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HIGGS MECHANISM IN LIGHT-FRONT QUANTIZED FIELD THEORY

by

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Abstract

The spontaneous symmetry breaking of continuous symmetry in light-front quantized scalar field theory is studied following the *standard* Dirac procedure for constrained dynamical systems. A non-local constraint is found to follow. The values of the constant background fields (zero modes) at the tree level, as a consequence, are shown to be given by minimizing the light-front energy. The zero modes are shown to commute with the non-zero ones and the isovector built from them is seen to characterize a (non-perturbative) vacuum state and the corresponding physical sector. The infinite degeneracy of the vacuum is described by the continuum of the allowed orientations of this background isovector in the isospin space. The symmetry generators in the quantized field theory annihilate the vacuum in contrast to the case of equal-time quantization. Not all of them are conserved and the conserved ones determine the surviving symmetry of the quantum theory Lagrangian. The criterian for determining the background isovector and the counting of the number of Goldstone bosons goes as in the equal-time case. A demonstration in favour of the absence of Goldstone bosons in two dimensions is also found. Finally, we extend the discussion to an understanding of the Higgs mechanism in light-front frame.

Key-words: Light-cone quantization; Gauge theory; Higgs mechanism; Dirac method.

1. The possibility of building a Hamiltonian formulation of relativistic dynamics on light-front surface, $\tau = (t + z) = const.$, was pointed out by Dirac [1] and rediscovered by Weinberg [2] in the guise of old-fashioned perturbation theory in the infinite-momentum frame. Since the longitudinal momentum k^+ turns out to be necessarily positive and is conserved, the vacuum structure is much simpler and it is conjectured that the non-perturbative effects may be easier to handle in the light-front framework. The perturbative field theory is, in fact, much simplified and with the introduction of the discretized light-cone quantization (DLCQ) [3] it has developed into a useful tool to handle the non-perturbative calculations as well. Other interesting developments are the recent studies on Light-front Tamm-Dancoff Field Theory [4] to study non-perturbative effects and the begining of a systematic study of perturbative renormalization theory [5].

The problem of non-perturbative vacuum structure, say, in the presence of a spontaneous symmetry breaking scalar potential, Higgs mechanism, the fermionic condensates, and other related problems, in the light-front framework, however, has remained without a clear understanding even at the tree level.

We address here to a description of the spontaneous symmetry breaking of a continuous symmetry in scalar field theory and obtain non-trivial vacuum properties [6] and then apply it to describe the Higgs mechanism. It is well known that any theory written in terms of light-front coordinates describes a constrained dynamical system. A method for constructing the canonical framework is the *standard* Dirac procedure [7] widely tested in the context of gauge and other constrained systems. It was applied *without any modifications* to describe the spontaneously broken reflection symmetry in $1+1$ dimensions [8]. We will extend its application to the problem at hand. We remark that it will in the least be very embarassing for the usual Dirac procedure if we were forced to modify it, as suggested in some recent papers, for handling the simple degenerate systems under consideration. The procedure leads to nonlocal constraints. Such constraints which are expected to occur in other theories written in light-front coordinates as well seem to have been overlooked. In our case they lead to a tree level description of spontaneous symmetry breaking analogous to the well known one in the case of equal-time framework. It is worth remarking that in view of the Coleman's theorem [9] some transverse dimensions must be present and we do confirm their necessity in our context and an argument in favour of the absence of Goldstone bosons in two dimensions seems to follow. We will not discretize the modes in order not to introduce spurious zero modes and finite volume effects. The Dirac procedure may also be used with discretized modes (box quantization) and the same conclusions are reached in the limit of the continuuum case.

2. The scalar field Lagrangian with a global isospin symmetry in light-frame coordinates is

$$\int_{-\infty}^{\infty} dx d^2\bar{x} \left[\dot{\phi}_a\phi'_a - \frac{1}{2}(\partial_i\phi_a)(\partial_i\phi_a) - V(\phi)\right], \tag{1}$$

where the real scalar fields ϕ_a, $a = 1,2...$ are the components of an isospin-multiplet ϕ, $V(\phi)(\geq 0)$ is the potential, and for simplicity, in our context, we may assume it not to involve any derivatives of the fields. Here an overdot and a prime indicate the partial derivations with respect to the light-front coordinates $\tau \equiv x^+ = (x^0 + x^3)/\sqrt{2}$ and $x \equiv x^- = (x^0 - x^3)/\sqrt{2}$ respectively, $\bar{x} \equiv (x^1, x^2)$ represents the transverse directions, and $d^4x = d\tau dx\, d^2\bar{x}$. The Euler-Lagrange equation of motion, $2\dot{\phi}'_a = -V'_a(\phi) + \partial_i\partial_i\phi_a$, where $i = 1,2$ and $V'_a \equiv \delta V(\phi)/\delta\phi_a$, shows that the classical solutions, for instance, $\phi_a = const.$, are possible to obtain. We seperate the zero mode along the longitudinal x direction, and write $\phi_a(\tau, x, \bar{x}) = \omega_a(\tau, \bar{x}) + \varphi_a(\tau, x, \bar{x})$, such that φ_a has no longitudinal zero mode and its integral over the space coordinate x vanishes. It is then easily seen that the Lagrangian density may be written as

$$\mathcal{L} = [\, \dot{\varphi}_a\varphi'_a - \frac{1}{2}(\partial_i\phi_a)(\partial_i\phi_a) - V(\phi)\,], \tag{2}$$

which is of first order in $\dot{\varphi}_a$, contains no kinetic term for the zero mode, and consequently describes a constrained dynamical system. Indicating by $p_a(\tau, \bar{x})$ and $\pi_a(\tau, x, \bar{x})$, the momenta conjugate to ω_a and φ_a respectively, the primary constraints are $p_a(\tau, \bar{x}) \approx 0$ and $\Phi_a \equiv \pi_a - \varphi'_a \approx 0$ while the canonical Hamiltonian is obtained to be

$$H_c = \int_{-\infty}^{\infty} dx d^2\bar{x} \left[V(\phi) + (1/2)(\partial_i\phi_a)(\partial_i\phi_a)\right]. \tag{3}$$

where $\approx$ stands for the weak equality [7].

We postulate now the standard Poisson brackets at equal τ, viz, $\{p_a(\bar{x}), \omega_b(\bar{y})\} = -\delta^2(\bar{x}-\bar{y})$, $\{\pi_a(x,\bar{x}), \varphi(y,\bar{y})\} = -\delta(x-y)\delta^2(\bar{x}-\bar{y})$ and define an extended hamiltonian

$$H' = H_c + \int d^2\bar{x}\, \nu_a(\bar{x})\, p_a(\bar{x}) + \int d^3x\, u_a(x,\bar{x})\, \Phi_a(x,\bar{x}), \tag{4}$$

where ν_a and u_a are Lagrange multipliers and we suppress the coordinate τ for convenience of writing. On requiring the persistency in τ of the primary constraints, we obtain

$$\dot{p}_a(\bar{x}) \;=\; \{p_a(\bar{x}), H'\} = -\int_{-\infty}^{\infty} dx \left[V'_a(\phi) \;-\; \partial_i\partial_i\phi_a\right] \equiv -\beta_a(\bar{x}) \approx 0, \tag{5}$$

$$\dot{\Phi}_a \;=\; \{\Phi_a, H'\} \;=\; V'_a(\phi) \;-\; 2\,u'_a \approx 0\,. \tag{6}$$

From (5) we obtain an interaction dependent and nonlocal secondary constraint $\beta_a \approx 0$, which is the same as that follows also on integrating the Euler-Lagrange equation, as already pointed out in ref. [8], if we assume suitable assymptotic boundary conditions on the fields, while (6) is a consistency requirement for determining u_a. Next we extend the Hamiltonian to

$$H'' \;=\; H' + \int d^2\bar{x}\, \mu_a(\bar{x})\, \beta_a(\bar{x}), \tag{7}$$

and check again the persistency of all the constraints encountered above making use of H''. We check that no more secondary constraints are generated if we set $\mu_a \approx 0$ and we are left only with consistency requirements for determining the multipliers.

The constraints $p_a \approx 0, \beta_a \approx 0$, and $\Phi_a \approx 0$ are easily verified to be second class [7]. They may be implemented in the theory by defining Dirac brackets and this may be performed iteratively. We have

$$\begin{aligned} C_{ab}(\bar{x},\bar{y}) &\equiv \{\beta_a(\bar{x}), p_b(\bar{y})\} \\ &= \left[\, L[-\,\delta_{ab}\,\partial_i\partial_i + V''_{ab}(\omega)\,] + \frac{1}{2!}V''''_{abcd}(\omega)\,\int dx\,\varphi_c\varphi_d + ...\right]\delta^2(\bar{x}-\bar{y}), \end{aligned} \tag{8}$$

$$\{\beta_a(\bar{x}),\beta_b(\bar{y})\} = \{p_a(\bar{x}),p_b(\bar{y})\} = 0, \tag{9}$$

where we have made a Taylor expansion in (8) and set $L = 2\pi\delta(0)$. The Dirac bracket with respect to the pair $p_a \approx 0$, $\beta_a \approx 0$ may be defined by

$$\{f,g\}^* = \{f,g\} - \int\int d^2\bar{u}d^2\bar{v}\left[\{f,p_a(\bar{u})\}C^{-1}_{ab}(\bar{u},\bar{v})\{\beta_b(\bar{v}),g\} - (\beta \leftrightarrow p)\right] \tag{10}$$

which does satisfy $\{\beta_a,g\}^* = 0$ and $\{p_a,g\}^* = 0$ for any functional g of the canonical variables. We may set now $\beta_a = 0$ and $p_a = 0$ as strong relations. The second term in the bracket (10) introduces modifications only to the standard Poisson brackets of ω_a with π_a apart from serving to eliminate p_a from the theory. We, however, show below that, in our context, this modification is also vanishing and consequently the Dirac brackets of the surviving canonical variables at this stage coincide with the standard Poisson brackets we started with.

On making a Taylor expansion in the constraint $\beta_a = 0$ we find

$$L\left[V'_a(\omega) - \partial_i\partial_i\omega_a\right] + \frac{1}{2!}V'''_{abc}(\omega)\int dx\varphi_b\varphi_c + ... = 0. \tag{11}$$

In the limit when $L \to \infty$ it leads to

$$\left[V'_a(\omega) - \partial_i\partial_i\omega_a\right] = 0 \tag{12}$$

which is also seen to minimize the the expression (3) for the light-front energy. The expression (3) indicates also that its value gets lowered when the zero modes do not depend on $\bar{x}$. They are then determined by solving $V'_a(\omega) = 0$ and would be shown to be relevant for a tree level description of the ground state.

We will consider for definiteness the potential $V(\phi) = (\lambda/4)(\phi_a\phi_a - m^2/\lambda)^2$, with $\lambda > 0$ and a wrong sign for the mass term to allow for spontaneous symmetry breaking. We require, $V'_a(\omega) = (\lambda\omega^2 - m^2)\omega_a = 0$, where $\omega^2 \equiv \omega_a\omega_a$. In the assymmetric or broken phase $\omega^2 = (m^2/\lambda)$ while in the symmetric phase or when the potential has the correct sign for the mass term $\omega_a = 0$. In the latter case the leading term in (8) is,

$-L(\partial_i\partial_i+m^2)\,\delta_{ab}\delta^2(\bar{x}-\bar{y})$, whose inverse vanishes when $L\rightarrow\infty$. For the assymetric case the leading term is $C_{ab}(\bar{x},\bar{y}) = L\,[-\delta_{ab}\partial_i\partial_i + 2m^2\,P_{ab}]\delta^2(\bar{x}-\bar{y})$ where $P_{ab}=\omega_a\omega_b/\omega^2$ is a projection operator. In the $L\rightarrow\infty$ limit the last term in (10) again drops out in the $\{\pi_a,\omega_b\}^*$ bracket.

The remaining constraint $\Phi_a\approx 0$ is second class by itself, $\{\,\Phi_a(x,\bar{x}),\Phi_b(y,\bar{y})\,\} = -2\,\delta_{ab}\,\delta^2(\bar{x}-\bar{y})\,\partial_x\delta(x-y)$. In view of the discussion in the preceeding paragraphs it may be implemented through the following (final) Dirac bracket

$$\{f,g\}_D = \{f,g\} + \frac{1}{4}\int\int d^3u d^3v\,\{f,\Phi_a(\bar{u})\}\,\epsilon(u-v)\delta^2(\bar{u}-\bar{v})\,\{\Phi_a(\bar{v}),g\}. \qquad (13)$$

We may now set also $\pi_a = \varphi'_a$ as a strong relation and all the constraints are now implemented leaving behind the constraint (12) and removing p_a and π_a from the theory. The expression for the Hamiltonian now reduces to that given in (3) and we find from (13)

$$\{\omega_a(\bar{x}),\omega_b(\bar{y})\}_D = \{\omega_a(\bar{x}),\varphi_b(y,\bar{y})\}_D = 0 \qquad (14)$$

$$\{\,\varphi_a(x,\bar{x}),\varphi_b(y,\bar{y})\,\}_D = -\frac{1}{4}\delta_{ab}\,\epsilon(x-y)\delta^2(\bar{x}-\bar{y}) \qquad (15)$$

The presence in the case of spontaneously broken continuous symmetry, of the transverse directions, was crucial for showing that the zero modes have vanishing Dirac brackets with the non-zero ones. For example, in $1+1$ dimensional field theory, we are unable to demonstrate this result due to the lack of such an extra space dimension. In fact, we find $C_{ab} = 2Lm^2P_{ab}$ which contains in it a projection operator which can not be inverted. There is, however, no problem with the spontaneous symmetry breaking of a discrete symmetry in this case as was shown in ref. [8]. We obtain thus another demonstration, in our framework, in favour of the absence of Goldstone bosons in two dimensions [9].

The quantized theory commutation relations for the corresponding field operators are obtained by the correspondence $i\{f,g\}_D \rightarrow [f,g]$ and we may need also an appropriate prescription for ordering the operators in order to obtain a successful quantized field theory. The zero mode operators are seen to commute among themselves and with the non-zero (longitudinal) modes contained in φ_a. They are thus proportional to the identity operator and the constants of proportionality are found by solving (12). They behave as background fields. The zero modes, in general, may be operators, for example, in the case of the bosonized version of the Schwinger model quantized again by the *standard* Dirac procedure [10]. The model is obtained by functionally integrating out the fermion field and introducing a scalar field in the theory to keep the action local. The chiral symmetry of the initial Lagrangian goes over to the symmetry with respect to the shift by a constant in the scalar field. In order to be able to implement it at the quantized level, a zero mode from the only other available field in the model, viz, the gauge field, must form a a canonically conjugate pair with the zero mode operator of the scalar field.

The light-front commutation relations of the field operator φ_c may be realized in momentum space through the expansion

$$\frac{1}{(\sqrt{2\pi})^3}\int dk d^2\bar{k}\;\frac{1}{\sqrt{2k}}\,\theta(k)\Big[\,a_c(k,\bar{k})\,e^{-i(kx+\bar{k}.\bar{x})}+\,a_c(k,\bar{k})^\dagger\,e^{i(kx+\bar{k}.\bar{x})}\Big]\,, \tag{16}$$

where the operators satisfy $[a_b(k,\bar{k}),a_c(k',\bar{k}')^\dagger]=\delta_{bc}\delta(k-k')\delta^2(\bar{k}-\bar{k}')$ while others are vanishing. Here $\bar{k}=(k^1,k^2)$ indicates the transverse components while k is the longitudinal component of the light-front momentum. Only the $k>0$ modes occur in the expansion. The vacuum state is defined to be annihilated by the destruction operators, $a(k,\bar{k})|vac\rangle=0$. The normal ordering with respect to the creation and destruction operators, which is interaction independent, may be introduced. The longitudinal momentum density is $:\varphi'_a\varphi'_a:$ and we find $[P^+,a_b(k,\bar{k})]=-k\,a_b(k,\bar{k})$, $[P^+,a_b(k,\bar{k})^\dagger]=k\,a_b(k,\bar{k})^\dagger$ which gives a justification for the above normal ordering.

The description at the tree level of the ground state, when the symmetry is spontaneously broken, goes as follows. A particular solution, $(\omega_1,\omega_2,\omega_3...)$, of $(\lambda\omega^2-m^2)=0$ defines a preferred direction in the isospace out of a continuous set of permitted orientations. It characterizes a particular (non-perturbative) vacuum state, $\langle|\phi_a|\rangle_{vac}=\omega_a$. The Fock space of the corresponding physical sector in the quantized theory is built by applying the particle creation operators on this vacuum state. The infinite degeneracy of the vacuum is described, in the light-front framework, by the continuum of the allowed orientations in the isospin space of the background isovector. In the symmetric phase $\omega_a = 0$ and there is no preferred direction to select. In the broken phase, when the vacuum expectation values of ϕ_a select out a certain fixed direction in isospace, the potential expressed in terms of the fields φ_a reveals that the surviving symmetry (ω_a remain fixed) is of lesser dimension than the initial one and we obtain Goldstone bosons in the theory as is shown below. There are no operators in the theory which will take us from one sector to another. We obtain the same results if we use functional integral method as applied to second class constraints. In our example, we find also $P^- = H = \int dx : V(\phi) :$ and $P^-|vac, \omega^2 = m^2/\lambda\rangle = 0$ while in the $\omega_a = 0$ phase we obtain an infinite value. We also note that the normal ordered expression of the constraint (11) when applied to the vacuum state again leads to the expression (12). This follows, in the light-front framework, from the normal ordering, the positivity of the longitudinal momenta, and its conservation. Our procedure to handle the non-local constraint may thus be considered justified. The full implication of the operator constraint $\beta_a = 0$ needs further study.

We discuss now the isospin symmetry field generators. The global invariance of (2) under the isospin group at the classical level gives rise to conserved isospin currents and the corresponding field theory generators. Before the constraints are implemented they are clearly given by

$$G_\alpha(\tau) = \int d^2\bar{x}\, p(\bar{x})\, t_\alpha\, \omega(\bar{x}) + \int dx d^2\bar{x}\, \pi(x,\bar{x})\, t_\alpha\, \varphi(x,\bar{x}) \tag{17}$$

Here t_α are the hermitian and antisymmetric matrix generators of the isospin group. After the constraints have been implemented, the field theory generators in the quantized theory are now given by ($p_a = 0$, $\pi_a = {\varphi'}_a$)

$$G_\alpha(\tau) = -i \int d^2\bar{x}\, dx\, {\varphi'}_a(x,\bar{x})\, (t_\alpha)_{ab}\, \varphi_b(x,\bar{x}) = \int d^2\bar{k}\, dk\, \theta(k) a_a(k,\bar{k})^\dagger\, (t_\alpha)_{ab}\, a_b(k,\bar{k}) \tag{18}$$

so that (18) is already normal ordered; the infinite term arising on normal ordering vanishes identically. In the quantized theory, the isospin symmetry generators, in the case of the light-front quantization, thus, annihilate the vacuum state, independent of the form of the potential. This is in contrast to the case of equal-time quantization where some generators do not annihilate the vacuum when the symmetry is spontaneously broken. We find, on using the expressions (14) and (15) for the corresponding operators in quantized theory, that $[G_\alpha, \varphi_a] = -(t_\alpha)_{ab}\varphi_b$, $[G_\alpha, \omega_a] = 0$, and $[G_\alpha, G_\beta] = i f_{\alpha\beta\gamma} G_\gamma$, which are consistent with the generators annihilating the vacuum state. The selected preferred direction remains unaltered under isospin rotations in the quantized theory.

It is now clear that for a vacuum state specified by a particular background isovector, satisfying $\lambda\omega^2 = m^2$, there may survive a set of linearly independent field generators which still commute with the Hamiltonian (3), e.g., are conserved. They are evidently found by solving $(\tilde{t}_\alpha)_{ab}\omega_b = 0$ where $\tilde{t}_\alpha$ are appropriate linearly independent combinations, depending on the ω_a, of the original matrix generators. The corresponding operators $\tilde{G}_\alpha$ generate the surviving symmetry of the Lagrangian written in terms of the quantized fields φ_a, with ω_a regarded as fixed constants, and whose number is also seen to be independent of the particular vacuum state selected. The counting of the number of Goldstone bosons may be done following the arguments as in the conventional equal-time quantization case [11]. The number of such bosons (ignoring the case of pseudo-Goldstone bosons) is the difference in the number of generators of the original and the final isospin symmetry group of the Lagrangian.

3. The discussion of the *Higgs mechanism* [12] may now be given. For definiteness, we consider the Georgi-Glashow model [13] with $SO(3)$ isospin gauge symmetry where the iso-triplet of the real scalar fields transforms according to the adjoint representation. The gauged Lagrangian is written as

$$\mathcal{L} = (\mathcal{D}_-\phi)_a(\mathcal{D}_+\phi)_a - \frac{1}{2}(\mathcal{D}_i\phi)_a(\mathcal{D}_i\phi)_a - V(\phi) + \frac{1}{2}[F^a{}_{+-}F^a{}_{+-} + 2F^a{}_{-i}F^a{}_{+i} - F^a{}_{12}F^a{}_{12}], \tag{19}$$

where the the components of the gauge field are $(A^+ = A_- = (A^0 + A^3)/\sqrt{2},\ A^- = A_+ = (A^0 - A^3)/\sqrt{2},\ A^1, A^2)$, $F^a{}_{\mu\nu} = \partial_\mu A^a{}_\nu - \partial_\nu A^a{}_\mu + g\epsilon_{abc}A^b{}_\mu A^c{}_\nu$, and $\mathcal{D}$ indicates a gauge covariant derivative. The Lagrangian density is gauge invariant and through a continuous application of gauge transformations, ignoring the case when solitons are present, we may bring, at any space-time point, the scalar multiplet to the form $(0, 0, \phi_3 \equiv \phi)$, e.g., we will work in the unitary gauge. The potential then takes the form, $V(\phi) = (\lambda/4)(\phi^2 - m^2/\lambda)^2$.

When the gauge coupling g is vanishing, the gauge field is decoupled from the scalar field and also the self coupling among the components of the iso-multiplet of the gauge field are absent. Each isospin component $A^a{}_\mu$ of the gauge field can be quantized independently, in the light-front frame, like a free electromagnetic field following the Dirac method. The scalar field Lagrangian may be quantized independently and in the broken symmetry phase the constraint (12) at the tree level for describing the ground state requires $\omega^2 = m^2/\lambda$ where $\phi = \omega + \varphi$. On examining the quadratic term in φ we find that this field becomes massive with the mass square given by $2m^2$. When the gauge coupling g is made nonvanishing we find that some of the iso-components of the gauge field acquire masses. The quadratic terms in the gauge field arise from the terms involving the gauge covariant derivatives. We find that each one of the gauge fields $A^1{}_\mu$ and $A^2{}_\mu$ acquires a mass while $A^3{}_\mu$ remains massless. The last one may be identified with the photon field. If we regard g as an electric charge, the photon may be shown to couple with the charged massive vector boson fields defined by, $(A^1{}_\mu \pm iA^2{}_\mu)/\sqrt{2}$, while the massive Higgs field φ remains neutral. The would-be-Goldstone bosons are used up to give rise to the longitudinal modes of the two massive vector bosons. The surviving SO(2) or U(1) symmetry is related to the fact that in the gauge adopted above we still are left with a symmetry with respect to the rotation in (1,2) isospace. The Higgs mechanism for other gauge groups may be similarly described and even in other choices of the gauge.

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References

[1.] P.A.M. Dirac, Rev. Mod. Phys. **21**, 392 (1949).

[2.] S. Weinberg, Phys. Rev. **150**, 1313 (1966).

[3.] H.C. Pauli and S.J. Brodsky, Phys. Rev. D **32**, 1993 (1985); Phys. Rev. D **32**, 2001 (1985).
For a complete list of references see, for example, S.J. Brodsky and G.P. Lepage, in *Perturbative Quantum Chromodynamics*, edited by A.H. Mueller, World Scientific, Singapore, 1989 and S.J. Brodsky and H.C. Pauli, SLAC preprint SLAC-PUB-5558/91.

[4.] K.G. Wilson, in *Lattice '89*, Proceedings of the International Symposium, Capri, Italy, 1989, edited by R. Petronzio *et al.* [Nucl. Phys. B (proc. Suppl.)] **17** (1990); R.J. Perry, A. Harindranath, and K.G. Wilson, Phys. Rev. Lett. **65**, 2959 (1990).

[5.] D. Mustaki, S. Pinsky, J. Shigemitsu and K. Wilson, Phys. Rev. D **43**, 3411 (1991).

[6.] The problem has been under constant attention. See [3] and *Sources for light-front physics* compiled by A. Harindranath at Ohio State. Some recent papers are
A. Harindranath and J.P. Vary, Phys.Rev. D **36**, 1141 (1987);
S. Glazek, Phys. Rev. D **38**, 3277 (1988);
G. McCartor, Z. Phys. C **41**, 271 (1988);
F. Lenz, M. Thies, S. Levit, and K. Yazaki, Ann. Phys. **208** 1 (1991);
G. McCartor and D.G. Robertson, SMU preprint SMUHEP/91-04;
Th. Heinzl, St. Krusche, and E. Werner, Z. Phys. A **334**, 443 (1989) and Regensburg preprint TPR-91-23.
K. Hornbostel, preprint, Cornell University, CLNS 91/078 (1991).

[7.] P.A.M. Dirac, Canad. J. Math. **1**, 1 (1950); *Lectures in Quantum Mechanics*, Benjamin, New York, 1964.

[8.] P.P. Srivastava, *On spontaneous symmetry breaking mechanism in light-front quantized field theory*, preprint, Ohio State University, November 1991.

[9.] S. Coleman, Commun. Math. Phys. **31**, 259 (1973).

[10.] P.P. Srivastava, *in preparation. A modified* Dirac procedure was used by Th. Heinzl, St. Krusche, and E. Werner, Phys. Lett. B **256**, 55 (1991).

[11.] S. Weinberg, Phys. Rev. Lett. **29**, 1698 (1972).

[12.] P.W. Higgs, Phys. Lett. **12**, 132 (1964); Phys. Rev. Lett. **13**, 508 (1964); F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964).

[13.] H. Georgi and S. Glashow, Phys. Rev. Lett. **13**, 168 (1964).