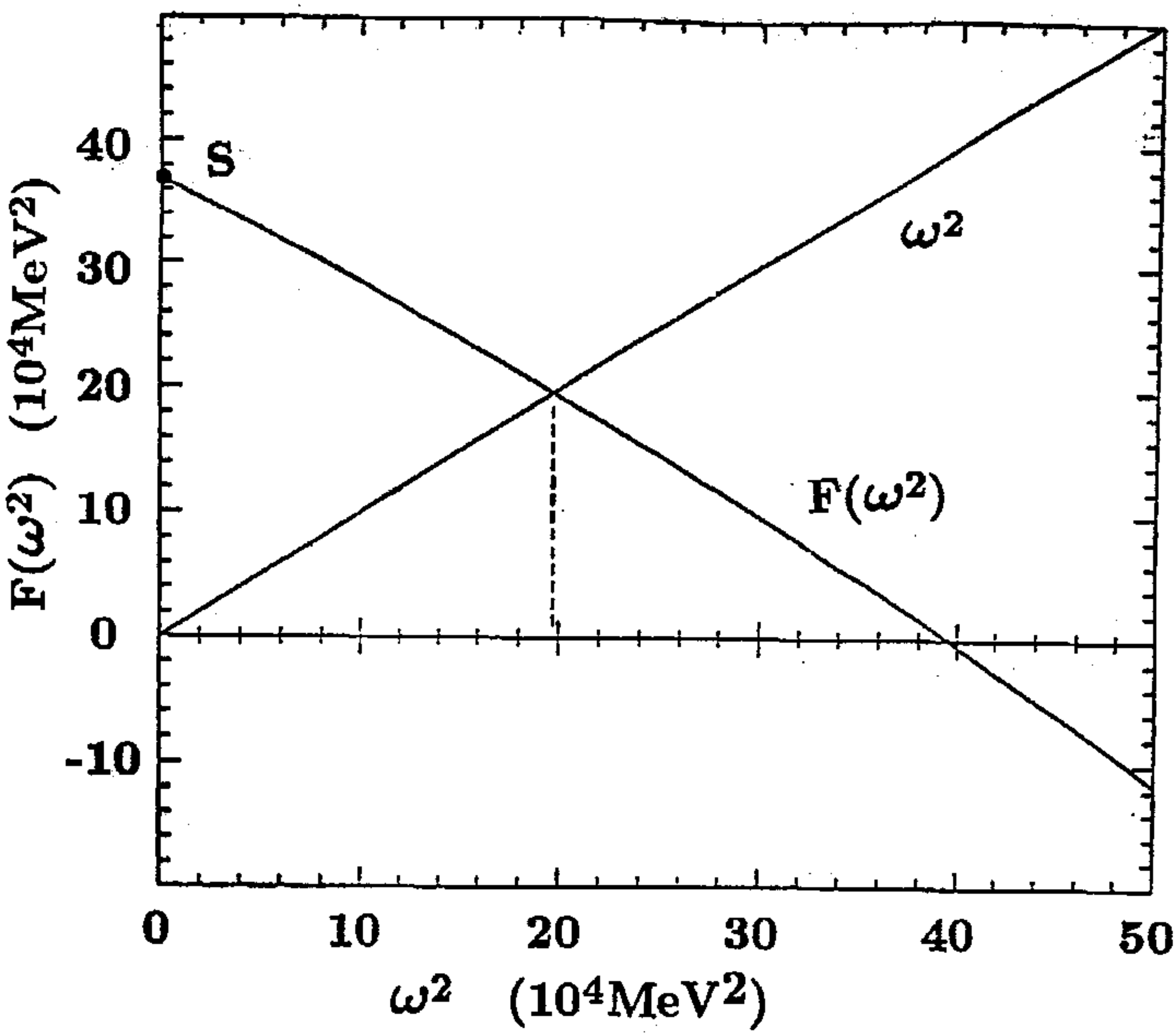
2.00E+02
1.00E+02
5.00E+01
0.0
4.00E+02
6.00E+02
1400
1000
600
200
M´ (MeV)
1.0
0.5
0.0
-0.5
-1.0
*10^5
Z (MeV^2)

Fig. 2



40
30
20
10
0
-10
S
ω^2
$F(\omega^2)$
$F(\omega^2)$ $(10^4 MeV^2)$
0
10
20
30
40
50
ω^2 $(10^4 MeV^2)$

Fig. 3

Table 1

Model \ λ	I	II	III	IV
λ	0	0	80	80
Dirac sea	excluded	included	excluded	included
g_σ	8.73	7.21	5.79	5.78
g_ω	11.66	8.90	4.95	4.93
M*(MeV)	522	675	818	818
K(MeV)	546	468	224	224
Stability	No	Yes	No	Yes

References

1) B.D. Serot and J.D. Walecka,
Advances in Nuclear Physics 16 (1985) 1.

2) S.A. Chin, Ann. Phys. 108 (1977) 301.

3) J.Boguta and A.R.Bodmer, Nucl. Phys. A292 (1977) 413,
L.I.Schiff, Phys.Rev. 84 (1951) 1,
T. Negishi and T.Kohmura, Phys. Rev. C19 (1979) 253.
The influence of the $\lambda\varphi^4$ interaction on a system of nucleons in positive energy states has been studied in these references.