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1-Loop Analysis of the Photon Self-Energy due to 3D-Gravity

by

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Abstract

A Maxwell-Chern-Simons field is minimally coupled to 3D-gravity. Feynman rules are written down and 1-loop corrections to the gauge-field self-energy are calculated. Transversality is verified and gauge-field dynamical mass generation does not take place.

Key-words: Topological gravity; Quantum gravity; 3D-field theories.

The study of topologically massive Yang-Mills and gravity theories has raised a great deal of interest after the classical paper by Deser, Jackiw and Templeton [1]. Relevant results, such as the finiteness of Chern-Simons theory in Landau gauge [2] and the renormalisability of 3D-gravity [3, 4], have contributed significantly for the broadening of research on a number of aspects of D=3-gauge field theories.

In a previous work [5], we have addressed to the proposal of extending the Barnes-Rivers projectors to account for 3D-gravity. As a byproduct of our study, we propose to carry out perturbative calculations for Chern-Simons gravity keeping the usual splitting of the metric field [3]. For that, we perform the minimal coupling of a Maxwell-Chern-Simons field to Einstein-Chern-Simons gravity and concentrate our attention on the 1-loop-corrected Abelian gauge field self-energy. The main motivation behind our calculation regards the analysis of the possibility of gauge-field dynamical mass generation [1, 6] through its gravitational coupling.

Let us now begin by considering the Lagrangean we adopt to describe the minimal coupling between the gauge and gravity sectors in D=3 :

$$\mathcal{L} = \mathcal{L}_{E.C.S.} + \mathcal{L}_{M.C.S.}\ , \tag{1}$$

where the first term on the right hand side denotes the Einstein-Chern-Simons Lagrangean,

$$\mathcal{L}_{E.C.S.} = \frac{-1}{2\kappa^2}\sqrt{-g}\,\mathcal{R} + \frac{1}{\mu}\varepsilon^{\lambda\mu\nu}\Gamma^{\rho}{}_{\lambda\sigma}(\partial_\mu\,\Gamma_{\rho}{}^{\sigma}{}_{\nu} + \frac{2}{3}\Gamma_{\mu}{}^{\sigma}{}_{\varphi}\,\Gamma_{\nu}{}^{\varphi}{}_{\rho})\ , \tag{2}$$

whereas the second term stands for the Maxwell-Chern-Simons term, [3]

$$\mathcal{L}_{M.C.S.} = \frac{-1}{4}\sqrt{-g}\;g^{\mu\alpha}g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} + \frac{M_{ph.}}{2}\varepsilon^{\mu\lambda\nu}A_\mu\,D_\lambda\,A_\nu\ . \tag{3}$$

Adopting the viewpoint of expanding the metric field around the flat-space geometry, [4]

$$g^{\mu\nu}(x) = \eta^{\mu\nu} - \kappa h^{\mu\nu}(x), \tag{4}$$

where $h^{\mu\nu}$ is the field variable defining the expansion, and κ is the Planck constant, one can read off the propagators and the Feynman rules describing the interaction in perturbative field theory.

[3] $D_\lambda A_\nu \equiv \partial_\lambda A_\nu - \Gamma^{\rho}_{\nu\lambda}A_\rho$ is the covariant derivative under gen. coord. transformations.
[4] diag. $\eta^{\mu\nu} \equiv (+\,;-\,,\cdots,-)$.

As fairly-well investigated [3, 5], on the basis of the field parametrisation (4), the propagator for topologically massive gravity, in Feynman gauge, read as below :

$$\begin{aligned}< h_{\mu\nu}(-q)\, h_{\kappa\lambda}(q) > &= \frac{-i}{64\, q^2\, [q^2 - M^2_{gr.}]} \{32\, i\, M_{gr.} q^\alpha\, [\varepsilon_{\mu\alpha\lambda}\Theta_{\kappa\nu} + \varepsilon_{\mu\alpha\kappa}\Theta_{\lambda\nu} + \\ &+ \varepsilon_{\nu\alpha\lambda}\Theta_{\kappa\mu} + \varepsilon_{\nu\alpha\kappa}\Theta_{\lambda\mu}] + \\ &- 64 M^2_{gr.}\, [\eta_{\mu\kappa}\eta_{\nu\lambda} + \eta_{\mu\lambda}\eta_{\nu\kappa} - 2\eta_{\mu\nu}\eta_{\kappa\lambda}] + \\ &- 64 q^2\, [\eta_{\mu\kappa}\omega_{\nu\lambda} + \eta_{\mu\lambda}\omega_{\nu\kappa} + \eta_{\nu\kappa}\omega_{\mu\lambda} + \\ &+ \eta_{\nu\lambda}\omega_{\mu\kappa} + \Theta_{\mu\nu}\Theta_{\kappa\lambda} - 2\eta_{\mu\nu}\omega_{\kappa\lambda} - 2\eta_{\kappa\lambda}\omega_{\mu\nu}]\}\,, \qquad (5)\end{aligned}$$

where $\Theta_{\mu\nu}$ and $\omega_{\mu\nu}$ are respectively the usual transverse and longitudinal projectors in the space of vectors and $M_{gr.} \equiv (\frac{\mu}{8\kappa^2})$.

The well-known Maxwell-Chern-Simons photon propagator [1], in Feynman gauge, take the form :

$$< A^a(-p)\, A^b(p) > = \frac{-i}{(p^2 - M^2_{ph.})} \left\{ \eta^{ab} - \frac{M_{ph.}}{p^2} \left[M_{ph.} \frac{p^a p^b}{p^2} - i\varepsilon^{a\alpha b}\, p_\alpha \right] \right\} . \qquad (6)$$

As we are coupling a bosonic field to 3D-gravity, the affine connection, $\Gamma_\mu{}^\nu{}_\rho$, appearing in the gravitational covariant derivative can be taken torsion-free; it is therefore identified with the Christoffel symbol. So, the last term at the RHS of eq.(3) shall not contribute to the interaction terms. Hence, the Feynman rules required for the computation of the 1-loop photon self-energy can be found by analysing the trilinear and quadrilinear parts stemming from the Maxwell term minimally coupled to gravity :

$$\mathcal{L}^{(3)}_{M.} = \frac{-\kappa}{4} \left(\frac{1}{2}\eta^{\mu\alpha}\eta^{\nu\beta} h^\varphi{}_\varphi(x) - \eta^{\mu\alpha} h^{\nu\beta}(x) - \eta^{\nu\beta} h^{\mu\alpha}(x) \right) F_{\mu\nu}\, F_{\alpha\beta}\,, \qquad (7)$$

$$\begin{aligned}\mathcal{L}^{(4)}_{M.} &= \frac{-\kappa^2}{4} \Big(h^{\mu\alpha}(x) h^{\nu\beta}(x) - \frac{1}{2}\eta^{\mu\alpha} h^{\nu\beta}(x) h^\varphi{}_\varphi(x) + \\ &- \frac{1}{2}\eta^{\nu\beta} h^{\mu\alpha}(x) h^\varphi{}_\varphi(x) - \frac{1}{4}\eta^{\mu\alpha}\eta^{\nu\beta} h^{\varphi\sigma}(x) h_{\varphi\sigma}(x) + \\ &+ \frac{1}{8}\eta^{\mu\alpha}\eta^{\nu\beta} h^\varphi{}_\varphi(x) h^\sigma{}_\sigma(x) \Big)\, F_{\mu\nu}\, F_{\alpha\beta}\,. \qquad (8)\end{aligned}$$

The 3- and 4-vertex Feynman rules can easily be read off from Lagrangean (3). They are drawn in Fig.(1) and their respective expressions, in momentum space, look as given below :

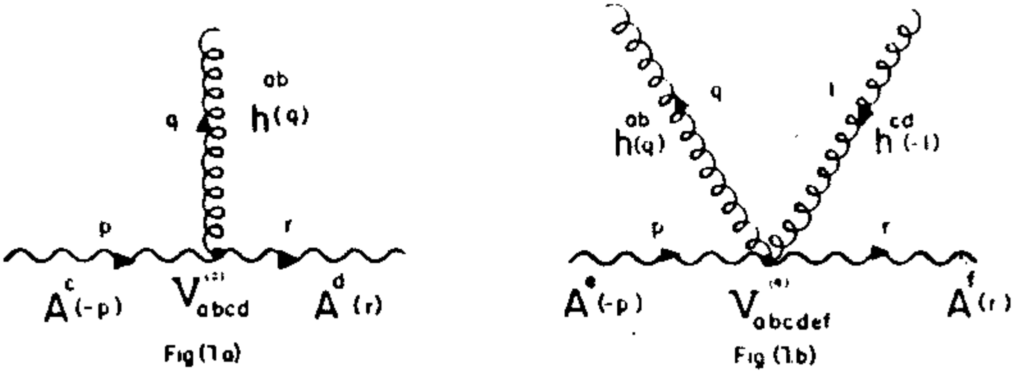

Fig.(1)
Graviton-photon vertices.

$$\begin{aligned}\mathcal{V}^{(3)}_{abcd} &= \frac{i\kappa}{2}\{(r\cdot p)\eta_{ab}\eta_{cd} - \eta_{ab}r_c p_d + \\ &+2p_a r_c\eta_{bd} + 2r_b p_d\eta_{ac} + \\ &-2(r\cdot p)\eta_{ac}\eta_{bd} - 2p_a r_b\eta_{cd}\} \end{aligned} \tag{9}$$

where $p = q + r$,

and

$$\begin{aligned}\mathcal{V}^{(4)}_{abcdef} &= i\kappa^2\{2p_c r_d\eta_{ae}\eta_{bf} - 2p_c r_b\eta_{ae}\eta_{df} + \\ &-(p\cdot r)\eta_{ae}\eta_{bf}\eta_{cd} + p_f r_b\eta_{ae}\eta_{cd} + \\ &+p_a r_e\eta_{bf}\eta_{cd} - p_a r_b\eta_{cd}\eta_{ef} + \\ &-\frac{1}{2}(p\cdot r)\eta_{ef}\eta_{ac}\eta_{bd} + \frac{1}{2}p_f r_e\eta_{ac}\eta_{bd} + \\ &+\frac{1}{4}(p\cdot r)\eta_{ef}\eta_{ab}\eta_{cd} - \frac{1}{4}p_f r_e\eta_{ab}\eta_{cd}\}\ , \end{aligned} \tag{10}$$

where $p + l = q + r$.

As long as the photon self-energy is concerned, there are only two Feynman diagrams contributing at the 1-loop approximation. They are both shown in Fig.(2):

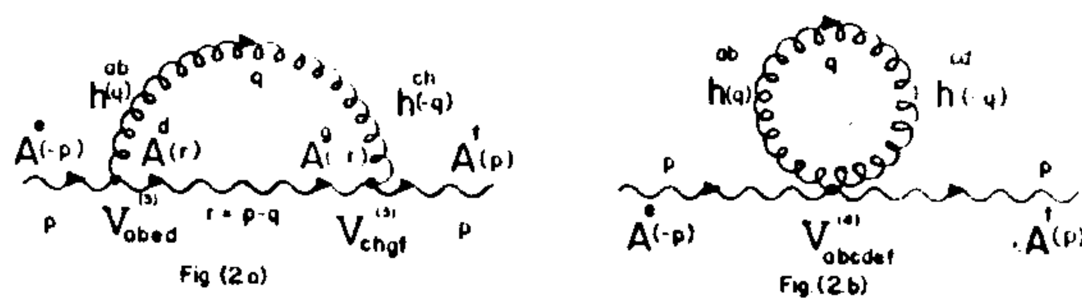

Fig.(2)
1-loop contributions to photon self-energy.

The diagram presented in Fig.(2.a) yields the following contributions to the self-energy :

$$\begin{aligned}\mathcal{I}^{(3)}_{ef}(p) &= \int \frac{d^3q}{(2\pi)^3}\, \mathcal{V}^{(3)}_{abcd}(p;q) < h^{ab}(-q)\, h^{ch}(q) > \times \\ &\quad \times < A^d(-(p-q))\, A^g(p-q) > \mathcal{V}^{(3)}_{chgf}(p;q)\,. \end{aligned} \tag{11}$$

The tadpole graph of Fig.(2.b) is given by the expression :

$$\mathcal{I}^{(4)}_{ef}(p) = \int \frac{d^3q}{(2\pi)^3}\, \mathcal{V}^{(4)}_{abcdef}(p) < h^{ab}(-q)\, h^{cd}(q) > . \tag{12}$$

Now, we simply replace in eqs.(11) and (12) the expressions previously derived for the propagators and vertices. The explicit evaluation of the loop integrals given above are algebrically extremely laborious [5]. The $\mathcal{I}^{(3)}-$ integral generates 1512 terms, while the $\mathcal{I}^{(4)}-$ integral other 140. A careful analysis reveals that 54

[5] The algebraic manipulation of these loop integrals and the task of exhautive simplifications would not be doable without the use of the software 'FORM'.

independent momentum-space integrals [6] can be identifyed among the generated terms. There appear integrals exhibiting up to 5 loop-momenta in the numerator (for the sake of illustration, a representative 5-momentum integral is presented in the appendix).

Dimensional regularisation procedure is adopted to solve the 1-loop integrals [7]. Clearly, since we are working in 3 dimensions, all these integrals turn out to be finite. However, since the main motivation of this work concerns the investigation of gauge-field dynamical mass generation through the coupling to gravity, the evaluation of the Feynman integrals is an important step in order to fix the answer for the graphs in terms of the external momentum p^{μ}.

The explicit calculations were carried out [8] and the final result is that no topological mass term for the gauge field A^{μ} appears at 1-loop upon the minimal coupling of this field to the gravitational sector. This amounts to saying that the pole of the propagator given in eq.(6) is not shifted after 1-loop corrections are taken into account. Also, we should mention that the $U(1)-$Ward identity is satisfied: the total 1-loop contribuition to the gauge field self-energy diagram has been checked to be transverse.

Having understood that there is no dynamical mass generation for the $A^{\mu}-$field minimally coupled to gravity, we are now contemplating interesting non-minimal couplings and we are trying to analyse whether in these cases a gauge-field mass generation may take place. These results shall soon be reported elsewhere [8].

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[6] As this paper is intended to be a short letter, these Feynman integrals shall be presented in a further work.

APPENDIX :
Typical 5-momentum integral :

$$\int \frac{d^3q}{(2\pi)^3} \frac{p^2 \; q^\mu q^\nu q^\rho q^\sigma q^\lambda}{q^4(q^2 - M^2_{gr.})[(p-q)^2 - M^2_{ph.}](p-q)^2} =$$

$$= \frac{p^2}{m^2_{gr.}} \left\{ \frac{1}{m^2_{gr.}} \frac{1}{m^2_{ph.}} \frac{i}{(4\pi)^{\frac{3}{2}}} \sqrt{\pi} \int_0^1 dx \left[-x^3 (\mathcal{A}_1^{\frac{1}{2}} - \mathcal{A}_2^{\frac{1}{2}} - \mathcal{A}_3^{\frac{1}{2}} + \mathcal{A}_4^{\frac{1}{2}}) \, \mathcal{B}_1^{\mu\nu\rho\sigma\lambda}(p) + \right.\right.$$

$$+ \; x^5 (\mathcal{A}_1^{-\frac{1}{2}} - \mathcal{A}_2^{-\frac{1}{2}} - \mathcal{A}_3^{-\frac{1}{2}} + \mathcal{A}_4^{-\frac{1}{2}}) \; \mathcal{B}_2^{\mu\nu\rho\sigma\lambda}(p) \; +$$

$$\left. + \; \frac{1}{3} x (\mathcal{A}_1^{\frac{3}{2}} - \mathcal{A}_2^{\frac{3}{2}} - \mathcal{A}_3^{\frac{3}{2}} + \mathcal{A}_4^{\frac{3}{2}}) \; \mathcal{B}_3^{\mu\nu\rho\sigma\lambda}(p) \; \right] +$$

$$- \; \frac{1}{m^2_{ph.}} \frac{i}{(4\pi)^{\frac{3}{2}}} \sqrt{\pi} \int_0^1 dx \, (1-x) \left[\frac{1}{2} x^3 (\mathcal{A}_3^{-\frac{1}{2}} - \mathcal{A}_4^{-\frac{1}{2}}) \; \mathcal{B}_1^{\mu\nu\rho\sigma\lambda}(p) \quad + \right.$$

$$\left.\left. + \; \frac{1}{2} x^5 (\mathcal{A}_3^{-\frac{3}{2}} - \mathcal{A}_4^{-\frac{3}{2}}) \; \mathcal{B}_2^{\mu\nu\rho\sigma\lambda}(p) - \frac{1}{2} x (\mathcal{A}_3^{\frac{1}{2}} - \mathcal{A}_4^{\frac{1}{2}}) \; \mathcal{B}_3^{\mu\nu\rho\sigma\lambda}(p) \; \right] \right\} ,$$

where the coefficients $\mathcal{A}_i$ read as below :

$$\begin{aligned}
\mathcal{A}_1 &\equiv \left[x(1-x)\cdot p^2 - M^2_{ph.}\cdot x - M^2_{gr.}\cdot(1-x)\right] ; \\
\mathcal{A}_2 &\equiv \left[x(1-x)\cdot p^2 - M^2_{gr.}\cdot(1-x)\right] , \\
\mathcal{A}_3 &\equiv \left[x(1-x)\cdot p^2 - M^2_{ph.}\cdot x\right] , \\
\mathcal{A}_4 &\equiv \left[x(1-x)\cdot p^2\right] ,
\end{aligned}$$

and the tensors $\mathcal{B}_i^{\mu\nu\rho\sigma\lambda}(p)$ take the forms :

$$\begin{aligned}
\mathcal{B}_1^{\mu\nu\rho\sigma\lambda}(p) &\equiv \left(\eta^{\mu\lambda}p^\nu p^\rho p^\sigma + \eta^{\nu\lambda}p^\mu p^\rho p^\sigma + \eta^{\rho\lambda}p^\mu p^\nu p^\sigma + \right. \\
&+ \; \eta^{\sigma\lambda}p^\mu p^\nu p^\rho + \eta^{\mu\nu}p^\rho p^\sigma p^\lambda + \eta^{\nu\sigma}p^\mu p^\rho p^\lambda + \\
&+ \; \eta^{\rho\sigma}p^\mu p^\nu p^\lambda + \eta^{\mu\rho}p^\nu p^\sigma p^\lambda + \eta^{\nu\rho}p^\mu p^\sigma p^\lambda + \\
&+ \left. \; \eta^{\mu\sigma}p^\rho p^\nu p^\lambda\right) , \\
\mathcal{B}_2^{\mu\nu\rho\sigma\lambda}(p) &\equiv \left(p^\mu p^\nu p^\rho p^\sigma p^\lambda\right) , \\
\mathcal{B}_3^{\mu\nu\rho\sigma\lambda}(p) &\equiv \left(\eta^{\mu\nu}\eta^{\rho\lambda}p^\sigma + \eta^{\mu\nu}\eta^{\sigma\lambda}p^\rho + \eta^{\nu\sigma}\eta^{\mu\lambda}p^\rho + \right. \\
&+ \; \eta^{\nu\sigma}\eta^{\rho\lambda}p^\mu + \eta^{\rho\sigma}\eta^{\mu\lambda}p^\nu + \eta^{\rho\sigma}\eta^{\nu\lambda}p^\mu + \\
&+ \; \eta^{\mu\rho}\eta^{\nu\lambda}p^\sigma + \eta^{\mu\rho}\eta^{\sigma\lambda}p^\nu + \eta^{\nu\rho}\eta^{\mu\lambda}p^\sigma + \\
&+ \; \eta^{\nu\rho}\eta^{\sigma\lambda}p^\mu + \eta^{\mu\sigma}\eta^{\rho\lambda}p^\nu + \eta^{\mu\sigma}\eta^{\nu\lambda}p^\rho + \\
&+ \left. \; \eta^{\mu\nu}\eta^{\rho\sigma}p^\lambda + \eta^{\nu\rho}\eta^{\mu\sigma}p^\lambda + \eta^{\mu\rho}\eta^{\nu\sigma}p^\lambda\right) .
\end{aligned}$$

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