ISSN 0029-3865

CBPF-NF-010/91

CHIRAL-BOSONS WITH GREEN-SCHWARZ SUPERSYMMETRY†

by

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†Preprint do IF-UFRJ-01/91

Abstract

Supersymmetric extension of Floreanini and Jackiw formulation for chiral boson is constructed adapting the Green-Schwarz procedure as applied to the string theory. Dirac brackets which implement the two second class constraints are also constructed.

Key-words: Chiral-bosons; Supersymmetry.

PACS: 11.10.Ef, 11.30.Pb, 11.30.Rd

1. Chiral bosons are relevant in the construction of many string theory models [1]. The bosonic formulation of Siegel [2] to describe them makes use of an auxiliary gauge field. The gauge symmetry, however, becomes anomalous at the quantum level and the action must be modified [3] which makes its coupling to gravity difficult. Another description of the chiral boson was proposed by Floreanini and Jackiw (FJ) [4] which does not require any auxiliary field and it seems amenable to the coupling with gravity [5] and supergravity [6]. We propose to discuss here a supersymmetric extension of the FJ formulation adapting to our case the procedure of Green and Schwarz (GS) [7] as applied to the string theory. The canonical Hamiltonian formulation is constructed following the Dirac's method [8].

2. The GS supersymmetric formulation of the string theory is basically obtained by replacing the quantity $\partial_a X^\mu$ of the bosonic string action by a supercoordinate Z_a^μ which for the case $N=1$ is

$$Z_a^\mu = \partial_a X^\mu - i\bar{\theta}\Gamma^\mu\partial_a\theta\,. \tag{1}$$

Here, $a=1,2$ is an world-sheet index and $\mu = 0,\cdots,D-1$, where D is the spacetime dimension. θ_α is a real D-dimensional spinor. The quantity Z_a^μ is invariant under the following global supersymmetry transformations

$$\begin{aligned}\delta\theta_\alpha &= \epsilon_\alpha \\ \delta X^\mu &= -i\,\bar{\theta}\Gamma^\mu\epsilon\,.\end{aligned} \tag{2}$$

We incorporate this procedure in the FJ theory by means of the following replacement (*)

$$\partial_\mu\phi \longrightarrow Z_\mu = \partial_\mu\phi + i\theta\partial_\mu\theta\,. \tag{3}$$

(*) Our convention is as follows: The Minkowski spacetime metric is $\eta_{\mu\nu} = \text{diag.}(-1,1)$ and the gamma matrices Γ^μ satisfy $\{\Gamma^\mu,\Gamma^\nu\} = 2\,\eta^{\mu\nu}$.

This corresponds to the GS supercoordinate for $D = 1$, where the two-dimensional world-sheet is occupied by the two-dimensional spacetime of the chiral boson theory. Here, θ is just a real anticommuting field.

The supersymmetric extension of FJ Lagrangian is

$$\begin{aligned}\mathcal{L} &= Z_o Z_1 - Z_1 Z_1 \\ &= \dot{\phi}\phi' - \phi'\phi' + i\,\theta\dot{\theta}\phi' + i\,\theta\theta'\left(\dot{\phi} - 2\phi'\right).\end{aligned} \tag{4}$$

One can verify that Lagrangian (4) is actually invariant under the following global transformations

$$\begin{aligned}\delta\theta &= \epsilon \\ \delta\phi &= i\,\theta\epsilon\,.\end{aligned} \tag{5}$$

It might be opportune to mention that we have not included a Wess-Zumino term [1] into the Lagrangian (4) because it would not be invariant under transformations (5).

The Euler-Lagrange equations of motion obtained from the Lagrangian (4) are

$$\phi'' - \dot{\phi}' - i\,\theta\dot{\theta}' + i\,\theta\theta'' = 0 \tag{6a}$$

$$\theta(\phi'' - \dot{\phi}') + \theta'(2\phi' - \dot{\phi}) - \dot{\theta}\phi' = 0\,. \tag{6b}$$

Combining these two equations and using the well-known properties of the anticommuting variables, we get

$$\theta'\left(\phi' - \dot{\phi}\right) - \left(\theta' - \dot{\theta}\right)\phi' = 0\,. \tag{7}$$

We note that the usual chiral condition $\dot{\phi} = \phi'$ together with $\dot{\theta} = \theta'$ are consistent with expression (7).

3. From the Lagrangian (4), we find that the canonical momenta conjugate to ϕ and θ are

$$\begin{aligned} p &= \frac{\partial \mathcal{L}}{\partial \dot{\phi}} \\ &= \phi' + i\,\theta\theta' \qquad (8a) \\ \pi &= \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \\ &= -i\,\theta\phi'\,, \qquad (8b) \end{aligned}$$

where we are using left derivative for the fermionic field. The relations above are primary constraints [8]. Let us denote them by

$$\Phi = p - \phi' - i\,\theta\theta' \qquad (9a)$$

$$\chi = \pi + i\,\theta\phi'\,. \qquad (9b)$$

The Poisson brackets of these constraints are

$$\begin{aligned} \{\Phi(x,t), \Phi(y,t)\} &= -2\,\frac{\partial}{\partial x}\,\delta(x-y) \\ \{\chi(x,t), \chi(y,t)\} &= -2i\,\phi'(x)\,\delta(x-y) \\ \{\chi(x,t), \Phi(y,t)\} &= -2i\,\theta(y,t)\,\frac{\partial}{\partial y}\,\delta(x-y)\,. \qquad (10) \end{aligned}$$

As one observes, constraints Φ and χ are second class [8]. Hence, since all primary constraints are second class we can affirm that there are no secondary constraints.

We now calculate the Dirac brackets. Let us do this iteratively. First we use the constraint Φ. We need the inverse of

$$\begin{aligned} C(x,y) &= \{\Phi(x), \Phi(y)\} \\ &= -2\,\frac{\partial}{\partial x}\,\delta(x-y)\,. \qquad (11) \end{aligned}$$

From now on we drop the explicity use of the time parameter t. The inverse $C^{-1}(x,y)$ is

$$C^{-1}(x,y) = -\frac{1}{2}\,\eta(x-y)\,, \tag{12}$$

where $\eta(x-y)$ is the step function. Using the definition of the Dirac brackets [8], we obtain the preliminar star-brackets

$$\begin{aligned}
\{\phi(x),p(y)\}^* &= \frac{1}{2}\,\delta(x-y)\\
\{\phi(x),\phi(y)\}^* &= -\frac{1}{2}\,\eta(x-y)\\
\{p(x),p(y)\}^* &= \frac{1}{2}\,\frac{\partial}{\partial x}\,\delta(x-y)\\
\{\theta(x),\pi(y)\}^* &= -\,\delta(x-y)\\
\{\theta(x),\theta(y)\}^* &= 0\\
\{\pi(x),\pi(y)\}^* &= \frac{3}{2}\,\theta\theta'\,\delta(x-y) + 2\,\theta'(x)\,\theta'(y)\,\eta(x-y)\\
\{\phi(x),\theta(y)\}^* &= 0\\
\{\phi(x),\pi(y)\}^* &= \frac{i}{2}\,\theta\,\delta(x-y) - i\,\theta'(y)\,\eta(x-y)\\
\{p(x),\theta(y)\}^* &= 0\\
\{p(x),\pi(y)\}^* &= \frac{i}{2}\,\theta'\,\delta(x-y) - \frac{i}{2}\,\theta(x)\,\frac{\partial}{\partial x}\,\delta(x-y)\,.
\end{aligned} \tag{13}$$

Now we pass to use the constraint χ. It is necessary to know the inverse of

$$\begin{aligned}
\tilde{C}(x,y) &= \{\chi(x),\chi(y)\}^*\\
&= 2\,(\theta\theta' - i\phi')\,\delta(x-y) + 2\,\theta'(x)\,\theta'(y)\,\eta(x-y)
\end{aligned} \tag{14}$$

This is given by

$$\tilde{C}^{-1}(x,y) = \frac{1}{2\phi'}\left(i + \frac{\theta\theta'}{\phi'}\right)\delta(x-y) + \frac{\theta'(x)\,\theta'(y)}{2\,\phi'(x)\,\phi'(y)}\,\eta(x-y) \tag{15}$$

The attainment of the final Dirac brackets of this theory is just a matter of a (tedious) algebraic calculation. The result is

$$
\begin{aligned}
\{\phi(x), p(y)\}_D =& \frac{1}{2}\left(1 - i\,\frac{\theta\theta'}{\phi'}\right)\delta(x-y)\\
\{\phi(x), \phi(y)\}_D =& -\frac{1}{2}\left(1 - i\,\frac{\theta(x)\theta'(x)}{\phi'(x)} + i\,\frac{\theta(y)\theta'(y)}{\phi'(y)} - \frac{\theta(x)\theta'(x)\theta(y)\theta'(y)}{\phi'(x)\phi'(y)}\right)\eta(x-y)\\
\{p(x), p(y)\}_D =& \frac{1}{2}\,\frac{\partial}{\partial x}\,\delta(x-y)\\
\{\theta(x), \pi(y)\}_D =& -\,\delta(x-y) + i\,\frac{\theta'(y)\theta'(x)}{\phi'(x)}\,\eta(y-x)\\
\{\theta(x), \theta(y)\}_D =& -\,\frac{1}{2\phi'}\left(i + \frac{\theta\theta'}{\phi'}\right)\delta(x-y) - \frac{\theta'(x)\theta'(y)}{2\phi'(x)\phi'(y)}\,\eta(x-y)\\
\{\pi(x), \pi(y)\}_D =& \frac{1}{2}\,\phi'\,\delta(x-y) + \left(1 + \frac{1}{2}\,\eta(x-y)\right)\theta'(x)\theta'(y)\\
\{\phi(x), \theta(y)\}_D =& -\,\frac{\theta}{2\phi'}\,\delta(x-y) + \frac{1}{2}\left(1 + i\,\frac{\theta(x)\theta'(x)}{\phi'(x)}\right)\frac{\theta'(y)}{\phi'(y)}\,\eta(x-y)\\
\{\phi(x), \pi(y)\}_D =& -\,\left(i + \frac{3\,\theta(x)\theta'(x)}{\phi'(x)}\right)\theta'(y)\,\eta(x-y)\\
\{p(x), \theta(y)\}_D =& -\,\frac{\theta'}{2\phi'}\,\delta(x-y)\\
\{p(x), \pi(y)\}_D =& -\,\frac{i}{2}\,\theta(x)\,\frac{\partial}{\partial x}\,\delta(x-y)\,.
\end{aligned}
\tag{16}
$$

In order to check these calculations, we may implement the constraints Φ and χ in the reverse order. In this case the inverse of

$$
\begin{aligned}
D(x,y) =& \{\chi(x), \chi(y)\}\\
=& -\,2\,i\,\phi'(x)\,\delta(x-y)
\end{aligned}
\tag{17}
$$

is

$$
D^{-1} = \frac{i}{2\phi'}\,\delta(x-y)
\tag{18}
$$

and the preliminar brackets are

$$
\begin{aligned}
\{\phi(x),p(y)\}^* &= \delta(x-y)\\
\{\phi(x),\phi(y)\}^* &= 0\\
\{p(x),p(y)\}^* &= 0\\
\{\theta(x),\pi(y)\}^* &= -\frac{1}{2}\,\delta(x-y)\\
\{\theta(x),\theta(y)\}^* &= \frac{-i}{2\phi'}\,\delta(x-y)\\
\{\pi(x),\pi(y)\}^* &= \frac{i}{2}\,\phi'\,\delta(x-y)\\
\{\phi(x),\theta(y)\}^* &= 0\\
\{\phi(x),\pi(y)\}^* &= 0\\
\{\theta(x),p(y)\}^* &= -\frac{\theta(x)}{2\phi'(x)}\,\partial\,\delta(x-y)\\
\{p(x),\pi(y)\}^* &= \frac{i}{2}\,\theta(y)\,\partial_y\,\delta(x-y)\,.
\end{aligned}
\tag{19}
$$

Corresponding now to

$$
\begin{aligned}
\tilde{D}(x,y) &= \{\Phi(x),\Phi(y)\}^*\\
&= -2\left(1 - i\,\frac{\theta(x)\theta'(x)}{\phi'(x)} - i\,\frac{\theta(y)\theta'(y)}{\phi'(y)}\right)\partial_x\,\delta(x-y)
\end{aligned}
\tag{20}
$$

we find

$$
\tilde{D}^{-1}(x,y) = -\frac{1}{2}\left(1 + i\,\frac{\theta(x)\theta'(x)}{\phi'(x)} + i\,\frac{\theta(y)\theta'(y)}{\phi'(y)} - i\,\frac{\theta(x)\theta'(x)\theta(y)\theta'(y)}{\phi'(x)\phi'(y)}\right)\eta(x-y) \tag{21}
$$

which leads to the same final Dirac brackets as those given by (16).

The consistency of this procedure is easily verified by calculating the equations of motion. These are

$$
\begin{aligned}
\dot{\phi} &= \{\phi, H(t)\}_D = \phi'\\
\dot{\theta} &= \{\theta, H(t)\}_D = \theta'
\end{aligned}
\tag{22}
$$

Here we use the fact that inside the Dirac bracket we have effectively $H = \int dx\,(p\phi' - \pi\theta') = \int dx\,\pi^2$ on treating the constraints as strong relations.

5.Conclusion

We have adapted the GS supersymmetry of strings to the FJ theory of chiral-bosons. It may be opportune to mention that the use of this kind of supersymmetry in chiral-bosons leads to a completely different result when the usual supersymmetry is considered, which was already studied by Bellucci, Brooks and Sonnenschein in a previous paper [9]. We have also constructed the canonical Hamiltonian formulation of the extended theory by following the Dirac procedure.

Another interesting aspect which remains to be analized is concerned to the influence of the GS supersymmetry in the quantum algebra of the Siegel theory [2]. This point is more subtle because the elimination of the second class constraint, related to the fermionic variable, leads to some problems which we do not know how to solve yet. Possible results shall be reported elsewhere.

Acknowledgments

This work was supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico - CNPq (Brazilian Reserach Agency). J.B.-N. would like to thank R. Amorim for many fruitful discussions and P.P.S. acknowledges with thanks constructive discussions with S. Ferrara and L. Alvarez-Gaumé.

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