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LINEARIZATION OF HOLOMORPHIC MAPPINGS ON LOCALLY CONVEX SPACES+

by

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Introduction

Throughout this paper the letters E and F represent locally convex spaces, always assumed complex and Hausdorff, and the letter U represents a nonvoid open subset of E. $\mathcal{L}(E;F)$ denotes the vector space of all continuous linear mappings from E into F, whereas $\mathcal{H}(U;F)$ denotes the vector space of all holomorphic mappings from U into F.

Mazet [16] proved the existence of a complete locally convex space $G(U)$ and a mapping $\delta_U \in \mathcal{H}(U;G(U))$ with the following universal property: For each complete locally convex space F and each mapping $f \in \mathcal{H}(U;F)$, there is a unique mapping $T_f \in \mathcal{L}(G(U);F)$ such that $T_f \circ \delta_U = f$. To prove this result Mazet introduced the notion of cotopological space and exploited the duality between cotopological spaces and locally convex spaces.

In this paper we present a different proof of the Mazet linearization theorem, based on a result on inductive limits of Banach spaces. The rest of the paper is devoted to the study of some aspects of the interplay between the spaces $\mathcal{H}(U;F)$ and $\mathcal{L}(G(U);F)$, with applications to the study of holomorphically barrelled domains, holomorphically Mackey domains, holomorphic continuation, analytic sets and holomorphic convexity.

This paper is organized as follows. In Section 1 we give a sufficient condition for an inductive limit of Banach spaces to admit a representation as a dual space, thus extending a result obtained by Mujica [17]. In Section 2 we use that result to prove the Mazet linearization theorem. In addition our proof shows that the correspondence $f \to T_f$ is a topological isomorphism between $\mathcal{H}(U;F)$ and $\mathcal{L}(G(U);F)$, when the former space is equipped with the topology τ_δ introduced independently by Coeuré [6] and Nachbin [19], and the latter space is equipped with the limit topology τ_ℓ introduced by Nachbin [21].

In Section 3 we show the existence of a dense subspace $G_0(U)$ of $G(U)$ such that $\delta_U \in \mathcal{H}(U;G_0(U))$, and which has the following universal property: For each complete locally convex space F and each Gateaux holomorphic mapping $f : U \to F$, there is a unique linear mapping $T_f : G_0(U) \to F$ such that $T_f \circ \delta_U = f$. Moreover, T_f is continuous if and only if f is continuous.

In Section 4, following the terminology of Barroso et al. [1], we study holomorphically barrelled domains, holomorphically Mackey domains, etc. We show that an open set U is holomorphically barrelled (resp. holomorphically Mackey) if and only if the corresponding space $G_0(U)$ is barrelled (resp. Mackey). We do not know whether similar results hold for holomorphically bornological or holomorphically infrabarrelled domains.

In Section 5 we give a result on vector–valued holomorphic continuation. We show that when V is an open subset of E including U, then the following conditions are equivalent: (a) the restriction mapping $\mathcal{H}(V;F) \to \mathcal{H}(U;F)$ is a bijection for every complete locally convex space F; (b) the spaces $G(U)$ and $G(V)$ are canonically topologically isomorphic; (c) the restriction mapping $(\mathcal{H}(V;F), \tau_\delta) \to (\mathcal{H}(U;F), \tau_\delta)$ is a topological isomorphism for every complete locally convex space F. This extends results of Coeuré [6], Hirschowitz [11] and Schottenloher [26] [27].

Finally, in Section 6 we study when the image of the mapping δ_U is an analytic set. We show that if U is an open subset of a complete locally convex space E, then $\delta_U(U)$ is an analytic subset of a suitable open set $V \subset G(U)$. We also show that if U is an open subset of a (DFM)–space E, then $\delta_U(U)$ is an analytic subset of $G(U)$ if and only if U is holomorphically convex. This last result was obtained by Mazet [15] when E is finite dimensional.

We remark that for the sake of simplicity we have stated our results for open subsets of locally convex spaces, but the results remain true for Riemann domains over locally convex spaces.

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1. Inductive Limits of Banach Spaces

We follow the standard terminology from the theory of topological vector spaces, as found for instance in the books of Horváth [12] or Schaefer [25].

We systematically use the following notation, introduced by Grothendieck [10]. If A is a convex balanced, bounded subset of a locally convex space E, then E_A denotes the vector subspace of E generated by A, and normed by the Minkowski functional p_A of A. If V is a convex, balanced 0–neighborhood in E, then p_V is a seminorm in E, and E_V denotes the normed space $(E, p_V) \,/\, p_V^{-1}(0)$.

We denote by E'_b the strong dual of E, and by E'_i the inductive dual of E, that is, the dual E' of E, endowed with the locally convex inductive limit topology defined by

$$E'_i = \operatorname{ind}_V E'_{V^0} = \operatorname{ind}_V (E_V)'_b,$$

where V varies among the convex, balanced 0–neighborhoods in E, and V^0 denotes the polar of V in E'. The inductive dual of a locally convex space has been studied by Berezanskii [2], Bierstedt [3] and Floret [9].

We denote by τ_ℓ the limit topology on $\mathcal{L}(E;F)$, introduced by Nachbin [21], and defined by

$$(\mathcal{L}(E;F), \tau_\ell) = \operatorname{proj}_W \operatorname{ind}_V \mathcal{L}(E_V; F_W),$$

where V and W vary among the convex, balanced 0–neighborhoods in E and F, respectively. Observe that when $F = \mathbb{C}$ then (E', τ_ℓ) coincides with E'_i.

A key tool in this paper is the following result, which was established by Mujica in [17] for countable inductive limits, but the proof given there works equally well in the general case.

1.1. Theorem. *Let $E = \operatorname{ind} E_\alpha$ be the locally convex inductive limit of a family of Banach spaces, directed under inclusion.*

(a) *Assume there exists a (Hausdorff) locally convex topology τ on E such that the closed unit ball B_α of each E_α is τ–compact. Let F be the complete locally convex space of all linear forms on E whose restrictions to each B_α are τ–continuous, equipped with the topology of uniform convergence on all the sets B_α. Then the evaluation mapping $J : E \to F'_i$ is a topological isomorphism.*

(b) *If, in addition,* E *has a basis of* τ*-closed, convex, balanced* 0*-neighborhoods, then* $F'_i = F'_b$ *and* E *is topologically isomorphic to the strong dual of* F.

2. Linearization of Holomorphic Mappings

We follow the standard terminology from complex analysis in locally convex spaces, as found for instance in the books of Dineen [8] or Pérez Carreras and Bonet [23].

Let U be an open subset of a locally convex space E. If F is a Banach space and $\mathcal{U} = (U_j)$ is a countable open cover of U, then $\mathcal{H}^\infty(\mathcal{U};\, F)$ denotes the Fréchet space

$$\mathcal{H}^\infty(\mathcal{U};\, F) = \{f \in \mathcal{H}(U;\, F) : \sup_{x \in U_j} \|f(x)\| < \infty \quad \text{for every} \quad j\},$$

equipped with the topology of uniform convergence on all the sets U_j. The topology τ_δ on $\mathcal{H}(U;\, F)$, introduced independently by Coeuré [6] and Nachbin [19], is the locally convex inductive limit topology defined by

$$(\mathcal{H}(U;\, F),\, \tau_\delta) = \text{ind}_{\mathcal{U}} \mathcal{H}^\infty(\mathcal{U};\, F).$$

If F is a complete locally convex space, then the topology τ_δ on $\mathcal{H}(U;\, F)$ is defined by

$$(\mathcal{H}(U;\, F),\, \tau_\delta) = \text{proj}_W (\mathcal{H}(U;\, \widetilde{F}_W),\, \tau_\delta),$$

where W varies among the convex, balanced 0–neighborhoods in F. When $F = \mathbb{C}$ we write $\mathcal{H}(U)$ instead of $\mathcal{H}(U; \mathbb{C})$ and $\mathcal{H}^\infty(\mathcal{U})$ instead of $\mathcal{H}^\infty(\mathcal{U}; \mathbb{C})$.

By using a theorem of Ng [22], which characterizes dual Banach spaces, Mujica [18] obtained a linearization theorem for bounded holomorphic mappings. We now use Theorem 1.1, which may be regarded as a generalization of the Ng theorem, to prove the linearization theorem of Mazet stated in the introduction.

2.1. Mazet Linearization Theorem [16]. *Let* U *be an open subset of a locally convex space* E*. Then there are a complete locally convex space* $G(U)$ *and a mapping* $\delta_U \in \mathcal{H}(U;\, G(U))$ *with the following universal property: For each complete locally convex space* F *and each mapping* $f \in \mathcal{H}(U;\, F)$*, there is a unique mapping* $T_f \in \mathcal{L}(G(U);\, F)$ *such that* $T_f \circ \delta_U = f$*. This property characterizes* $G(U)$ *uniquely up to a topological isomorphism.*

Proof. If $\mathcal{U} = (U_j)$ is a countable open cover of U, and $\alpha = (\alpha_j)$ is a sequence of strictly positive numbers, then we set

$$B^\alpha_{\mathcal{U}} = \{f \in \mathcal{H}^\infty(U) : \sup_{x \in U_j} |f(x)| \le \alpha_j \quad \text{for every} \quad j\}.$$

Then, with Grothendieck's notation, we have that

$$\mathcal{H}^\infty(\mathcal{U}) = \mathrm{ind}_\alpha \mathcal{H}^\infty(\mathcal{U})_{B^\alpha_\mathcal{U}}$$

and therefore

$$(\mathcal{H}(U),\, \tau_\delta) = \mathrm{ind}_{\mathcal{U},\alpha} \mathcal{H}(U)_{B^\alpha_\mathcal{U}}.$$

Furthermore, it follows from the Ascoli theorem that each $B^\alpha_\mathcal{U}$ is a compact subset of $(\mathcal{H}(U),\, \tau_c)$, where τ_c denotes the compact–open topology. Let $G(U)$ denote the complete locally convex space of all linear forms on $\mathcal{H}(U)$ whose restrictions to each $B^\alpha_\mathcal{U}$ are τ_c–continuous, endowed with the topology of uniform convergence on all the sets $B^\alpha_\mathcal{U}$. Then it follows from Theorem 1.1 that the evaluation mapping

$$J : (\mathcal{H}(U),\, \tau_\delta) \to G(U)'_i$$

is a topological isomorphism. Observe that

$$(\mathcal{H}(U),\, \tau_c)' \subset G(U) \subset (\mathcal{H}(U),\, \tau_\delta)'.$$

Let $\delta_U : x \in U \to \delta_x \in G(U)$ denote the evaluation mapping, that is, $\delta_x(f) = f(x)$ for all $x \in U$ and $f \in \mathcal{H}(U)$. Since

$$Jf \circ \delta_U(x) = \delta_x(f) = f(x) \tag{2.1}$$

for all $x \in U$ and $f \in \mathcal{H}(U)$, and since $J : \mathcal{H}(U) \to G(U)'$ is surjective, we see that the mapping $\delta_U : U \to G(U)$ is weakly holomorphic, and in particular Gateaux holomorphic. To show that δ_U is continuous, we begin by observing that the topology of $G(U)$ is generated by the seminorms

$$p^\alpha_\mathcal{U}(u) = \sup\{|u(f)| : f \in B^\alpha_\mathcal{U}\}.$$

Since $p^\alpha_\mathcal{U}(\delta_x) \le \alpha_j$ for every $x \in U_j$ and every j, we conclude that δ_U is amply bounded, and therefore continuous. Thus $\delta_U \in \mathcal{H}(U;\, G(U))$.

We next claim that the evaluations δ_x, with $x \in U$, generate a dense vector subspace of $G(U)$. For otherwise, by the Hahn–Banach theorem, we could find a nonzero $T \in G(U)'$ such that $T(\delta_x) = 0$ for every $x \in U$. But since $J : \mathcal{H}(U) \to G(U)'$ is surjective, we would have that $T = Jf$ for some $f \in \mathcal{H}(U)$. But then we would have that

$$f(x) = \delta_x(f) = T(\delta_x) = 0$$

for every $x \in U$, and hence $T = 0$, a contradiction.

We finally show that the pair $(G(U),\, \delta_U)$ has the required universal property. We distinguish three cases.

(a) If $f \in \mathcal{H}(U)$, then we define $T_f = Jf$. It follows from (2.1) that $T_f \circ \delta_U = f$, and the uniqueness of T_f follows from the fact that the evaluations δ_x generate a dense subspace of $G(U)$.

(b) If $f \in \mathcal{H}(U;\, F)$, where F is a Banach space, then we define $T_f : G(U) \to F''$ by

$$(T_f u)(\psi) = T_{\psi \circ f}(u) = u(\psi \circ f) \tag{2.2}$$

for all $u \in G(U)$ and $\psi \in F'$. Clearly T_f is linear. To show that T_f is continuous, observe that the set

$$B = \{\psi \circ f : \psi \in F', \quad \|\psi\| \leq 1\}$$

is locally bounded, and is therefore included in $B^{\alpha}_{\mathcal{U}}$ for suitable $\mathcal{U}$ and α. Thus $\|T_f u\| \leq 1$ for every u in the polar $(B^{\alpha}_{\mathcal{U}})^0$ of $B^{\alpha}_{\mathcal{U}}$ in $G(U)$. Thus $T_f \in \mathcal{L}(G(U); F'')$. On the other hand

$$(T_f\delta_x)(\psi) = \delta_x(\psi \circ f) = \psi \circ f(x)$$

for all $x \in U$ and $\psi \in F'$. Hence $T_f\delta_x = f(x) \in F$ for every $x \in U$. Since the evaluations δ_x generate a dense subspace of $G(U)$, it follows that $T_f u \in F$ for every $u \in G(U)$. Thus $T_f \in \mathcal{L}(G(U); F)$ and $T_f \circ \delta_U = f$, as asserted. The uniqueness of T_f follows as in (a).

(c) Finally let $f \in \mathcal{H}(U; F)$, where F is a complete locally convex space. Then F can be represented as a projective limit of Banach spaces, namely $F = \text{proj}\, \tilde{F}_W$, where W varies among the convex, balanced 0–neighborhoods in F. Let $\pi_W : F \to \tilde{F}_W$ and $\pi_{WW'} : \tilde{F}_{W'} \to \tilde{F}_W$ denote the canonical mappings, when $W' \subset W$. Since $\pi_W \circ f \in \mathcal{H}(U; \tilde{F}_W)$, (b) yields a unique $T_W \in \mathcal{L}(G(U); \tilde{F}_W)$ such that $T_W \circ \delta_U = \pi_W \circ f$. One can readily see that $\pi_{WW'} \circ T_{W'} = T_W$ whenever $W' \subset W$. Hence there exists $T \in \mathcal{L}(G(U); F)$ such that $\pi_W \circ T = T_W$ for every W, and it follows that $T \circ \delta_U = f$, as asserted. The uniqueness of T follows again as in (a) or (b).

Since the uniqueness of $G(U)$, up to a topological isomorphism, is a direct consequence of the universal property, the proof of the theorem is complete.

2.2. Remark. In the proof of Theorem 2.1 we saw that the evaluations δ_x, with $x \in U$, generate a dense subspace of $G(U)$. The same proof shows the following:

(a) If D is a dense subset of U, then the evaluations δ_x, with $x \in D$, generate a dense subspace of $G(U)$. In particular, $G(U)$ is separable if E is separable.

(b) If V is an open subset of U which meets each connected component of U, then the evaluations δ_x, with $x \in V$, generate a dense subspace of $G(U)$.

In the proof of Theorem 2.1 we saw that the mapping

$$f \in (\mathcal{H}(U), \tau_\delta) \to T_f \in G(U)'_i$$

is a topological isomorphism. More generally, we have the following result.

2.3. Proposition. *Let E and F be locally convex spaces, with F complete, and let U be an open subset of E. Then the mapping*

$$f \in (\mathcal{H}(U;\, F),\, \tau_\delta) \to T_f \in (\mathcal{L}(G(U);\, F),\, \tau_\ell)$$

is a topological isomorphism.

To prove this proposition we need the following lemma.

2.4. Lemma. *Let U be an open subset of a locally convex space E, and let F be a Banach space. Let $\mathcal{U} = (U_j)$ be a countable open cover of U, let $\alpha = (\alpha_j)$ be a sequence of strictly positive numbers, and let*

$$B_{\mathcal{U}}^{\alpha}(F) = \{f \in \mathcal{H}(U;\, F) : \sup_{x \in U_j} \|f(x)\| \leq \alpha_j \quad \text{for every } j\}.$$

Then for a mapping $f \in \mathcal{H}(U;\, F)$ the following conditions are equivalent:
(a) $f \in B_{\mathcal{U}}^{\alpha}(F)$.
(b) $\psi \circ f \in B_{\mathcal{U}}^{\alpha}$ *for every* $\psi \in F'$ *with* $\|\psi\| \leq 1$.
(c) $\|T_f u\| \leq 1$ *for every* $u \in (B_{\mathcal{U}}^{\alpha})^0$.

Proof. To begin with observe that the set $B_{\mathcal{U}}^{\alpha}$ is convex, balanced and τ_c–closed. Hence $B_{\mathcal{U}}^{\alpha}$ is $\sigma(\mathcal{H}(U),\ (\mathcal{H}(U),\, \tau_c)')$–closed, and therefore $\sigma(\mathcal{H}(U),\ G(U))$–closed, since $(\mathcal{H}(U),\, \tau_c)' \subset G(U)$. Thus $(B_{\mathcal{U}}^{\alpha})^{00} = B_{\mathcal{U}}^{\alpha}$, by the bipolar theorem.

The equivalence (a) $\Leftrightarrow$ (b) is obvious. As we saw in the proof of Theorem 2.1, the implication (b) $\Rightarrow$ (c) follows from (2.2). And finally, if (c) holds, then it follows from (2.2) that $\psi \circ f \in (B_{\mathcal{U}}^{\alpha})^{00} = B_{\mathcal{U}}^{\alpha}$ for every $\psi \in F'$ with $\|\psi\| \leq 1$. Thus (c) $\Rightarrow$ (b) and the proof of the lemma is complete.

Proof of Proposition 2.3. We first assume that F is a Banach space. Then, with Grothendieck's notation, we have that

$$(\mathcal{H}(U;\, F),\, \tau_\delta) = \operatorname{ind}_{\mathcal{U},\alpha} \mathcal{H}(U;\, F)_{B_{\mathcal{U}}^{\alpha}(F)}.$$

On the other hand, since the polars $(B_{\mathcal{U}}^{\alpha})^0$ form a basis of convex, balanced 0–neighborhoods in $G(U)$, we have that

$$(\mathcal{L}(G(U);\, F),\, \tau_\ell) = \operatorname{ind}_{\mathcal{U},\alpha} \mathcal{L}(G(U)_{(B_{\mathcal{U}}^{\alpha})^0};\, F).$$

Since, by Lemma 2.4, the correspondence

$$f \in \mathcal{H}(U;\, F)_{B_{\mathcal{U}}^{\alpha}(F)} \to T_f \in \mathcal{L}(G(U)_{(B_{\mathcal{U}}^{\alpha})^0};\, F)$$

is an isometry, the desired conclusion follows when F is a Banach space.

Finally, the case in which F is a complete locally convex space can be easily reduced to the preceding case by representing F as the projective limit of the Banach spaces $\tilde{F}_W$. The details are left to the reader.

The following result is also a direct consequence of Lemma 2.4.

2.5. Proposition. *Let E and F be locally convex spaces, with F complete, and let U be an open subset of E. Then a family $(f_i) \subset \mathcal{H}(U; F)$ is amply bounded if and only if the corresponding family $(T_{f_i}) \subset \mathcal{L}(G(U); F)$ is equicontinuous.*

If E and F are locally convex spaces, and $m \in \mathbb{N}$, then $\mathcal{P}(^mE; F)$ denotes the vector space of all continuous m-homogeneous polynomials from E into F. If U is an open subset of E, $f \in \mathcal{H}(U; F)$ and $a \in U$, then $P^m f(a) \in \mathcal{P}(^mE; F)$ denotes the mth term in the Taylor series expansion of f at a.

2.6. Proposition. *Let U be an open subset of a complete locally convex space E. Then E is topologically isomorphic to a complemented subspace of $G(U)$.*

Proof. By Theorem 2.1 there exists $T \in \mathcal{L}(G(U); E)$ such that $T \circ \delta_U(x) = x$ for every $x \in U$. Fix $a \in U$ and let $S = P^1\delta_U(a) \in \mathcal{L}(E; G(U))$. We claim that $T \circ S(t) = t$ for every $t \in E$. Indeed, given $t \in E$ choose $r > 0$ such that $a + \zeta t \in U$ for every $\zeta \in \mathbb{C}$ with $|\zeta| \leq r$. By the Cauchy integral formula

$$S(t) = \frac{1}{2\pi i} \int_{|\zeta|=r} \frac{\delta_U(a + \zeta t)}{\zeta^2} \, d\zeta,$$

and therefore

$$T \circ S(t) = \frac{1}{2\pi i} \int_{|\zeta|=r} \frac{a + \zeta t}{\zeta^2} \, d\zeta = t,$$

as asserted.

3. Linearization of Gateaux Holomorphic Mappings

Let U be an open subset of a locally convex space E. By Theorem 2.1, for each finite dimensional subspace M of E, there is a unique mapping $\pi_M \in$

$\mathcal{L}(G(U \cap M); G(U))$ such that the following diagram is commutative:

$$
(3.1) \qquad \begin{array}{ccc}
U \cap M & \xrightarrow[\sigma_M]{} & U \\
\downarrow \delta_{U\cap M} & & \downarrow \delta_U \\
G(U \cap M) & \xrightarrow[\pi_M]{} & G(U)
\end{array}
$$

If M and N are finite dimensional subspaces of E, with $M \subset N$, then, again by Theorem 2.1, there is a unique mapping $\pi_{NM} \in \mathcal{L}(G(U \cap M); G(U \cap N))$ such that the following diagram is commutative:

$$
\begin{array}{ccc}
U \cap M & \xrightarrow[\sigma_{NM}]{} & U \cap N \\
\downarrow \delta_{U\cap M} & & \downarrow \delta_{U\cap N} \\
G(U \cap M) & \xrightarrow[\pi_{NM}]{} & G(U \cap N)
\end{array}
$$

Whence it follows that $\pi_N \circ \pi_{NM} = \pi_M$ whenever $M \subset N$. Let $G_0(U)$ denote the subspace

$$G_0(U) = \bigcup_M \pi_M(G(U \cap M)),$$

where M varies over the finite dimensional subspaces of E, and equip $G_0(U)$ with the topology induced by $G(U)$.

3.1. Theorem. *Let U be an open subset of a locally convex space E. Then:*

(a) *$G_0(U)$ is a dense subspace of $G(U)$.*

(b) $\delta_U \in \mathcal{H}(U; G_0(U))$.

(c) *For each complete locally convex space F and each Gateaux holomorphic mapping $f : U \to F$, there is a unique linear mapping $T_f : G_0(U) \to F$ such that $T_f \circ \delta_U = f$. Moreover, T_f is continuous if and only if f is continuous.*

Proof. (a) It follows from the commutativity of diagram (3.1) that $\delta_x \in G_0(U)$ for every $x \in U$. Since the evaluations δ_x, with $x \in U$, generate a dense subspace of $G(U)$, we conclude that $G_0(U)$ is a dense subspace of $G(U)$.

(b) We already know that $\delta_U \in \mathcal{H}(U; G(U))$ and $\delta_U(x) \in G_0(U)$ for every $x \in U$. To prove that $\delta_U \in \mathcal{H}(U; G_0(U))$ it suffices to show that $P^m \delta_U(a) \in \mathcal{P}(^m E; G_0(U))$ for every $a \in U$ and $m \in \mathbb{N}$. Fix $m \in \mathbb{N}$, $a \in U$ and $t \in E$. Let $r > 0$ such that $a + \zeta t \in U$ for all $\zeta \in \mathbb{C}$ with $|\zeta| \leq r$, and let M be the subspace of E generated by a and t. Then it follows from the commutativity of diagram (3.1) that

$$P^m \delta_U(a)(t) = \frac{1}{2\pi i} \int_{|\zeta|=r} \frac{\delta_U(a+\zeta t)}{\zeta^{m+1}} \, d\zeta$$

$$= \frac{1}{2\pi i} \int_{|\zeta|=r} \frac{\pi_M \circ \delta_{U\cap M}(a+\zeta t)}{\zeta^{m+1}} \, d\zeta = \pi_M(P^m \delta_{U\cap M}(a)(t))$$

and therefore $P^m \delta_U(a)(t) \in \pi_M(G(U \cap M)) \subset G_0(U)$.

(c) By Theorem 2.1, for each finite dimensional subspace M of E there is a unique mapping $T_M \in \mathcal{L}(G(U \cap M); F)$ such that $T_M \circ \delta_{U\cap M} = f \circ \sigma_M$. If M and N are finite dimensional subspaces of E, with $M \subset N$, then the following diagram is commutative:

$$\begin{array}{ccccc}
U \cap M & \xrightarrow{\sigma_{NM}} & U \cap N & \xrightarrow{\sigma_N} & U \\
\downarrow \delta_{U\cap M} & & \downarrow \delta_{U\cap N} & & \downarrow f \\
G(U \cap M) & \xrightarrow[\pi_{NM}]{} & G(U \cap N) & \xrightarrow[T_N]{} & F
\end{array}$$

Whence it follows that $T_N \circ \pi_{NM} = T_M$ whenever $M \subset N$. Hence there is a unique linear mapping $T : G_0(U) \to F$ such that $T \circ \pi_M = T_M$ for each finite dimensional subspace M of E. Since $T_M \circ \delta_{U\cap M} = f \circ \sigma_M$ for every M, it follows that $T \circ \delta_U = f$. If $S : G_0(U) \to F$ is a linear mapping such that $S \circ \delta_U = f$, then it follows that $S \circ \pi_M = T_M = T \circ \pi_M$ for every M, and therefore $S = T$.

Finally, if T is continuous, then $f = T \circ \delta_U$ is certainly continuous. And if f is continuous, then if follows from Theorem 2.1 that T is continuous. This completes the proof.

The following result is then a direct consequence of Proposition 2.5.

3.2. Proposition. *Let E, F be locally convex spaces, with F complete, and let U be an open subset of E. Then a family $(f_i) \subset \mathcal{H}(U; F)$ is amply bounded* *if and only if the corresponding family* $(T_{f_i}) \subset \mathcal{L}(G_0(U);\, F)$ *is equicontinuous.*

In the following result, which parallels Proposition 2.6, no completeness hypothesis on E is necessary.

3.3. Proposition. *Let* U *be an open subset of a locally convex space* E. *Then* E *is topologically isomorphic to a complemented subspace of* $G_0(U)$.

Proof. Fix $a \in U$ and let $S = P^1\delta_U(a) \in \mathcal{L}(E;\, G_0(U))$. If $\tilde{E}$ is the completion of E, then, by Theorem 2.1, there exists $T \in \mathcal{L}(G(U);\, \tilde{E})$ such that $T\circ\delta_U(x) = x$ for every $x \in U$. The proof of Proposition 2.6 shows that $T \circ S(t) = t$ for every $t \in E$. Thus to complete the proof it suffices to show that $T(G_0(U)) \subset E$. Now, again by Theorem 2.1, for each finite dimensional subspace M of E, there is a unique mapping $T_M \in \mathcal{L}(G(U \cap M);\, M)$ such that $T_M \circ \delta_{U\cap M}(x) = x$ for every $x \in U \cap M$. Whence it follows that $T \circ \pi_M = T_M$ and therefore $T \circ \pi_M(u) \in M$ for every $u \in G(U \cap M)$ and every finite dimensional subspace M of E. Hence $Tu \in E$ for every $u \in G_0(U)$, thus completing the proof.

3.4. Remark. By using ε-products, Schottenloher [28] obtained linearization theorems for Gateaux holomorphic mappings and for hypoholomorphic mappings. Ryan [24] obtained similar results by using tensor products. We could also obtain Schottenloler's results with the help of Theorem 1.1, by introducing appropiate substitutes for the topology τ_δ on the spaces of Gateaux holomorphic functions or hypoholomorphic functions.

3.5. Remark. If $G_0(U)$ is equipped with the locally convex inductive limit topology τ_i defined by

$$(G_0(U),\, \tau_i) = \mathrm{ind}_M\, G(U \cap M),$$

where M varies over the finite dimensional subspaces of E, then one can readily see that $\delta_U : U \to (G_0(U),\, \tau_i)$ is Gateaux holomorphic, and the space of Gateaux holomorphic mappings from U into F can be canonically identified with $\mathcal{L}((G_0(U),\, \tau_i);\, F)$. This gives back one of the linearization theorems of Schottenloher mentioned in the preceding remark. However, the results in Section 4 will show that it is more useful to equip $G_0(U)$ with the topology induced by $G(U)$.

4. Holomorphically Barrelled Domains

4.1. Proposition. *Let U be an open subset of a locally convex space E. Then, for each $u \in G_0(U)$, there are a finite dimensional subspace M of E, a compact set $K \subset U \cap M$, and a constant $\rho > 0$ such that $u \in \rho\overline{\Gamma}\delta_U(K)$, where $\overline{\Gamma}A$ denotes the closed, convex balanced hull of A.*

This proposition follows at once from the following lemma.

4.2. Lemma. *Let U be an open subset of a finite dimensional, locally convex space E. Then, for each compact set $L \subset G(U)$, there are a compact set $K \subset U$ and a constant $\rho > 0$ such that $L \subset \rho\overline{\Gamma}\delta_U(K)$.*

Proof. Since E is finite dimensional, each bounded subset of $(\mathcal{H}(U), \tau_c)$ is locally bounded, $\tau_\delta = \tau_c$ on $\mathcal{H}(U)$, and $(\mathcal{H}(U), \tau_c)$ is a Fréchet–Montel space. It follows that $G(U) = (\mathcal{H}(U), \tau_c)'_b$ and the evaluation mapping

$$J : (\mathcal{H}(U), \tau_c) \to G(U)'_b$$

is a topological isomorphism. Let L be a compact subset of $G(U)$. Then the polar L^0 is a 0–neighborhood in $G(U)'_b$, and the set $J^{-1}(L^0)$ is a 0–neighborhood in $(\mathcal{H}(U), \tau_c)$. Hence there are a compact set $K \subset U$ and a constant $r > 0$ such that

$$\{f \in \mathcal{H}(U) : \sup_{x \in K} |f(x)| \leq r\} \subset \{f \in \mathcal{H}(U) : T_f \in L^0\}.$$

After writting $f = T_f \circ \delta_U$ we see that $r(\delta_U(K))^0 \subset L^0$ and therefore

$$L \subset L^{00} \subset \frac{1}{r}(\delta_U(K))^{00} = \frac{1}{r}\overline{\Gamma}\,\delta_U(K),$$

as asserted.

4.3. Proposition. *Let E, F be locally convex spaces, with F complete, and let U be an open subset of E. Then, for a family $(f_i) \subset \mathcal{H}(U; F)$, the following conditions are equivalent:*

(a) *(f_i) is bounded on the finite dimensional compact subsets of U.*

(b) *(T_{f_i}) is pointwise bounded in $\mathcal{L}(G_0(U); F)$.*

(c) *(T_{f_i}) is bounded on all the sets of the form $\pi_M(L)$, where M varies over the finite dimensional subspaces of E, and L varies over the compact subsets of $G(U \cap M)$.*

Proof. (a) $\Rightarrow$ (b): This is a direct consequence of Proposition 4.1.

(b) $\Rightarrow$ (c): Assume (T_{f_i}) is pointwise bounded in $\mathcal{L}(G_0(U); F)$, and let M be a finite dimensional subspace of E. Then $(T_{f_i} \circ \pi_M)$ is pointwise bounded in $\mathcal{L}(G(U \cap M); F)$. Since $G(U \cap M)$ is a (DFM)–space, and in particular barrelled, $(T_{f_i} \circ \pi_M)$ is equicontinuous, and therefore bounded on the compact subsets of $G(U \cap M)$.

(c) $\Rightarrow$ (a): This follows from the commutativity of diagram (3.1).

Let U be an open subset of a locally convex space E. We shall say that U is *holomorphically barrelled* if for each locally convex space F (or equivalently, for $F = \mathbb{C}$), a family $(f_i) \subset \mathcal{H}(U; F)$ is amply bounded whenever (f_i) is bounded on the finite dimensional compact subsets of U. We shall say that U is *holomorphically infrabarrelled* if the same condition holds, but replacing "finite dimensional compact subsets of U" by "compact subsets of U". With the terminology of Barroso et al. [1], a locally convex space E is holomorphically barrelled (resp. holomorphically infrabarrelled) if and only if each open subset of E is holomorphically barrelled (resp. holomorphically infrabarrelled). By combining Propositions 3.2 and 4.3 we get at once the following result:

4.4. Theorem. *Let U be an open subset of a locally convex space E. Then U is holomorphically barrelled if and only if $G_0(U)$ is a barrelled space.*

If U is holomorphically infrabarrelled, then it is easy to see that $G_0(U)$ is infrabarrelled, but we do not know whether the converse is true.

We shall say that U is *holomorphically bornological* if for every locally convex space F (or equivalently, for every Banach space F), a mapping $f : U \to F$ belongs to $\mathcal{H}(U; F)$ whenever f is Gateaux holomorphic and f is bounded on the compact subsets of U. With the terminology of Barroso et al. [1], a locally convex space E is holomorphically bornological if and only if each open subset of E is holomorphically bornological. If U is holomorphically bornological, then it is easy to see that $G_0(U)$ is bornological, but we do not know whether the converse is true.

Finally, we shall say that U is *holomorphically Mackey* if for every complete locally convex space F (or equivalently, for every Banach space F), a mapping $f : U \to F$ belongs to $\mathcal{H}(U; F)$ whenever $\psi \circ f \in \mathcal{H}(U)$ for every $\psi \in F'$. With the terminology of Barroso et al. [1], a locally convex space E is holomorphically Mackey it and only if each open subset of E is holomorphically Mackey.

4.5. Theorem. *Let U be an open subset of a locally convex space E. Then*

U is holomorphically Mackey if and only if $G_0(U)$ is a Mackey space.

Proof. ($\Rightarrow$) Let F be a Banach space, and let $T : G_0(U) \to F$ be a mapping such that $\psi \circ T \in G_0(U)'$ for every $\psi \in F'$. Let $f = T \circ \delta_U : U \to F$. Then $\psi \circ f = (\psi \circ T) \circ \delta_U \in \mathcal{H}(U)$ for every $\psi \in F'$. Since U is holomorphically Mackey, $f \in \mathcal{H}(U; F)$. By Theorem 3.1, $T \in \mathcal{L}(G_0(U); F)$ and $G_0(U)$ is a Mackey space.

($\Leftarrow$) Let F be a Banach space, and let $f : U \to F$ be a mapping such that $\psi \circ f \in \mathcal{H}(U)$ for every $\psi \in F'$. Then f is Gateaux holomorphic and by Theorem 3.1 there is a unique linear mapping $T : G_0(U) \to F$ such that $T \circ \delta_U = f$. If $\psi \in F'$ then $(\psi \circ T) \circ \delta_U = \psi \circ f \in \mathcal{H}(U)$ and therefore $\psi \circ T \in G_0(U)'$, again by Theorem 3.1. Since $G_0(U)$ is a Mackey space, $T \in \mathcal{L}(G_0(U); F)$. Hence $f \in \mathcal{H}(U; F)$ and U is holomorphically Mackey.

We next summarize the two problems posed in this section.

4.6. Problems. Let U be an open subset of a locally convex space E.

(a) If $G_0(U)$ is infrabarrelled, does it follow that U is holomorphically infrabarrelled?

(b) If $G_0(U)$ is bornological, does it follow that U is holomorphically bornological?

For other results and open problems concerning these matters the reader is referred to the book of Pérez Carreras and Bonet [23] and to the recent paper of Bonet et al. [4].

5. Holomorphic Continuation

The following theorem complements results of Coeuré [6], Hirschowitz [11] and Schottenloher [26] [27].

5.1. Theorem. *Let U and V be open subset of a locally convex space E, with $U \subset V$. Let $S : G(U) \to G(V)$ be the unique continuous linear mapping such that the following diagram is commutative:*

$$\begin{array}{ccc} U & \xrightarrow{\quad\sigma\quad} & V \\ \delta_U \big\downarrow & & \big\downarrow \delta_V \\ G(U) & \xrightarrow[S]{\qquad} & G(V) \end{array}$$

Consider the following conditions:

(a) *The restriction mapping $\mathcal{H}(V) \to \mathcal{H}(U)$ is a bijection.*

(b) *The restriction mapping $\mathcal{H}(V; F) \to \mathcal{H}(U; F)$ is a bijection for every complete locally convex space F (or equivalently, for every Banach space F; or equivalently, for every Banach space F of the form $F = \ell^\infty(I)$).*

(c) *The mapping $S : G(U) \to G(V)$ is a topological isomorphism.*

(d) *The restriction mapping $(\mathcal{H}(V; F), \tau_\delta) \to (\mathcal{H}(U; F), \tau_\delta)$ is a topological isomorphism for every complete locally convex space F (or equivalently, for every Banach space F; or equivalently, for every Banach space F of the form $F = \ell^\infty(I)$).*

Then conditions (b), (c) and (d) are always equivalent, and each of then implies condition (a). If V is holomorphically Mackey, then condition (a) is equivalent to the other conditions.

Proof. To begin with we observe that since every complete locally convex space F can be canonically represented as the projective limit of the Banach spaces $\tilde{F}_W$, condition (b) for every complete locally convex space F is equivalent to condition (b) for every Banach space F. And since every Banach space F can be identified with a closed subspace of $\ell^\infty(I)$, for a suitable set I, an application of the identity principle shows that condition (b) for every Banach space F is equivalent to condition (b) for every Banach space F of the form $F = \ell^\infty(I)$. Similar remarks apply to condition (d). Thus to prove the theorem it is sufficient to consider conditions (b) or (d) for every complete locally convex space F.

(b) $\Rightarrow$ (c): By (b) there exists $g \in \mathcal{H}(V; G(U))$ such that $g \circ \sigma = \delta_U$. By Theorem 2.1 there exists $T \in \mathcal{L}(G(V); G(U))$ such that $T \circ \delta_V = g$. To prove (c) we shall show that

$$T \circ S(u) = u \quad \text{for every} \quad u \in G(U) \tag{5.1}$$

and

$$S \circ T(v) = v \quad \text{for every} \quad v \in G(V). \tag{5.2}$$

On one hand

$$T \circ S \circ \delta_U = T \circ \delta_V \circ \sigma = g \circ \sigma = \delta_U,$$

and (5.1) follows, since the evaluations δ_x, with $x \in U$, generate a dense subspace of $G(U)$. On the other hand

$$S \circ T \circ \delta_V \circ \sigma = S \circ g \circ \sigma = S \circ \delta_U = \delta_V \circ \sigma,$$

and (5.2) follows, since, by Remark 2.2 (b) the evaluations δ_x, with $x \in U$, generate a dense subspace of $G(V)$.

(c) $\Rightarrow$ (d): By Proposition 2.3, each of the mappings

$$g \in (\mathcal{H}(V; F), \tau_\delta) \to T_g \in (\mathcal{L}(G(V); F), \tau_\ell)$$

and

$$f \in (\mathcal{H}(U; F), \tau_\delta) \to T_f \in (\mathcal{L}(G(U); F), \tau_\ell)$$

is a topological isomorphism. On the other hand, since $S : G(U) \to G(V)$ is a topological isomorphism, the mapping

$$T \in (\mathcal{L}(G(V); F), \tau_\ell) \to T \circ S \in (\mathcal{L}(G(U); F), \tau_\ell)$$

is a topological isomorphism too. And since $T_g \circ S = T_{g \circ \sigma}$ for every $g \in \mathcal{H}(V; F)$, we conclude that the mapping

$$g \in (\mathcal{H}(V; F), \tau_\delta) \to g \circ \sigma \in (\mathcal{H}(U; F), \tau_\delta)$$

is a topological isomorphism, as we wanted.

It is obvious that (d) $\Rightarrow$ (b) and (b) $\Rightarrow$ (a). And finally, the implication (a) $\Rightarrow$ (b) when V is holomorphically Mackey, follows from a result of Nachbin [20].

To end this section we state some old problems that still remain open.

5.2. Problems. Let U and V be open subsets of a locally convex space E, with $U \subset V$ and V connected.

(a) If the restriction mapping $R : \mathcal{H}(V) \to \mathcal{H}(U)$ is surjective, does it follow that R is a topological isomorphism for the topology τ_δ?

(b) In Theorem 5.1, is condition (a) always equivalent to the other conditions?

(c) Let F be a Banach space and let $g : V \to F$ be a weakly holomorphic mapping whose restriction to U is holomorphic. Does it follow that g is holomorphic?

By the aforementioned result of Nachbin [15], a positive solution to problem 5.2(c) would imply a positive solution to problem 5.2(b). And obviously a positive solution to problem 5.2(b) would imply a positive solution to problem 5.2(a).

6. The Image of the Universal Mapping

Let U be an open subset of a locally convex space E. We recall that a set $X \subset U$ is said to be a *holomorphic* or *analytic* subset of U if for each $x \in U$ there are an open neighborhood V of x in U, a locally convex space F, and a mapping $f \in \mathcal{H}(V; F)$ such that $X \cap V = f^{-1}(0)$.

6.1. Theorem. *Let U be an open subset of a complete locally convex space E. Then there are an open subset V of $G(U)$ and a mapping $f \in \mathcal{H}(V; G(U))$ such that $\delta_U(U) = f^{-1}(0)$. In particular, $\delta_U(U)$ is a holomorphic subset of V.*

Proof. By Theorem 2.1 there exists $T \in \mathcal{L}(G(U); E)$ such that $T \circ \delta_U(x) = x$ for every $x \in U$. If we define $V = T^{-1}(U)$, then it is clear that $\delta_U(U) \subset V$. If we define $f \in \mathcal{H}(V; G(U))$ by $f(v) = v - \delta_U \circ T(v)$, then it is clear that $\delta_U(U) = f^{-1}(0)$.

6.2. Corollary. *If E is a complete locally convex space, then there is a mapping $f \in \mathcal{H}(G(E); G(E))$ such that $\delta_E(E) = f^{-1}(0)$. In particular, $\delta_E(E)$ is a holomorphic subset of $G(E)$.*

Let X and Y be Hausdorff topological spaces. According to Bourbaki [5], a mapping $f : X \to Y$ is said to be *proper* if f is continuous and closed and if $f^{-1}(L)$ is a compact subset of X for each compact set $L \subset Y$. By [5, p. I. 72, Proposition 2], a continuous injective mapping $f : X \to Y$ is proper if and only if f is a homeomorphism between X and a closed subset of Y.

6.3. Proposition. *Let U be an open subset of a locally convex space E. Then δ_U is a homeomorphism between U and a linearly independent subset of $G(U)$.*

Proof. We know that δ_U is continuous. If $\tilde{E}$ denotes the completion of E, then we know there exists $T \in \mathcal{L}(G(U); \tilde{E})$ such that $T \circ \delta_U(x) = x$ for every $x \in U$. Whence it follows that δ_U is injective and that $\delta_U^{-1} : \delta_U(U) \to U$ is continuous too. We finally show that $\delta_U(U)$ is a linearly independent subset of $G(U)$. Suppose there are distinct points $x_1, \ldots, x_n \in U$ and $\alpha_1, \ldots, \alpha_n \in \mathbb{C}$ such that $\sum_{k=1}^{n} \alpha_k \delta_{x_k} = 0$. By the Hahn–Banach theorem, for each pair of indices j, k with $j \neq k$, we can find $\varphi_{jk} \in E'$ such that $\varphi_{jk}(x_j - x_k) = 1$. Then the polynomials $P_1, \ldots, P_n$ defined by

$$P_j(x) = \prod_{\substack{k=1 \\ k \neq j}}^{n} \varphi_{jk}(x - x_k)$$

satisfy the conditions $P_j(x_j) = 1$ and $P_j(x_k) = 0$ whenever $k \neq j$. By applying the functional $\sum_{k=1}^{n} \alpha_k \delta_{x_k} = 0$ to the polynomials $P_1, \ldots, P_n$, we get that $\alpha_1 = \cdots = \alpha_n = 0$, as we wanted.

6.4. Corollary. *Let U be an open subset of a complete locally convex space E. Then the following conditions are equivalent:*

(a) *$\delta_U(U)$ is a holomorphic subset of $G(U)$.*
(b) *$\delta_U(U)$ is a closed subset of $G(U)$.*
(c) *$\delta_U : U \to G(U)$ is a proper mapping.*

Proof. The equivalence (a) $\Leftrightarrow$ (b) follows from Theorem 6.1, whereas the equivalence (b) $\Leftrightarrow$ (c) follows from Proposition 6.3.

6.5. Proposition. *Let U be an open subset of a (DFM)–space E. Then, for each compact set $L \subset G(U)$, there are a compact set $K \subset U$ and a constant $\rho > 0$ such that $L \subset \rho \overline{\Gamma} \delta_U(K)$.*

Proof. Dineen [7] proved that the bounded subsets of $(\mathcal{H}(U), \tau_c)$ are locally bounded, that $\tau_\delta = \tau_c$ on $\mathcal{H}(U)$, and that $(\mathcal{H}(U), \tau_c)$ is a Fréchet–Montel space. Hence $G(U) = (\mathcal{H}(U), \tau_c)'_b$ and the proof of Lemma 4.2 applies.

6.6. Theorem. *Let U be an open subset of a (DFM)–space E. Then the following conditions are equivalent:*

(a) *$\delta_U(U)$ is a holomorphic subset of $G(U)$.*
(b) *$\delta_U(U)$ is a closed subset of $G(U)$.*
(c) *$\delta_U : U \to G(U)$ is a proper mapping.*
(d) *U is holomorphically convex.*

Proof. By Corollary 6.4, conditions (a), (b) and (c) are equivalent.

(c) $\Rightarrow$ (d): If K is a compact subset of U, then one can readily see that

$$\delta_U^{-1}(\overline{\Gamma}\,\delta_U(K)) = \widehat{K}_{\mathcal{H}(U)}, \tag{6.1}$$

where

$$\widehat{K}_{\mathcal{H}(U)} = \{y \in U : |f(y)| \leq \sup_{x\in K} |f(x)| \quad \text{for every} \quad f \in \mathcal{H}(U)\}.$$

Hence $\widehat{K}_{\mathcal{H}(U)}$ is compact if $\delta_U : U \to G(U)$ is a proper mapping.

(d) $\Rightarrow$ (b): We first show that $\delta_U^{-1}(L)$ is a compact subset of U for each compact set $L \subset G(U)$. By Proposition 6.5 there are a compact set $K \subset U$ and a number $\rho \geq 1$ such that $L \subset \rho\,\overline{\Gamma}\,\delta_U(K)$. Now, the proof of (6.1) shows also that

$$\delta_U^{-1}(\rho\,\overline{\Gamma}\delta_U(K)) = \{y \in U : |f(y)| \leq \rho \sup_{x\in K} |f(x)| \quad \text{for every} \quad f \in \mathcal{H}(U)\}$$

and, by applying the inequality $|f(y)| \leq \rho \sup_{x\in K}|f(x)|$ to the function f^m, taking mth root, and letting m tend to infinity, we see that

$$\widehat{K}_{\mathcal{H}(U)} = \{y \in U : |f(y)| \leq \rho \sup_{x\in K} |f(x)| \quad \text{for every} \quad f \in \mathcal{H}(U)\},$$

and therefore

$$\delta_U^{-1}(\rho\,\overline{\Gamma}\delta_U(K)) = \widehat{K}_{\mathcal{H}(U)} \tag{6.2}$$

for every $\rho \geq 1$. Thus $\delta_U^{-1}(L) \subset \widehat{K}_{\mathcal{H}(U)}$, and since U is holomorphically convex, $\delta_U^{-1}(L)$ is compact, as asserted.

We finally prove that $\delta_U(U)$ is a closed subset of $G(U)$. It follows from the proof of Proposition 6.5 that $G(U)$ is a (DFM)–space, and therefore a k–space, either by a result of Dineen [7], or else by the classical Banach–Dieudonné theorem. Thus to show that $\delta_U(U)$ is a closed subset of $G(U)$, it suffices to prove that $\delta_U(U) \cap L$ is a closed subset of L for each compact set $L \subset G(U)$. But this follows from the first part of the proof, for $\delta_U(U) \cap L = \delta_U(\delta_U^{-1}(L))$. The proof of the theorem is now complete.

Mazet [15] obtained Theorem 6.6 when E is finite dimensional. We have followed his proof of the implication (c) $\Rightarrow$ (d), which works for arbitrary locally convex spaces. Mazet gave a constructive proof of the implication (d) $\Rightarrow$ (c), and then derived the implication (c) $\Rightarrow$ (a) from a theorem of Barlet and Mazet (see [13] and [14]) which generalizes the classical proper mapping theorem of Remmert.

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