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DYNAMICAL ETERNAL UNIVERSE SCENARIO*

by

M. NOVELLO

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

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*In honor of 70th birthday of professor Jayme Tiomno

ABSTRACT

We present an overview on cosmological models and some reasons to support the program of an Eternal Universe.

Key-words: Cosmological model; Eternal universe.

The recent results of the satellite COBE (Cosmic Background Explorer) gave origin to the reborn of an old question – which was thought to be solved – opposing competitive ideas concerning the finitude or the infinitude of our Universe.

The origin of such question goes back to the beginnings of the Relativistic Cosmology and can be traced directly to the stablishement of the notion of a unique global time that provides a common temporal characterization of all existing matter. This can be synthesized by uses of the so called Weyl's postulate which Robertson (1933) enunciates as being "... the assumption that the world lines of all matter belong to a pencil of geodesics which converges toward the past – the Universe is a coeherent whole rather than the fortuitous superposition of two or more incoherent parts".

If one accepts the idea of a unique global time then there appears immediately the question of the finitude or the infinitude of such (cosmic) time.

The most popular answers to these questions are the models known as Big-Bang and Steady State:

(i) Hot Big-Bang or Explosive FRW Universe

This model is based on the geometry proposed by Friedmann more than 60 years ago. The fundamental length is given by

$$ds^2 = dt^2 - A^2(t)\, d\sigma^2 \quad . \tag{1}$$

This geometry is spatially homogeneous and isotropic. It admits a global singularity (in which physical quantities diverges) which is identified with the "beginning of time". Such singularity is supposed to be not an odd consequence of the high degree of symmetries of the model but instead a common property implied by the validity of Einstein's General Relativity plus some additional hypothesis concerning the behavior of matter at large.

ii) Steady State or Stationary Eternal Universe Scenario (SEUS)

This model is based on the Sitter geometry,

$$ds^2 = dt^2 - e^{2Ht}\, d\sigma^2 \quad . \tag{2}$$

The metric is homogeneous not only on the 3-dimensional space but also in time. The global features of the Universe represented by this geometry do not experience any changes. The Universe should not have any history !

The sources of the HBB model is a perfect fluid; the source of SEUS is the energy of the vacuum ($T_{\mu\nu} = \Lambda g_{\mu\nu}$, Λ is the cosmological constant). Recently some equivalent alternative description of these sources have been presented.

These two models faced enourmous difficulties during the sixties, both from observational and theoretical arguments. The main criticism against SEUS was provided by the observation of the microwave background radiation (MBR) and its association to a global motion which was broadly interpreted as the final proof of the expansion of the Universe. This fact was explained in a very simple and direct way by the Hot Big-Bang (NF 058/88) ("standard") Model. Indeed, according to this HBB model, after its emergence out of an initial singular state — separate a finite time interval H_0^{-1} from us, where H_0 is the Hubble parameter — the Universe goes into an expanding phase with geometrical features, described through Einstein's equations of gravitation, corresponding to the spatially homogeneous and isotropic Friedman-Robertson-Walker (FRW) model, given by the line element (1).

Therefore, the observed cosmological redshift in the spectra emitted by distant objects (Hubble expansion) would result from the coupling between the eletromagnetic and gravitational fields: photons (described by the propagation 4-vector K_μ) move along paths identified to the null geodesics of the FRW geometry, and consequently the frequency $\omega = k_\mu V^\mu$ of the radiation, seen by an observer V^μ co-moving with the hipersurface of homogeneity, varies in inverse proprtion to the Universe radius A(t). Thus, the origin of the observed redshift is attributed to the expansion of the Universe.

On the other hand the existence of the MBR quoted above presently characterized by the spectrum of a black-body in thermal equilibrium at temperature $T = 2.7^{\circ}K$, is understood as the remnant of a hot, dense primordial phase of the cosmic evolution. Indeed, according to the standard model, in the initial stages of the cosmic expansion electromagnetic radiation becomes the main responsible for the curvature of space-time;

furthermore, its black-body character would be preserved along the entire history of the Universe. This would happen since the photon energy density is proportional to the fourth power of the temperature, $\rho_\gamma \sim T^4$, and at early times should prevail over the contribution of the matter content, for $\rho_M \sim T^3$. Moreover, since the law of energy conservation giges that $\rho_\gamma \sim A^{-4}$, it follows that the equilibrium temperature T of the radiation varies in inverse proportion to the Universe radius $A(t)$, in the same way as the frequency ω. This in turn implies the miracle that the thermal spectrum of the radiation, described by the black-body distribution $dN_\omega = [\exp \frac{\omega}{T} - 1]^{-1}$, would be preserved throughout the expanding era (at least as long as these photons interact through gravitation only), once $\frac{\omega}{T}$ = constant.

Another far-reaching consequence of the standard approach concerns total entropy conservation along the evolution of the Universe, which stems from the assumed thermalized photon configuration and also from the fact that the total number of photons in the Universe must be a constant, if particle-antiparticle annihilation is neglected — which seems to be quite reasonable under ordinary conditions. In effect, from the conclusion that the energy density of the photons evolves according to $\rho_\gamma \sim A^{-4}$ it follows that the photon number density $n = \frac{N}{V}$ varies in time exclusively in virtue of the expansion of the Universe. Setting $\theta = \frac{\dot{V}}{V}$ one gets that $\dot{n} + n\theta = 0$, and hence that $\dot{N} = \frac{dN}{dt} = 0$. If this is indeed true, then the ratio n_γ/n_B between the number densities of photons and baryons existing in the Universe becomes an universal constant, which is estimated today to be of the order of 10^9. Thus in the standard scenario this number stands for an intrinsic characteristic imprinted on the actual Universe, requiring either an explanation on the basis of a more fundamental principle or else to be assigned to an inaccessible set of initial conditions. This question may be alternatively formulated in terms of the total entropy S of these photons, which in the same token should represent a "primordial" quantity.

Another difficulty of this HBB scenario comes precisely from the observed high degree of isotropy of the MBR. This can be put in the following way.

The Friedmann geometry contains horizons — regions in which no information can be enchanged (at least during some epochs). This is nothing but a consequence of the behavior of null geodesics (the paths of the photons) in this metric. It becomes then a crucial problem to conciliate such isotropy of the 3-dimensional space with the existence of horizons in a non-infinitely old Universe.

A simple way to solve this is to suppose the existence of a prior anisotropic era. We thus face another question: to find a very efficient mechanism of isotropization.

Curiously, when reading the literature and studying the distinct proposals and myriads of methods that have been used to analyse this question one is led to believe that isotropy is either a miracle, i.e., a kind of initial condition (once, as it has been claimed, the spectrum of anisotropic spatially homogeneous geometries ending in a Friedmannian stage is of measure null (Hawking) or, conversely, isotropization is an unavoidable property of the theory of general relativity complemented with a cosmological constant (... "the cosmological constant is the best isotropizer"... (Starobinsky).

In Appendix we present a way to solve the isotropization problem by means of a mechanism of phase transition. Neverthless, we still do not have confident evidences of the existence of such prior anisotropic era.

The Hot Big-Bang model has been strongly criticized during the 80's. A review of this and some tentations to save it, through the so called inflationary program, has been given in many publications[1] . Thus let us concentrate here on the program of an Eternal Universe.

The above arguments yield an almost complete unanimity that SEUS should not be maintained in its old version. Thus let us look into some modifications of it which have been analysed recently.

To save an eternal Universe from the main critics that have almost destroyed SEUS one must modify its stationary geometry into a dynamical one.

DEUS is the model which have been proposed to deal with such modification: in other words, the construction of a Dynamical Eternal Universe Scenario.

It has been claimed that in order to bypass the restrictions imposed by the socalled singularity theorems and to constructsuch eternal Universes one has to deal with some extravagant unusual properties of matter. However, this argument should be applied in the opposite way. Indeed, it is certainly a tremendous ammout of simplification to describe the myriads of particles, fields and conglomerates of matter with the highly idealized configuration of a perfect fluid, as it is commonly treated in the standard model.

There are many mechanisms proposed in the literature which avoids the presence of singularity in the cosmological metric. Elsewhere[2] we have presented some of them. Here we will limit our remarks to the case suggested in (3). Before entering in the details of this model, let us make some general comments on the properties which our cosmological metric should have.

An Eternal Universe, in order to be not involved on difficulties with assymptotic conditions, should be indentified both in the past and future infinity with the Minkowski geometry. Only in this case we are free from a vicious circle concerning unobservable special conditions of the gravitational field.

Besides, at these limits the Universe could, in a rather precise way, to be treated as a closed system.

The temperature of this system should not be in a permanent stable condition but otherwise it changes smoothly during the evolution of the geometry.

The model consists on a classical vector field (which can be identified with the electromagnetic field) coupled directly with the curvature of the space-time, the Lagrangian of which takes the form:

$$(3) \qquad L_{NS} = \sqrt{-g}\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{k}R + \sigma R\, W_\mu W^\mu\right\} .$$

In the standard HBB model, in the vicinity of the origin of the Universe (t = 0) the energy of the radiation gas dominates over other forms of matter. The effect of the minimal coupling is just to induce an average $\langle E_\mu E_\nu \rangle$ proportional to the metric $g_{\mu\nu}$. This in turn can be interpreted as a gas of photons (in equilibrium). In the present case of the non-standard Lagrangian L_{NS} there appears the possibility of a new class of cosmic solutions which should not be possible otherwise. This solution, in terms of the Friedmann coordinates, is given by (1) in which the radius of the Universe takes the form:

$$(4) \qquad A(t) = \sqrt{t^2+Q^2} .$$

Contrary to the case proposed in SEUS, here the expansion parameter θ that measures the time evolution of the 3-dimensional volume is not identified with a constant (Hubble constant) but changes with time.

Besides the assymptotic limit of the radius of the Universe (for $t \to \pm\infty$) whows that in these regions the Universe behaves as a structure which can be identified with Minkowski space-time described in terms of Milne coordinates. Thus as time goes on, from $t = -\infty$, the Universe changes smoothly its behavior from a thermodynamically closed system to an open one induced by gravity and the fluctuations of the W_μ field. After a finite amount of time the Universe behaves as an open system (once the geometry is not treated within the thermodynamical balance). The most important consequence of this situation concerning our interest here is the possibility to solve at once three crucial problems of cosmology, to wit :

(i) The existence of the MBR;

(ii) The avoidance of the cosmological singularity;

(iii) The relation between the cosmological and the thermodynamical arrow of time.

The standard HBB asserts that one of the most important observational quantity to be understood is precisely the ratio n_γ/n_B which we referred before. This ratio is supposed in the HBB to be a constant. It has been argued that the support for such assertion comes from Quantum Field Theory (QFT). The argument goes in the following way: although a non-stationary gravitational field can modify the total number of quanta of a given field, it does not changes this number if two conditions are simultaneously fulfilled:

(i) The gravitational field is conformally flat;

(ii) The equation of motion of the quanta is conformally invariant.

Thus, it is argued, once the geometry is given by a FRW form and the electromagnetic field is conformally invariant it follows by a simple uses of the above theorem that the total number of photons in the Universe N_γ is a constant.

However, in the model we are concerned here, the direct coupling of electromagnetic and gravitational fields, as given by L_{NS}, is not conformally invariant. Thus, N_γ is not a constant of motion. [It seems worth to make here a pause in order to remark that photons that obeys L_{NS} follow null geodesics. This is an important property once most of the informations concerning the Universe comes from photon's observations.]

Let us follow here Novello, Salim and Oliveira and review briefly the consequences for such non-conservation of the photon number in the standard HBB model.

After demonstrating the actual viability of photon production by the gravitational field associated to an expanding homogeneous Universe, the natural step ensuing is the attempt to quantify such process. However, instead of pursuing the usual approach and put under scrutiny the quantum evolution equation for the photon vacuum, we shall follow another line of inquiry: starting from the present, observed values of the MBR, we will proceed to investigate, through an statistical analysis, the possible influence of a variation in the number of photons upon the characteristics of such radiation field.

In condensed matter physics, changes in the number of elementary constituents of a given reaction are costumarily taken into account through the introduction of a "chemical potential" term [Landau-Lifshitz]. Nevertheless, the behaviour of photons interacting with matter is characterized by a null value of the chemical potential, once photons can be emitted or absorbed at any rate in an arbitrary reaction. Let us now pose the question: would such charactheristic still be valid for the photon-gravity interaction ?

Of course, the standard approach provides an inequivocal answer: if the concept of a vanishing chemical potential for the photon holds locally, then a straightforward use of the Equivalence Principle would extend its validity to any circumstances whichever. Due to the current trend of esteeming the Equivalence Principle as a <u>de facto</u> generator of physical laws, such statement became uncontroversial and thus acquired a generalized acceptance. However, as indicated in the previous section, a strict adhesion to empirical criteria leads to a wider conclusion, since the ensemble of observational data presently available on electromagnetic processes in gravitational fields does not suffice to establish the Hypothesis of Minimal Coupling (HMC) as the true only type of admissible coupling between these fields. Therefore, it seems worth to examine the alternative Hypothesis of Direct Coupling to the Curvature (HDCC) hereunto developed. We are thus led to consider the statistical distribution function of a boson gas endowed with a chemical potential μ :

$$(5) \qquad dN_\omega = \frac{1}{\exp[\frac{\omega-\mu}{T}] - 1} \quad .$$

According to the present HDCC scenario, the total number of photons in the Universe is affected by gravitational interaction; hence, in eq. (5) the total chemical potential μ may be conveniently split into two independent parts:

$$(6) \qquad \mu = \mu_0 \ (P,T) + \Delta\mu \quad ,$$

where $\Delta\mu$ is the gravitationally – induced contribution and $\mu_0(P,T)$ is the flat-space component which, in view of the arguments quoted above, should vanish. To proceed, one needs to consider the question: which is the presumable form of functional dependence of $\Delta\mu$ on the curvature ? Instead of attempting to answer this question for arbitrary configurations, we will restrict ourselves to the special cases in which space-time structure can be adequately represented by FRW geometries. Furthermore, as we have previously stated, in the present study photons will be treated as test-particles in a background gravitational field, thus contributing negligibly to the matter-energy content responsible for space-time curvature. Such premise requires that throughout the history of the Universe – even at primordial epochs of great compression – the energy density ρ_γ of the photons should remain very small. We shall see latter on that such regularization of the photon energy density, in the case of a FRW model, will follow as a natural consequence of photon number non-conservation.

Due to the spatial homogeneity of the FRW model, $\Delta\mu$ shall depend on cosmic time only. It seems reasonable to suppose that $\Delta\mu$ may be written as a combination of powers of the unique curvature parameter available, the expansion factor θ . This feature allows us to preliminarly consider the form

$$\Delta\mu = -b^2\theta \quad , \tag{7}$$

in which b^2 = const. and the minus sign arises from the bosonic nature of photons and from the fact that we presently live in an expanding era (i.e., $\theta > 0$). In this way, the arrow of time provided by the cosmic expansion coincides with the thermodynamical arrow, as we shall see soon. Remark that, since the inclusion of higher powers of θ does not affect qualitatively our forthcoming results, eq. (7) can be accepted at least as a good first approximation.

From Lagrangian L_{NS} it follows that photons travel along null geodesics. Also, as in the standard model, frequency ω

varies like $\omega \sim A^{-1}$; we will further assume that temperature T behaves in the same manner. The thermodynamical potential $\Omega = -PV$ is provided by the expression

$$(8) \qquad \Omega = \frac{VT}{\pi^2} \int_0^\infty \omega^2 \ln \left[1 - e^{-\frac{(\omega-\mu)}{T}} \right] d\omega \quad ,$$

in which μ is given by eq. (7). Calling $\beta \equiv -\frac{\mu}{T} = \frac{b^2\theta}{T}$, we thus obtain

$$(9) \qquad \Omega = \frac{-2}{\pi^2} VT^4 \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^4} \quad .$$

This kind of infinite series will appear frequently in the remaining of this paper; they will be denoted by

$$(10) \qquad L(\beta,s) \equiv \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^s} \quad .$$

Particularly interesting for our future developments is a regularization procedure which consists in the conversion of $L(\beta,s)$ into a power series in β, with coefficients proportional to the Riemann Zeta function

$$(11) \qquad \zeta(s) \equiv \sum_{m=1}^{\infty} \frac{1}{m^s} \quad ,$$

evaluated away from its poles. In order to apply this result, let us first recall the expression for the radius of the Universe in the standard HBB model, given by $A(t) = A_0 T^q$, where the parameter q varies in the range $0 < q < 1$. Accordingly, the functional dependence of β with respect to cosmic time goes like $\beta \sim t^{q-1}$.

Thus, for very long time intervals $(t \to \infty)$ factor β becomes extremely small, and we obtain

$$\text{(12)} \qquad \lim_{\beta\to 0} \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^4} = \zeta(4) \quad ,$$

where $\zeta(4)$ is the corresponding form assumed by Riemann's Zeta function. Therefore, potential Ω in this limiting condition approaches the value

$$\text{(13)} \qquad \Omega_\infty = \frac{-\pi^2}{45} VT^4 \quad .$$

Hence, we see that HDCC distribution eq. (9) converges assymptotically to a black-body spectrum. Since $\beta \ll 1$ implies that $\frac{\hbar\theta}{k_B T} \ll \frac{1}{b^2}$, where k_B is Boltzmann's constant, and present MBR observations show that cosmic photons behave very nearly like black-body radiation, we achieve the condition $b^2 \ll 10^{-5}$.

From eq. (9) for the potential Ω a straightforward calculation provides the expressions of other relevant thermodynamical quantities. In Table I below we list their values for both HMC and HDCC cases; remark that in the limit $\beta \to 0$ (that is, for large values of t) HDCC relations coincide with those derived from HMC. Particularly noteworthy is the preservation of the equation of state $P = \frac{1}{3}\rho$, which in the HMC case arises from the condition $T^\alpha_\alpha = 0$, and in the HDCC case from the vanishing, on the average, of the trace $T^\alpha_\alpha = \xi\ (RF^2 - 3 \Box F^2)$ of the electromagnetic energy-momentum tensor.

<u>TABLE I</u> - Thermodynamical quantities - comparison between HMC case (which includes the creation of phtons by the expansion of the Universe). Note that for large values of cosmic time (or $\frac{\theta}{T} << \frac{1}{b^2}$) HDCC formuli go into HMC ONES. Note also the definition $v = \frac{1}{n}$

Thermodynamical Quantities	HMC case (Photon gas minimally coupled to gravity)	HDCC case (Photon gas non-minimally coupled to gravity)
Energy density	$\rho = \frac{\pi^2}{15} T^4$	$\rho = \frac{6}{\pi^2} T^4 \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^4}$
Pressure	$P = \frac{1}{3} \rho$	$P = \frac{1}{3} \rho$
Total number of photons	$N = \frac{2}{\pi^2} \zeta(3) VT^3$	$N = \frac{2}{\pi^2} VT^3 \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^3}$
Free energy	$F = \frac{-\pi^2}{45} VT^4$	$F = \frac{-2}{\pi^2} VT^4 \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^3} + N\mu$
Total entropy	$S = \frac{-4F}{T}$	$S = \frac{-4F}{T} + 3 \frac{N\mu}{T}$
Specific entropy $s = \frac{S}{N}$	$s = \frac{4}{3} \frac{\rho v}{T}$	$s = \frac{1}{T} (\frac{4}{3} \rho v - \mu)$
Thermodynamical potential $\Omega = -PV$	$\Omega = F$	$\Omega = F - N\mu$
Evolution of the number density of photons $n = \frac{N}{V}$	$\dot{n} + n\theta = 0$	$\dot{n}+n\theta = \frac{-2b^2}{\pi^2} T^2 R_{00} \sum_{m=1}^{\infty} \frac{e^{-m\beta}}{m^2}$

Let us make some further comments on the results displayed at Table I. At the beginning of the Universe (i.e., at $t = 0$), the total number of photons N vanishes, as well as the total entropy S. Thus we arrive at the qualitatively new result we have announced: according to the HDCC approach developed here neither the total number of photons nor their total entropy are in fact conserved quantities, and so the ratio n_γ/n_B between the number of photons and baryons comprised in the Universe depends on the cosmic epoch. As the Universe expands, gravity's capacity to create photons progressively decreases, and at later times (i.e., for $\frac{\theta}{T} << \frac{1}{b^2}$) the total number of photons approaches a constant value (indeed, if we make use of the lower possible limit $b^2 \sim 10^{-5}$ we obtain that the present rate of photon creation is $\frac{\dot{N}}{N} \sim 10^{-70}$). The effects of the presence of a non-null chemical potential gradually vanish and the photon gas correspondingly acquires the black-body character manifested by current MBR observations. Thus in the HDCC scenario presented here the observed value of the entropy S does not constitute a crucial cosmological problem (as it has been claimed to be from the point of view of the standard approach), representing but a fortuitous value associated to the present epoch.

Let us now come back to DEUS:

The first important distinction here is that expression (7) for the chemical potential of the photon induced by gravity must be modified. This is due to the bosonic nature of the photon and to the existence of both a contracting and an expanding phase in the Eternal Universe.

Thus the first possible term in the chemical potential

μ is a quadratic one in θ . After inserting this in the formula for the thermodynamical quantities we find two main results:

i) The total entropy at the infinite past and the infinite future is the same;

ii) The total number of photons at the infinite past and the infinite future is the same.

Note however that N_γ is not a constant. This is not in contradiction with standard Thermodynamics, once in the analysis of the mechanism of creation/destructution of quanta one must take into account the gravitational field which acts as an external agent.

The present stage of development of the new cosmological scenario (DEUS) is still incomplete, thus requiring further analysis; neverthless, we believe that the results sketched above indicate that this model deserve to be studied at length.

APPENDIX - Phase Transition Induced by Gravity

In Landau's conventional theory a phase transition can occur between a liquid and a crystal if the fluid attains a certain critical temperature. Thus the whole process of phase characterization depends uniquely on a non-extensive quantity: the temperature.

In the case of gravity one can follows an alternative way, in which the role of the temperature is played by the expansion factor θ.

The proof of this is based on three conditions:

(i) The validity of Eintein's equations for the gravitational field

$$R^{\mu}{}_{\nu} - \frac{1}{2} R\gamma^{\mu}{}_{\nu} = -k\, T^{\mu}{}_{\nu} \quad .$$

(ii) The stress-energy tensor $T^{\mu}{}_{\nu}$ is of a generic stokesian form (see below).

(iii) The influence of gravity on the dependence of the free energy on the macroscopic order parameter is given by

$$(\Delta F)_{grav} = \gamma R_{\mu\nu}\sigma^{\mu\nu} \quad .$$

Let us pause for a while and comment on the conditions (ii) and (iii).

When considering the behavior of the cosmic fluid under anisotropic perturbations it seens a good strategy to follow the analysis of Gramsbergen et al.[6] in the use of Landau-de Gennes theory of the nematic-isotropic phase transition which assumes that it is the shear $\sigma^{\mu}{}_{\nu}$ of the fluid itself that is to be used as the macroscopic order parameter.

The generic form of $T^{\mu\nu}$ can be written as

$$T_{\mu\nu} = \rho V_{\mu}V_{\nu} - \frac{1}{3}\rho\ (g_{\mu\nu}-V_{\mu}V_{\nu}) + \pi_{\mu\nu}$$

The anisotropic tensor is a polynomial on the kinematical parameters of the cosmic fluid, that is

$$\pi_{\mu\nu} = (\alpha_0 + \alpha_1\theta + \beta\sigma^2)\sigma_{\mu\nu} + \delta\sigma_{\mu}{}^{\alpha}\sigma^{\alpha}{}_{\nu} + - \frac{1}{3}\delta\sigma_{\alpha\beta}\sigma^{\alpha\beta}(g_{\mu\nu} - V_{\mu}V_{\nu}) \quad .$$

If we look for the extremum(minimum) of ΔF using this expression for $T_{\mu\nu}$ we obtain the following results , concerning the relative values of θ and the critical values θ_c and θ_t , defined by:

$$\theta_c = \theta^* - \frac{3}{64}\frac{\delta^2}{a^2\beta}$$

$$\theta_t = \theta^* - \frac{1}{24}\frac{\delta^2}{a^2\beta}$$

in which $\theta^* = \frac{\alpha_0}{a^2}$.

i) If $\theta < \theta_c$ then the most favourable state is an isotropic phase;

ii) If $\theta_c < \theta < \theta_t$ the most favourable state is the isotropic phase but there is a local minimum corresponding to a small anisotropy.

iii) $\theta_t < \theta < \theta^*$.

The most favourable state is the anisotropic phase but there is a local minimum corresponding to an isotropic (less favourable) phase.

iv) $\theta^* < \theta$.

It corresponds to an anisotropic phase.

The above mechanism of phase transition induced by gravity and controlled by the expansion parameter can be applied to our Universe, in a direct way allowing a possible interpretation of the isotropisation era we live, if we accept the model of a dynamical eternal Universe which could, in some phase of its existence, be dominated by a non-isotropic mode of global excitation.

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