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by

M.A. dos SANTOS*+, J.C. de MELLO*& and P. PITANGA†

*Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

+Departamento de Física
Universidade Federal de Juiz de Fora
36100 - Juiz de Fora, MG - Brasil

&Departamento de Física
Universidade Federal Rural do Rio de Janeiro
Antiga Rodovia Rio-São Paulo, Km 47
23460 - Seropédica, RJ - Brasil

†Instituto de Física
Universidade Federal do Rio de Janeiro
Cidade Universitária
21944 - Rio de Janeiro, RJ - Brasil

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Universidade Federal do Rio de Janeiro
Cidade Universitária
21944 - Rio de Janeiro, RJ - Brasil

ABSTRACT

We analyze the Christ-Lee model in the framework of the symplectic projector method. The correspondent Schrödinger quantization procedure is also applied.

Key-words: Projector; Constrained systems; Physical variables.

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As is well known, one of the possibilities in treating constrained systems, as pointed out by Fradkin and Vilkovisky [1], is to reduce the phase-space in such a way that we work only with those variables which are gauge independents ("physical variables"). There are, however, great difficulties in finding a procedure which display those variables in a systematical way [2], which limits drastically their applicabilities.

We think that there is a trap for this problem in the framework of the simplectic projector developped by Pitanga and Amaral [3], which appears as an alternative technique to deal with classical Lagrangian and Hamiltonian constrained systems. There, the simplectic projector, whose matrix elements turns to be the fundamentals Dirac Brackets, when applied to the phase-space variables, produce, automatically, those "physical variables" which play a central role in Fradkin's development.

We want to exemplify here those viewpoints analyzing a non-relativistic, gauge invariant model, proposed by Christ and Lee [4], with a Lagrangian

$$L = \frac{1}{2}\left(\dot{x}_1{}^2 + \dot{x}_2{}^2\right) - \left(x_1\dot{x}_2 - \dot{x}_1 x_2\right)x_3 + \frac{1}{2}\,x_3^2\left(x_1^2+x_2^2\right) - V\left(x_1^2+x_2^2\right) \qquad (1)$$

That model was used by Costa and Girotti [5] as a check of the so-called Dirac Bracket Quantization Procedure (DBQP) where they had succeeded in obtain the physical variables, althought in a intuitive way.

All we need in the projector technique is to have a local vector space generated by the (second class) constraints of the theory.

Those second class constraints are: (see eq. (2.a), (2.b) (6.a) and (6.b) of ref. [5])

$$\phi_1 = p_3 = 0 \tag{2.a}$$

$$\phi_2 = p_2 - ep_1 \tag{2.b}$$

$$\phi_3 = x_2 - ex_1 \tag{2.c}$$

$$\phi_4 = x_3 = 0 \tag{2.d}$$

where $e = \mathrm{Tan}\frac{b}{c}$, and b,c are nonzero constants. Therefore, the simplectic local metric, defined by $g^{ij} = \{\phi^i, \phi^j\}$ turns to be:

$$g = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & (1+e^2) & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

The projector on the manifold provided by the constraints has components in free coordinates

$$P^{\mu\nu} = \varepsilon^{\mu\nu} - g_{ij}\,\varepsilon^{\mu\rho}\,\partial_\rho\phi^i\,\partial_\alpha\phi^j\,\varepsilon^{\alpha\mu} \tag{4}$$

(where $\varepsilon^{\mu\nu}$ is the global simplectic metric and g_{ij} is the inverse of g^{ij}) which are, in this case, given by

$$P = \begin{bmatrix} 0 & 0 & 0 & (1+e^2)^{-1} & e(1+e^2)^{-1} & 0 \\ 0 & 0 & 0 & e(1+e^2)^{-1} & e^2(1+e^2)^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -(1+e^2)^{-1} & -e(1+e^2)^{-1} & 0 & 0 & 0 & 0 \\ -e(1+e^2)^{-1} & -e^2(1+e^2)^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

When we look for the projected coordinates we find:

$$\chi_1^* = (1+e^2)^{-1}\chi_4 + e(1+e^2)^{-1}\chi_5 \tag{6.a}$$

$$\chi_2^* = e\chi_1^* \tag{6.b}$$

$$\chi_3^* = 0 \tag{6.c}$$

$$\chi_4^* = -(1+e^2)^{-1}\chi_1 - e(1+e^2)^{-1}\chi_2 \tag{6.d}$$

$$\chi_5^* = e\chi_4^* \tag{6.e}$$

$$\chi_6^* = 0 \tag{6.f}$$

where the correspondence with the canonical coordinates is:

$$(x_1, x_2, x_3, p_1, p_2, p_3) \leftrightarrow (\chi_1, \chi_2, \chi_3, \chi_4, \chi_5, \chi_6) \tag{7}$$

The meaning of the set (6) is the following: the manifold allowed by the constraints is a unidimensional one, where the motion is driven by a Hamiltonian whose form is derived from the canonical one

$$H = \frac{1}{2} p_1^2 + \frac{1}{2} p_2^2 + V(x_1^2+x_2^2) = \frac{1}{2}\chi_4^2 + \frac{1}{2}\chi_5^2 + V(\chi_1^2+\chi_2^2) \tag{8}$$

By taking their projection:

$$H^* = \frac{1}{2}\chi_4^* + \frac{1}{2}\chi_5^{2*} + V(\chi_1^{2*} + \chi_2^{2*}) \quad , \tag{9}$$

That is, $$H^* = \frac{1}{2}(1+e^2)\chi_4^* + V((1+e^2)\chi_1^{*2}) \tag{10}$$

To clarify, redefine

$$\begin{cases} x_* = (1+e^2)^{1/2}\,\chi_1^* & (11.a) \\ p_* = (1+e^2)^{1/2}\,\chi_4^* & (11.b) \end{cases}$$

In this way, we can write

$$H = \frac{1}{2}\,p^2 + V(x^2) \qquad (12)$$

Then, we have the canonical equations of motion:

$$\dot{X} = \{X,H\} = P \qquad (13.a)$$

$$\dot{P} = \{P,H\} = -\frac{\partial V}{\partial x} \qquad (13.b)$$

These results are in agreement with Costa-Girotti [5], obtained via DBQP. The canonical quantization procedure follows in the usual way.

As a check, we look for the Schrödinger quantization procedure for constrained systems proposed by Pitanga and Mundim [6]; there the Hamiltonian operator is given by

$$\hat{H} = \partial_\mu P^{\mu\nu}\partial_\mu + \hat{V} \qquad (14)$$

With our configuration space projector in the following form

$$P = \begin{bmatrix} (1-e^2)^{-1} & e(1+e^2)^{-1} & 0 \\ e(1+e^2)^{-1} & e^2(1+e^2)^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (15)$$

and recalling that $\partial_2 = e\,\partial_1$, we obtain

$$\hat{\hat{H}} = (1+e^2)\partial_1^2 + V((1+e^2)x_1^2)$$

Redefine now

$$\begin{cases} x = (1+e^2)^{1/2}\, x_1 \\ \partial_x = (1+e^2)^{1/2}\, \partial_1 \end{cases}$$

The Hamiltonian takes the form:

$$\hat{H} = \partial_x^2 + V(x^2) \quad ,$$

giving the same physics obtained in the canonical form.

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